

# the fellowship of maths



Volume 1

## CHANGE YOUR MIND

Get new knowledge and learn math from a completely new perspective.

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Math doesn't have to be boring. See how!

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Surprise your friends with fascinating facts!

## MATHS & ASTRONOMY

MATHS  
&  
ART

MATHS  
&  
MUSIC

MATHS  
&  
ARCHITECTURE

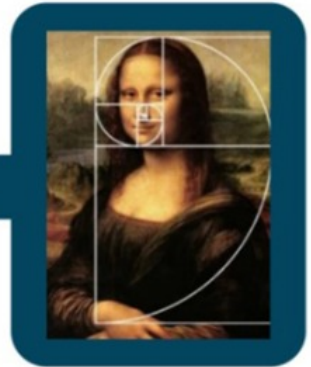
MATHS  
&  
SPORT

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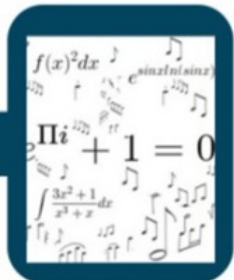
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Geometric elements in sculpture created a universal language and created a basic geometrical language such as square, triangle and circle taken from nature.

Forms are combined with forms based on more complex mathematical principles. It has presented an aesthetic abstract approach for the viewer and the artist.



This situation, which pushes the way of mathematical thinking, causes the sculptor to oppose the science of mathematics. It also increased their interest in the subject and led to an interdisciplinary interaction.



*Proportion is important in sculpture in order to convey the idea that is wanted to be expressed correctly and to reach the aesthetically beautiful.*

*Michelangelo's David is a perfect example to see that.*



The grid method has been used by artists for centuries as a tool to creating correct proportions. Renaissance artists, even Leonardo da Vinci, used the grid method. The grid method dates back to the ancient Egyptians. It is clearly a useful method for artists and aspiring artists alike.

The grid method is an inexpensive, low-tech way to reproduce or enlarge an image that you want to paint or draw. While the process is not as quick as using a projector or transfer paper, it does have the added benefit of helping to improve your drawing and observational skills. In a nutshell, the grid method involves drawing a grid over your reference photo, and then drawing a grid of equal ratio on your work surface (paper, canvas, wood panel, etc).

Then you draw the image on your canvas, focusing on one square at a time, until the entire image has been transferred. Once you're finished, you simply erase or paint over the grid lines.





## Perspective

Objects and Space can create different view shapes from different perspectives, in the simplest terms, as objects move away, their appearances become smaller by differentiating from their real appearances. This differentiation is perspective and perspective is closely related to mathematics.

Perspective is a technique heavily used in most visual arts. The painter Salvador Dali, who paints the things he sees in his dreams, is one of the most striking names of this technique. The names of the works seen in the images are "The Disintegration of the Persistence of Memory" and "Perspective".



*Salvador Dali*





## Symmetry

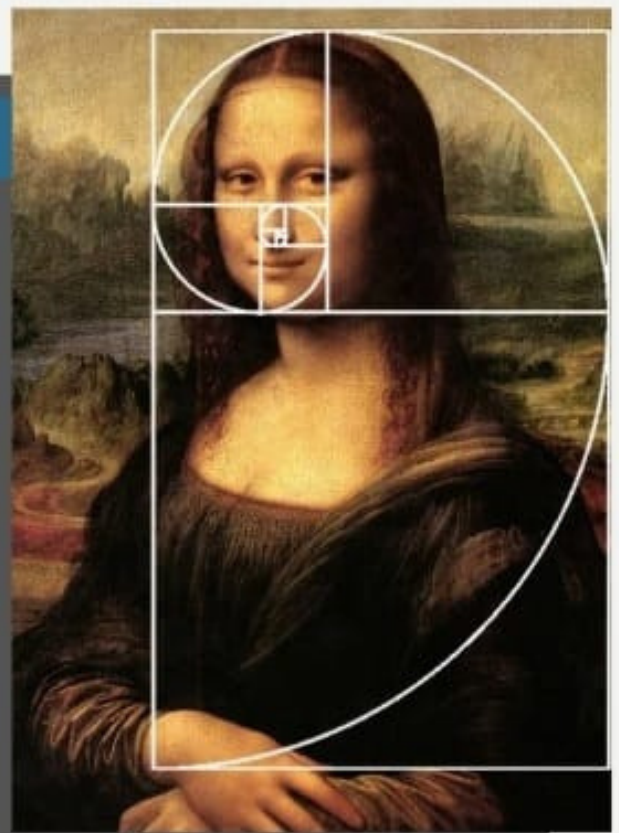
Davinci did not directly use symmetry in this work. He used symmetry to encrypt the thoughts that he had to hide due to the pressures of the period he drew and the inability of people to share their thoughts comfortably. So this drawing creates an air of mystery and interest thanks to mathematics.

