Sequence 4

Measure the angle between the solar rays and the vertical

Introduction

During this sequence, the children will have the occasion to get more familiar with the notion of angles. They will build a gnomon, emblem of the project, and will carry out their first measurement of angles between the solar rays and the vertical, like our scientist Eratosthenes. We advise you to read very carefully the technical assistance n°4 in order to evaluate the precision of your readings and to improve it as you go along.

During this sequence, you are going to make your firts measures with the gnomon Eratosthenes. We advise you to read carefully the technical assistance N° 4 in order to assess and improve its accuracy.

Notions

Notion of angle, of equality of angles. Schematization of the angle of the solar rays compared to the vertical. Approach of the notion of proportion. Using a protractor. Use of a chart of measurements. Comparison and interpretation of these measurements. Evolution of the angle of the solar rays during the year.

Preliminary : Eratosthenes 's measurements

After having discovered with Eratosthenes the moment of the solar midday, the children will dive into the first specific measurements to the project itself... Suggest to read them the short text following that will invite them to discover the notion of angles :

" *Eratosthenes having chosen to observe the shadow of an obelisk located near to the entrance of his library wanted to know more. He decided to measure precisely the angle bertween the solar rays and the obelisk of which he knew the height : around 8 meters. He waited for the sun to be at its highest in the sky, and measured its shadow and found exactly 1 meter. He concluded that the sun rays at that precise hour made an angle of 7.2 degrees with the obelisk, it was june 21 ...* "

Now get to your notebooks : will you be able to discover how Eratosthenes found the value of the angle and measre it yourselves ?

Summary of the sequence :

This sequence is composed of four parts, each one of one or two sessions, or a simple punctual activity at lunchtime (or else at the beginning of the afternoon since we just changed to summer time!).

1) Works of approach on angular measurements

Duration : 1 hour, entire class.

Location : classroom

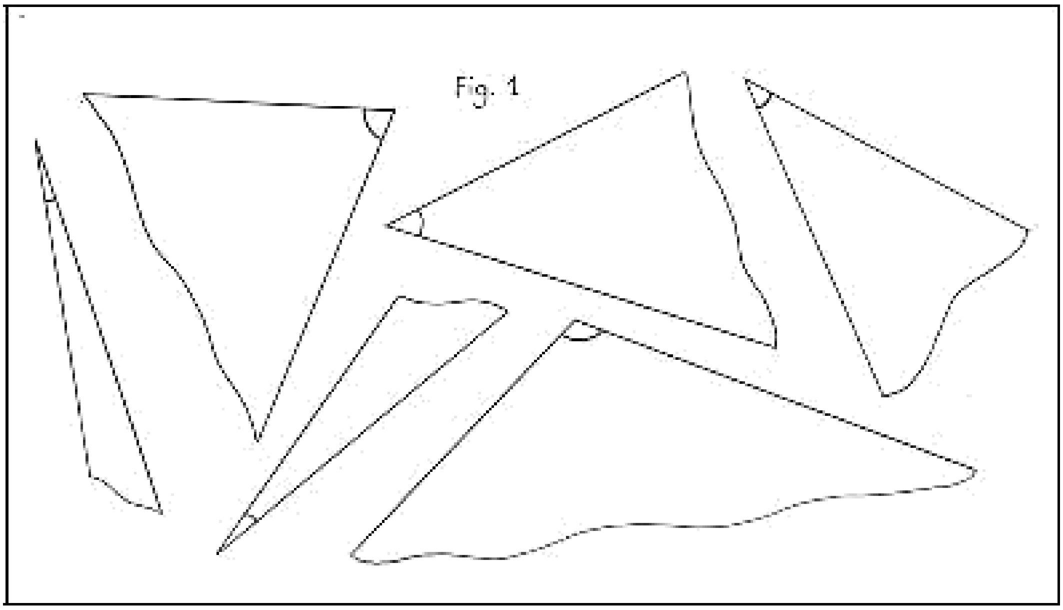
Equipment :

For each group of 3/5 pupils :
set squares,
rulers,
protractors,
tracing paper,
white paper and graph paper,
color bristol boards and cisors

Remark on the difficulties related to the concept of angle and activities to help it outby Valérie Munier, laboratory of didactics of science at Paris VII.

