Subject: **Useful properties of some special tetrahedrons.**

**Level -** from the sixth grade**. Materials:** a set of nets to be cut and fold without glue

1. Today we are going to work with solids. We start with regular terahedron – the simplest example of Platonic solid. Let’s try to calculate its volume. The problem is to get the length of its height which needs using the Pythagoras theoreme twice. We will try to do it in a more specatacular way just observing of the some properties of special tetrahedrons. Let’s build some models. It is going to be easy because I prepared some special nets which let us build solids without using glue. The idea comes from a great Polish Math educator professor Wacław Zawadowski, who is also an author of interesting manuals.
2. First step – we are going to learn how to create solids without glue basedon the example of regular tetrahedron.

We will do it in teams. Every team has to consist of 5 students. Please notice that the blue faces should be on the outside and the white or checkered should be inside. Let’s start with folding the paper along the lines between faces. It should be done precisely with scissors. Do not hurry. Do it precisely. Then try to form the tertrahedron. You can watch and follow me.

1. In the same way please build the other tetrahedrons .
2. Then try to submit received models to create a more known polyhedron.
3. EUREKA – we can create a cube.
4. The most important observation is that the volume of tetrahedron could be calculated as the difference between the volume of the cube and three tetrohedrons which volumes are easy to calculate.
5. Let’s calculate them then. At first sight it seems to be as difficult as the regular tetrahedron or even more difficult. But if you consider putting it to a special position with the base as a rectagular triangle it becomes much easier.

$V=\frac{1}{3}∙A∙h=\frac{1}{3}∙\frac{1}{2}∙a^{2}∙a, $where $a$ means the length of triangle’s cathetus . It is equal to the edge of the cube. So the volume of our solid is equal to the sixth part of a cube. It means also that the volume of tetrahedron is equal to the third part of a cube because

 $a^{3}-4∙\frac{1}{6}a^{3}=\frac{1}{3}a^{3}$

As a next step it could be interesting to express the value in units of edge of the tetrahedron named $b$.

It is easy looking at the rectangular triangle. The hypotenuse of it is just equal to the searched edge, so

$b=a\sqrt{2}$ and furher $ a=\frac{b}{\sqrt{2}} ⇒V=\frac{1}{3}a^{3}=\frac{1}{3}(\frac{b}{\sqrt{2}})^{3}=\frac{b^{3}\sqrt{2}}{12}$

1. I have prepared a special net of cube to pack your solids. You can open and close this model as a box. Good luck!