## Golden Ratio

Original dimensions of the painting, that is, both its width and height, inform us that it is the golden ratio. By drawing a rectangle where the face of the Mona Lisa is, this rectangle gives the golden ratio measurements.

When this rectangle, which includes this face part, is divided into two with a line at eye level, a rectangle suitable for the golden ratio system appears. This

painting is a very good example that mathematics does not consist of only 4 operations, even in drawings in a painting we see.



*Art is a science and all sciences are based on mathematics. There is no concept of humanity that does not consist of anything.*

**-Leonardo Da Vinci**







The art of tiles, which is one of the most beautiful examples of the effects of geometry on art, is one of the important decorative arts used today.

Geometric motifs are formed by the combination of geometric shapes in a certain order. Geometric compositions are placed on the interior and exterior surfaces of buildings. The outer surface of the Çinili Mosque in Kütahya, which is one of the best examples of this, is decorated with motifs formed by the combination of various geometric shapes.

Many mathematical patterns such as translation, reflection and rotation are used in the creation of patterns in tile art. In the tile decoration of the Ben Yusuf Madrasa in Morocco, the art of "Zellij" in Morocco, it is seen that the patterns created repeat at regular intervals and form a pattern.



Isometry is a transform function that preserves lengths. That is, the value of the distance between two points on the plane does not change as a result of the transformation. While the tulip in the example of the tile on the wall plate transforms from one state to another with the effect of a certain transformation function, there is a change in its size, position and orientation on the plate. The leaves and tulips growing at a certain rate in the piece are a good example of non-isometric transformations.

Geometry is the foundation of all painting

Albrecht Dürer





# Maths and Astronomy


How is math used in astronomy?

**Maths and astronomy** have been closely related since their beginning. One of the founders of mathematics, **Pythagoras**, theorized about the spheres to which each planet is attached. **Claudius Ptolemy**, in the second century BC, developed a geocentric mathematical model of the Solar system that was used until Columbus' time. **Copernicus** was a mathematician and astronomer who developed the heliocentric model of the Solar system. In the 17th century, **Johannes Kepler** studied the planet's orbits mathematically, while **Isaac Newton** discovered the laws of gravity and described the planets' motion in relation to one another.


**Astronautics** is the theory and practice of navigation beyond the Earth's atmosphere by manned and unmanned artificial objects. It is based on the study of human trajectories, navigation, exploration and survival in outer space. It contains the design and construction of the spacecraft and launchers that will launch them into orbit, or carry them to other planets, natural satellites, asteroids, comets or elsewhere in the cosmos.


It is a broad and highly complex field, due to the difficult conditions under which the devices to be designed must operate. Astronautics involves the collaboration of various scientific and technological disciplines, such as astronomy, mathematics, physics, robotics, electronics, computer science, bioengineering, medicine and materials science. Astronautics, in combination with astronomy and astrophysics, has given rise to new scientific disciplines such as astrodynamics, astrogeophysics and astrochemistry.





## Ideal Rocket Equation





$M$  = instantaneous mass of rocket     $A$  = exhaust area  
 $u$  = velocity of rocket     $p$  = exhaust pressure  
 $v$  = exhaust velocity     $p_0$  = atmospheric pressure  
 In time increment  $dt$ , exhausted mass =  $dm$      $dm = \dot{m} dt$   
 Change in momentum of system =  $M du - dm v$   
 Force on system =  $(p - p_0) A - Mg \cos \alpha$  (neglect drag)  
 Change in momentum = Impulse = Force  $dt$   
 $M du - dm v = [(p - p_0) A - Mg \cos \alpha] dt$   
 $M du = [(p - p_0) A + \dot{m} v] dt$  (neglect weight)  
 $V_{eq}$  = equivalent exhaust velocity =  $\frac{(p - p_0) A + v \dot{m}}{\dot{m}}$   
 $M du = V_{eq} \dot{m} dt = -V_{eq} dM$   
 $du = -V_{eq} \frac{dM}{M}$   
 $\Delta u = -V_{eq} \ln(M) \Big|_{mf}^{me}$      $MR = \text{propellant mass ratio} = \frac{mf}{me}$

$\Delta u = V_{eq} \ln\left(\frac{mf}{me}\right) = V_{eq} \ln MR = I_{sp} g_0 \ln MR$

Without mathematics, a rocket launch into space to send satellites or manned missions would not be possible. A concrete example (as you can see in the photo) of the mathematics that appears can be the equations associated with the flight of a rocket in which even the classical equations of Newton cannot be used since the mass varies.