During this sequence, the pupils will have to use the concept of angles. They often tend to define an angle from the data of two segments of the same length and origin. Therefore, two figures whios only difference is the length of the sides will appear to be representing two different angles. It will therefore be interesting to insist on the fact that the angle is defined rather by the measurement of the spread between two directions materialized by straight lines or segments of whatever length. They will then be able to represent the direction of the solar rays and of the vertical (the obelisk) with segments of variable lengths and check that the angle they measure with a protractor is the same in any case.

If you wish to make them materialize their angles, you can use bristol board and make angular sectors drawing a section of a circle to note the angle considered. That way they can have fun comparing an angle with others by sliding a board on top of the other, superposing the summits of the angles to be compared. They can also look around them for a place in the class where they can find natural angles with furniture, walls and slide their borad into any corner. They will notice that way that some angles are more or lesspointy, more or less open, and therefore more or less big.



They can also look around them, in the classroom, for several angles : but they will see that books, notebooks, furniture, walls, windows... all have equal angles, since they are straight angles, and that therefore it has nothing to do with the length of the sides.

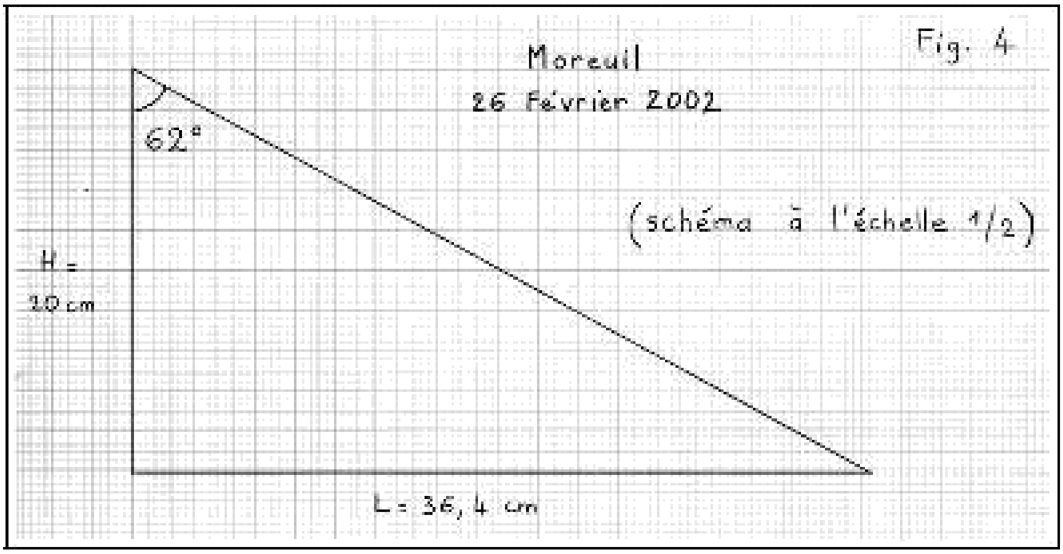
Drawing the angles of the solar rays - approach of proportionnality

The children wonder about the challenge given at the end of the text, and you suggest them to make a drawing representing the obelisk and its shadow, reaching with them an agreement on the scale and using the graph paperor the squared paper. However, there could be a question in the class : haw are we sure that the inclination of the rays will be the same as on the drawing? To put it differently, is the angle between the solar rays and the obeliskthe same when you change the scale or is it also divided by the scale factor? The children debate about this tricky question and have their own hypotheses, which they write down on their science notebook before trying to check them by making an experiment themselves by making different drawings.

You can ask the following question : " Will the shadow of a gnomon that is twice longer be twice longer at the same hour of the day? Will the angle be doubled? " The answer is of course yes to the first question and no to the second one. The research of the answers will make it easy tio tackle with the notion of proportion.

To the first question, the children will probably answer " yes, the shadow will probably be double " and they will check on a simple drawing on paper : here, simple sheets of 5 x 5 mm will do it perfectly. You can also carry out experiments with the sun to convince them better.

They start by drawing a little gnomon (8 cm) and choose the slope of the solar ray by counting the squares down and right to join two points of the ray. They chose the inclination of the rays on the sheet and can then easily reproduce a ray that is parallel to that one by counting the squares, a little farther. Drawingthe first gnomon, they draw and measure the shadow on the ground being careful about drawing the ground with the ruler, perpendicular to the gnomon. They therefore get a first rectangle triangle (a triangle that has one angle of 90 degrees).