The resolution of equations and the use of numerical approximation methods are what allow space exploration.

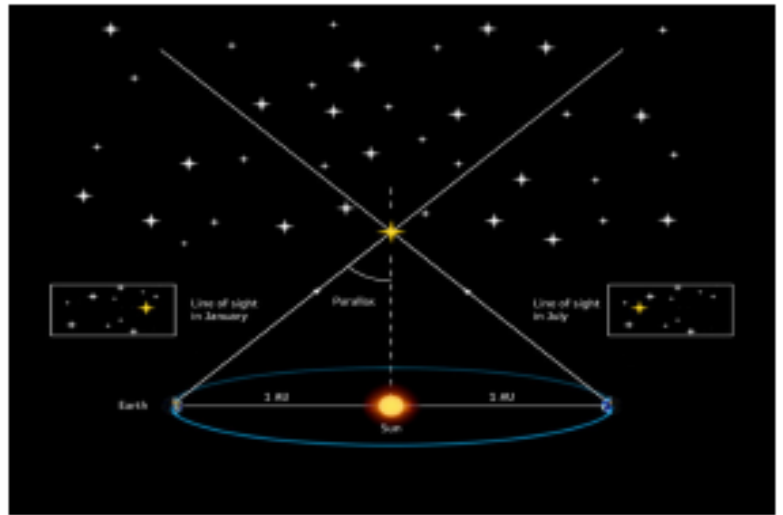


# MATHS AND ASTRONOMY

When you hear the word “astronomy” you probably think about the sun, the planets, the Solar System, the Milky Way... But, *have you ever wondered which are the bases that hold the universe’s knowledge up?*

Astronomy is a science that is closely related to mathematics, which is essential because it can establish laws and predict events. In terms of astronomy, maths have plenty of applications and play a crucial role.

We have always wanted to know the distances, speeds, number of objects in the universe such as stars. To be able to calculate all these parameters, a mathematical base is needed. So, we are going to talk about how, thanks to trigonometry, we obtain the distance between the sun and the moon or the size of the Earth, that we can calculate the probability that an asteroid could hit the earth, or with algebra discover the existence of certain planets.



Measuring the universe is quite complex and usual units are not often useful. There are enormous forces, distances and time much greater, so they cannot be directly measured. To measure the distance to close stars, a technique called **parallax** is used. This method measures the angle formed by far objects, the star observed and the Earth (in its solar orbit’s opposite points, like in January and July). The **diameter of the terrestrial orbit** is 300 kilometres. In this way, using trigonometry we can calculate the distance to the star.

The Earth’s orbit is not circular, rather an **elliptical orbit**. Being F1 and F2 two points and K a distance called “ellipse constant”, we can define an ellipse as the geometric place of the points whose sum of distances to F1 and F2 is equal to K.  $d(X,F1)+d(X,F2)=K$

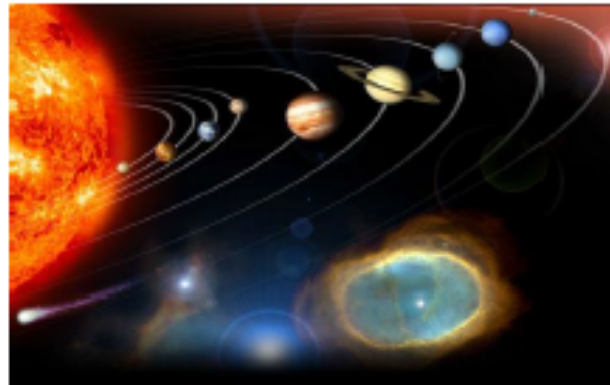
By using other mathematical techniques it is possible to estimate the diameters of **celestial bodies** and measure their **speed** through the celestial sphere. For example, the role played by **geometry** in astronomy is really important. In the cases like distances mentioned before, maths is indispensable. For larger distances, more sophisticated methods are needed, and more complex mathematical theories, like the **theory of relativity** which is fundamentally geometric theory, that has been crucial in the construction of the current explanatory models that we have of the universe.



Many astronomers use algebra in their work to achieve their goals. Without algebra things like the **Hubble Telescope** and the **discovery of Pluto** wouldn’t have been possible. Algebra is used to measure speeds and to graph movement rates. There are a few astronomers that represent these achievements which, without them and their usage of algebra technology and science, would not have been able to flourish. All in all, nowadays, our knowledge of the Universe is based on astronomers and mathematicians.

# distance between the planets

The distance of the planets to the Sun is measured in Astronomical Units (AUs) at two different orbital moments: perihelion and aphelion. Perihelion is the distance of a planet from the sun measured at its nearest orbital point; and aphelion at its farthest orbital point



Astronomers use parallax (in astronomy, the difference between the apparent positions of a star in the Earth's vault depending on the point from which it is observed) to calculate distances to objects in the Solar System and beyond. In the case of nearby objects, such as the planets and the Sun, they observe the same object from two different places on Earth – similar to looking through two eyes that are far apart.

To calculate the distance to a more distant object, such as a star, astronomers look at it from opposite sides of the Earth's orbit around the Sun. They could measure the position of the object relative to the other stars in July and then again in January, thus making their "two eyes" about 186 million miles apart!

Astronomers now have technologies to directly measure distances to other planets.

## Here you can see some distances between the planets

### the distance between the Sun and Venus

- According to science, Venus is separated from the Sun by 107 million km..

### the distance between the Sun and Jupiter

- The space separating these two bodies is only 741 million





## COMETS AND MATH



According to NASA, *comets* are large objects made of dust and ice that orbit the sun. These celestial bodies are leftovers from the formation of the solar system, roughly 4.6 billion years ago.

Two of the most commonly studied properties of comets are their *path* and their *period*. Some variables needed to determine the path and period of a comet are velocity, diameter, distance from the Sun and from the Earth, orbital eccentricity, their perihelion and aphelion and total length of their tail, as well as multiple angles, vectors and tangents related to their situation. Through advanced informatic programs which gather all of this information, astrophysics can easily study the orbit of comets millions of kilometers away from the Earth.

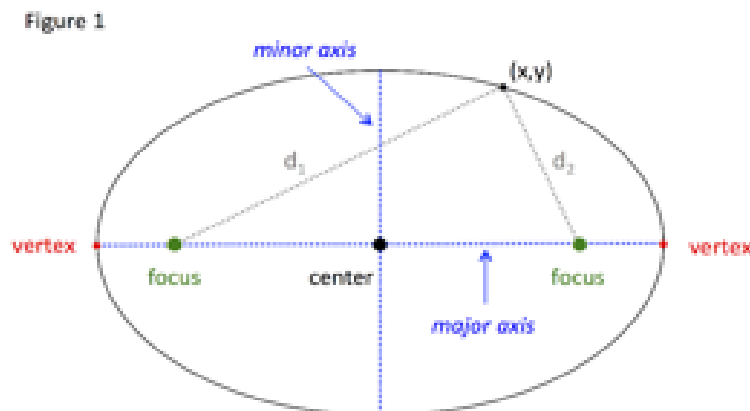
When studying celestial objects, *velocity* is one of the main studied properties. Velocity of comets is not constant, since it varies depending on their distance from the Sun. This occurs because comet's orbits are mostly elliptical (although they can also be parabolic or hyperbolic). Respecting the position of comets, they can be found on their *perihelion* when they are closer to the Sun, or on their *aphelion* when they are further from it. The closer the comets are to the central body, the faster it moves. To compute the velocity of a comet, *vis-viva equation* is used, which is described as:

$$v^2 = GM \left( \frac{2}{r} - \frac{1}{a} \right)$$

Where:

- $v$  is the relative speed of the two bodies, for example the Sun and a comet
- $r$  is the distance between the two bodies
- $a$  is the length of the semi-major axis of the elliptic orbit the comet describes ( $a > 0$  for elliptic orbits,  $a = \infty$  or  $1/a = 0$  for parabolic orbits and  $a < 0$  for hyperbolic ones)
- $G$  is the gravitational constant (not  $g$ , the gravity of Earth)
- $M$  is the mass of the central body, the mass of Sun for example

When talking about comets, we can distinguish mainly two types of periods: the *sidereal period* and the *synodic period*. While the *sidereal period* is the time a celestial body requires to complete its orbit a single time, *synodic period* is the time it takes for a comet to appear on the exact same spot when observed from a certain point of Earth. This is because the orbit described by comets is an *ellipse*, which in geometry is described as the locus of the points which addition of distances to two specific points, known as *focal points*, is equivalent to a constant.