Then they draw a gnomon next to it that is twice smaller and draw a solar ray parallel to the first one and that touches the tip of the gnomon, they draw it to the ground represented by a horizontal line and measure the shadow they obtain as well as the angle of the gnomon with the ray (they can use one of the Bristol cards or a protractor. If it proves to be difficult, ***they can make a mini-gnomon with Bristol or a mini-obelisk cutting a rectangle of 1 square of width and the desired number of squares for the length, and they will cut the tip sharp, making sure the length of the gnomon is 4 cm, half of the other one***. The can do the same with a bigger rectangle twice smaller, 4cm.

With the same solar ray or a parallel one, they look for the correct position of the gnomons for the foot to touch the ground and the tip to touch the solar ray. Then they draw the shadows and their measurements. They can also write down the value of the angle and notice that the angle hasn't changed whatever scale you choose. However, the shadows are in the same ratio : a gnomon that is twice as high gives a shadow that is twice as long and a gnomon that is twice smaller gives a shadow that is twice smaller too!

What would happen if the gnomn were 3 or 5 times bigger (or smaller) ? The shadows would be 3 or 5 times bigger (or smaller). What would happen if we used a solar ray that would be more inclined and if we carried out the experiment again? ***It would change nothing to the conclusions, it is independent from the inclination of the rays provided that the inclination does not change between the gnomons you compare (that was checked because the sun rays that reach us are parallel, cf Seq. 1)***

Conclusion : you can choose the scale you prefer to draw the gnomons and their shadows, it is only a matter of convention and it does not bother for the measurement of angles which is identical whatever scale you choose. You can now decide with them which scale to use to represent Eratosthenes's scale and try to find his angle. You will complete these activities initiating them to the reading of a protractor, they will then be able to measure any angle around them in the classroom.

---

2) Find Eratosthenes's angle measurement

Duration : 15 to 20 minutes for the sketch and the measurement of the angle ; 45 mn to 1h to use the chart of measurements.

Location : classroom

Equipment:

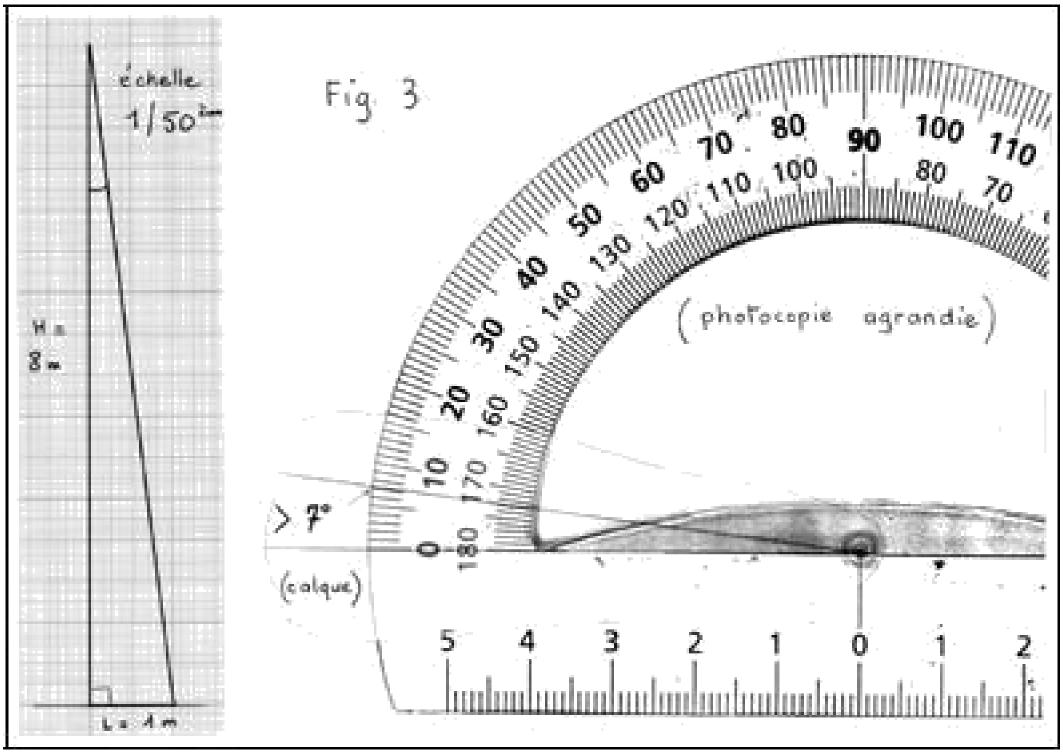
sheets of millimetered or squared paper,
tracing paper,
enlarged photocopies of a protractor,
sharpened pencils,
rulers,
cisors,
protractors.
Then, photocopies of the chart of measurements (See farther).

Find Eratosthenes's measurement

Distribute your pupils in groups of two. Deal out to each group a pair of cisors, color bristol boards, a piece of tracing paper, a protractor, a big sheet of squared or graph paper. On the latter, they must draw the obelisk, the ground and the shadow of June 21st . To respect the scale, you will agree to repesent one meter in the reality with one or two centimeters on the drawings. Then, they draw the ray joining the summit of the obelisk to the tip of the shadow, making an angle appear between the obelisk and the ray. Then they trace the angle on the color bristol board that they will cut and measure with the protractor. They must find an angle close to the value measured in Alexandria, around 7 degrees. They will notice that it is impossible to read an angle of 7.2 degrees with their protractor, that they can only go by 0.5 at the reading.

If the gap is important, they will have to ask themselves why their measurement is bad and look for the causes of the possible mistakes on the drawing : is the obelisk perpendicular to the ground (check with the set square), is the solar ray straight, is the angle section well cut?…

Last, going back to their drawings of the shadow at the real midday, ask them to determine the angles they measured that week. Then they will be able to compare to the value of Alexandria the closest to this date. They will note the difference (by a direct measurement, a substractio or superposing to angular sections that have a different color) and they will try to explain it. Remind them of their experiment about the "earth-balloon" (previous session). They will probably remember that the shadows of the gnomons grew bigger as long as you went away from Syena North or South. In the first case, they would point North and in the second case South. They will check with a new balloon and fake gnomons that taking Alexandria as a reference, and they will locate their town in comparison with Alexandria where the shadow of the gnomon will be chosen arbitrarily (use an Atlas). According to the location of your country, the conclusion of this modelisation is variable, the children observe in different cases a bigger shadow (North of the Tropic of Cancer, in Europe and North America, and South of the Tropic of Capricorn) or shorter if you are in between.



Option : A little practice!

In the following chart, erase a few values of angles or shadows and deal out photocopies to the pupils with the following sentence.

" We found the measurements that our great scientist could do during the year in Alexandria, they are given in the chart below, but some of them are missing… can you find them?"

Month	21 january	21 february	21 march	21 april	21 may	21 june	21 july	21 august	21 september	21 october	21 november	21 december
Length of the shadow of the obelisk in meters	9.9	7.2	4.8	2.8	1.6	1	1.5	2.8	4.7	7.2	9.9	11.3
Angle of the rays with the obelisk in degrees	51.1	41.8	31	19.3	11.1	7.2	10.8	19.2	30.6	42.0	51.2	54.7

They will have the opportunity to manipulate the protractor in order to fill in the empty boxes. It is a good practice before measuring their own angles. Then suggest them to try themselves to measure the angle between the solar rays and their gnomons at midday measuring the length of thhe shadow with great precision (they can start with the readings they already carried out if they were carefully). You can then compare your results with those of Eratosthenes and, like him, establish a list of regular measures throughout the year.

As a conclusion, the children will notice in the chart of measures that the length of the shadow at midday - and therefore the inclination of the solar rays - varies during the year in a given place. They will probably already have noticed the evolution during the ezxperiments from the beginning of the project. If it is not the case, the discovery will doubtlessly intreague them and they will want to know more about it. Suggest them to visit themselves if it is also the case in their town. For that purpose, they will have to measure precisely in the weeks and months to come the length of the shadow of their gnomon and deduce the angle that the solar rays make with this "gnomon-obelisk".
The next session will be dedicated to the making of a gnomon for the class.

---

3) Building the solar stick " Eratos "

Duration :several sessions to discuss, build, adjust, and test the solar stick

Location : classroom and sunny place afterwards.

Matériel :

For the whole class :
Chosen materials,
tools,
" double square
", water level,
compass.

Action in concert

A " full session " will take place to establish the specifications of the building of the solar stick " Eratos ". While taking into account the material constraints, notably the one on the dimensions of the support which should not exceed one meter, each child will argue and give his/her opinion.