## Celestial Mechanics

Celestial mechanics is defined as the branch of astronomy that studies the movements of celestial bodies in relation to the gravitational effects of other bodies, celestial mechanics applies the principles of physics to astronomical objects, such as stars and planets.

Celestial mechanics is concerned with calculating the orbit of a body that has been discovered recently and of which there are few observations. With three observations you can already calculate the orbit. A clear example of celestial mechanics would be calculating the position of a body at a given instant, knowing its orbit.

### **Origin of Celestial Mechanics**

Celestial mechanics studies have their origin in ancient astronomy, which was about observing the stars in order to guide navigators. Later, Newton in 1687, published the formula of the Universal Gravitation Law, which generates a system of differential equations. Thus, the problems that Kepler had formulated from an analytical or mathematical way began to be studied, giving rise to celestial mechanics.



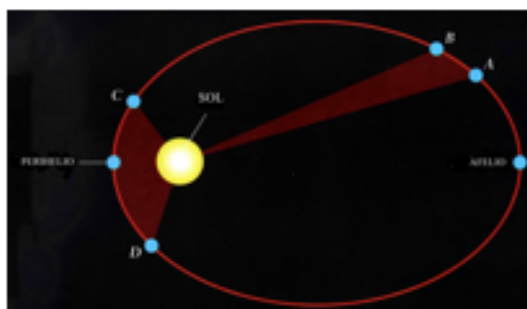
### **Celestial mechanics according to Kepler.**

The astronomer and mathematician named Johannes Kepler, was the one who created the laws on the movement of the planets in their orbit around the Sun, after having studied most of the existing theories. From Pythagoras to Copernicus, Kepler developed new modern laws of planetary orbits from his own physical principles.

These laws were three:

The first law was that Kepler stated that "every planet moves around the Sun in an orbit that is an ellipse". This is the formula used to calculate the eccentricity of the ellipse:

$$e = \frac{c}{a}$$



The second law was reached after checking the speed and movement of the planets through their orbits, which stated that "a straight line joining the Sun and a planet would cover equal areas in equal times."

The third and last law, produced a quantitative relationship between the orbital periods of planets and the size of their elliptical orbits. This law stated that the squares of the periods of the planets were in proportion to the cubes of the

semi-major axis of their orbits. This is the formula to calculate the third law:

$$\frac{T^2}{a^3} = K$$



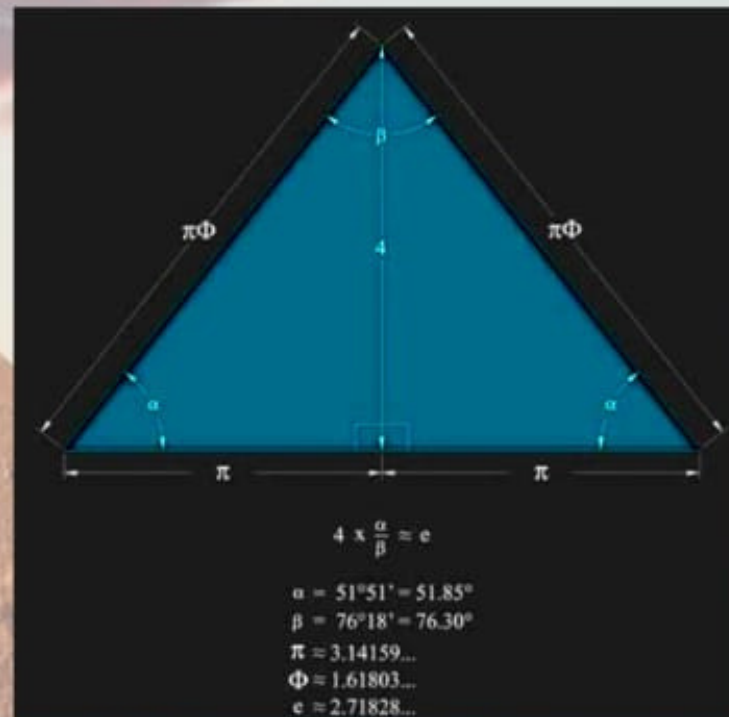
# MATHS AND ARCHITECTURE

*Mathematics plays a very important role in architecture as well as in every field in our lives. You can't design a building without mathematical calculations. For a better explanation, we would like to give you some examples about the relationship between mathematics and architecture. So let's take a look one of the best structures on the world;*

## THE GREAT PYRAMID OF GIZA

The Pyramid of Cheops is the oldest and largest of the three monumental pyramids located in the ancient "tomb city of Giza" surrounding Giza, which is now part of the Egyptian capital, Cairo. The Pyramid of Cheops, which is the tallest pyramid in the world, has a height of 146.7 meters and the height record has not been broken for 4000 years.