Taking into account the fact that the shadows are going to lengthen until December the 22nd (date when, in France, the shadows at noon solar local time are almost three times as high as the objects), and that the blur of the shadow increases when moving away from its base, some children will decide to build a solar stick rather small, 15 cm seeming reasonable to them. Others will want a bigger tool for the simple pleasure to decorate it as a little totem: they will not hesitate to propose a 30 cm height.

Hence a consensus will be established for a solar stick being 10 to 20 cm high, having a cylindrical section but with a small diameter, with a flat end, and being attached to a rectangular support of about

100 cm x 80 cm. Beware with metal stick that heat under the sun and might dilate if exposed for too long a time.

.

Building of the solar stick.

The pupils being widely trained to this type of activity and teeming with ideas always more and more surprising (which ones will be rigorously examined before any action!) we will not be long on this subject. However moderation will be required for the support: no treatment that could warp its surface nor decorations that could interfere with the readings of shadow! ***Always be careful with solar radiance that could dilate and even buckle the materials you use if the temperature at the time of the measurements reaches 30 degrees. The board must not undergo the risk to warp, it must remain flat and plane.***

Orientation and adjustment.

On the side of marking the orientation of the support, and then the adjustment of the perpendicularity of the stick and the horizontality of the support (see the part " Adjustment of the solar sticks " at the very end of the sequence 2), it is obvious that the biggest care will be taken to this triple operation, which one will have to be verified and repeated if need be, before each reading.

Testimony from the school in Rocquigny (08), Mr. Pouyet, about the building of the solar stick :

"Here is how we have built our solar sticks:
First the children have built an individual solar stick:
-a thick cardboard plate on which a stick (rod, long toothpick) is fixed and with which they have performed the first readings ... barely reliable. Very quickly, one has had to take into account various parameters (horizontality and verticality) and finally our solar stick is made of: -a plywood plate (2 cm thick) of 50 cm x 30 cm with a hole on one of its sides,
-a 15 cm screwed rod perpendicularly attached to the plate with nut and lock nut.
Then we have chosen a place in the schoolyard which is sunny around noon local solar time. We have checked its horizontality with the level and the measurements are all carried out at this place.
Hence we mark the length of the shadow of the solar stick on the plate and back in the classroom we reproduce at the scale ½ on millimetered paper the triangle formed by the solar rays, the shadow of the solar stick and the solar stick itself. We measure the angle obtained. All this does not take more than 10 minutes."

---

4) Angular measurements specific to the project with the solar stick "Eratos"

Duration : during several weeks, according to the weather, 15 min daily to read the shadow at noon local solar time, followed by a short session to determine the angle of the rays.

Location : sunny place, classroom.

Matériel :

For the whole class :

Angular measurements from the shadow:
the solar stick " Eratos ",
a roller meter,
several millimetered paper sheets,
yarn,
a protractor .

Preliminary: Tracing of the meridian line of your place. *You can refer to the corresponding optional slip. This plot is useful to precisely know* the time corresponding to noon at solar local time every day, by observing the shadow of your solar stick passing over this imaginary line which joins the two poles of our planet passing by your school. If you are short in time to completely realize these activities simply trace this meridian by writting down once the precise time of noon at local solar time (information given by the ephemerides of your place or on the website of the Bureau Des Longitudes : http://www.bdl.fr/…) and by extending on both sides the shadow of the solar stick at this given time. On the ground, you will put two reference marks on each side of the support so as to quickly orientate your instrument before each measurement.

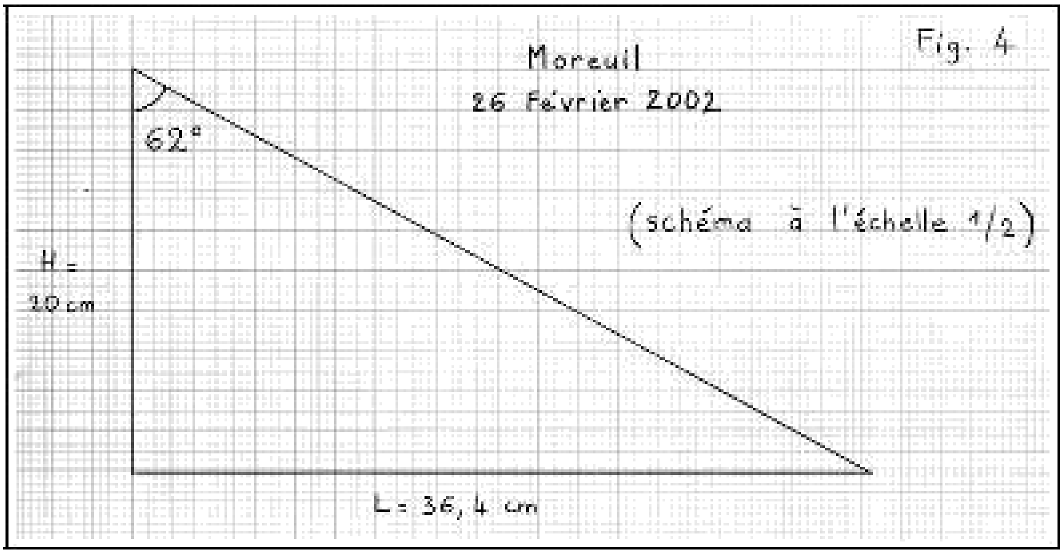
At last, here is the actual beginning of the operation Eratosthenes!

Experiment:

Every day, when the Sun shines at mid-day and when the shadow of the solar stick " Eratos " can be observed while meeting the meridian, one pupil comes and puts a very precise reference mark with a pencil at the end of the shadow. Then he/she carefully measures the length of the shadow.

Schema:

With a pencil having a very fine point, the child copies on a millimetered paper sheet the height of the solar stick and the length of the shadow. Then he/she traces the solar ray by a continuous line joining both ends. If the height of the solar stick leads to a very long shadow (in winter time), the children are going to overcome the difficulty: they will remember that the angle is going to be the same after reproducing the schema at the ½ scale for instance.



Measurement of the angle

One will only have to measure the angle with the help of a protractor and all the accuracy possible, that is to say at least within half a degree!
It will be helpful to ask that three pupils carry out the measuremet in parallel, from the same initial reference mark of course (This is why this reference mark is so important): if two or three of the results agree in a very "short" way, you will be allowed to consider that this result is valid. Otherwise one will have to re-do the schemas.

Some pupils will want to try to measure the angle "on the field ", that is to say by stretching a (very fine) thread from the end of the solar stick until the shadow's end and by using a protractor: they will see that the experiment is very ticklish especially when considering how to maintain the thread and how to adjust the protractor with respect to the rod of the solar stick! Nevertheless they will be able to compare their results with those obtained thanks to the schemas and to draw some conclusions.

Remarks:

- Long term observations: the children will notice that the marks on the meridian slowly move from one week to another, first moving away slowly towards the North until Christmas holidays, then starting to move back in January. They will deduce that the trajectory of the Sun, after reaching its lowest level in the sky in December, starts its slow climb. Then think of refering to the measurements in the Table of the shadows' readings in Alexandria and analyze with the children the similarities in the variations of shadow all through the year.
- If one day the shadow of the solar stick reaches the same length as the solar stick itself (this depends on your geographical position and more precisely on your latitude), the children will see that the angle is half the right angle, as when one double folds the corner of a sheet. Under the latitude of Bordeaux, that is to say 45°, this will happen on March the 21st, the equinox day, and obviously on September the 21st. About equinox, let us mention that schools of different latitudes will see on that day the angle of their own latitude! But we go too fast …).

Schema: With a pencil having a very fine point, the child copies on a millimetered paper sheet the height of the solar stick and the length of the shadow (mesured to within a millimeter, it is very important). Then he/she traces the solar ray by a continuous line joining both ends. If the height of the solar stick leads to a very long shadow (summertime in the southern hemisphere), the children are going to overcome the difficulty after reproducing the schema at the ½ scale for instance (they will remember that the angle is going to be the same whichever scale you choose, if they paid attention to the activities of approach about proportionnality).

Communications and exchanges

The values found for the angle will be carefully written down next to the corresponding date with the accurate time (civilian time - one can get this time from a watch adjusted on the speaking clock) and the place, in order to proceed to communications through Internet with all the schools involved in the project, and more particularly to exchange with correspondents when the groups will be defined. Don't forget to publish your measurements quite often and observe carefully the results of the other schools : the comparison between the measurements of the classes will lead to the next sequence and it will allow you to introduce the notion of geographical coordinates.