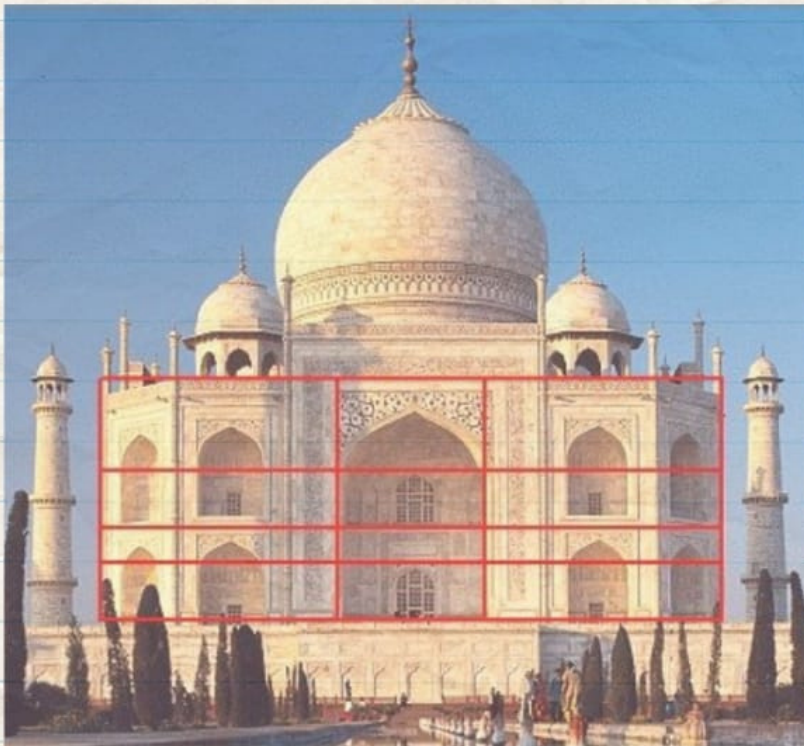
The height, floor area, and height coefficients of the Egyptian Pyramids contain the three most important constants in mathematics. These; pi, fi and e are the numbers. Researcher J.H. Cole made calculations with an angle of 51 degrees and 51 seconds. The Pi number is obtained by dividing the surface of the base of the Cheops Pyramid by twice the half of the monument. The total area of the four surfaces of the Cheops Pyramid is equal to the square of the pyramid height. The Cheops Pyramid is located at the center of the earth.





# ***THE TAJ MAHAL***

*The Taj Mahal is a mausoleum located in Agra, India, that was built under Mughal Emperor Shah Jahan in memory of his favorite wife, Mumtaz Mahal. While the white domed marble and tile mausoleum is most familiar, Taj Mahal is an integrated symmetric complex of structures that was completed around 1648. Ustad Ahmad Lahauri is generally considered to be the principal designer of the Taj Mahal.*



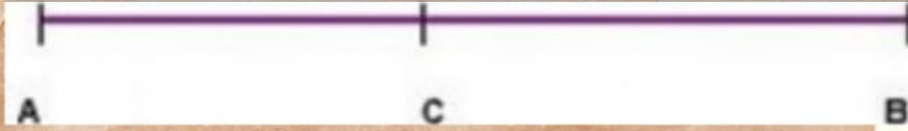
*A golden rectangle is a rectangle whose side lengths are in the golden ratio, one-to-phi, that is, approximately 1:1.618. A distinctive feature of this shape is that when a square section is removed, the remainder is another golden rectangle, that is, with the same proportions as the first. Square removal can be repeated infinitely, which leads to an approximation of the golden or Fibonacci spiral*

*There are two symmetrically identical structures on both sides of the mausoleum. The garden is arranged as a square divided into four on long streams with walking paths, fountains and ornamental trees. Even here, the reflections in the surrounding pools of water evoke a sense of symmetry. Regular shapes were used in the stones on the ground. For example Triangle, Semicircle, cubic, rectangular etc. shapes are used.*



# Golden Ratio in ARCHITECTURE

*In mathematics, two quantities are in the golden ratio if their ratio is the same as the ratio of their sum to the larger of the two quantities. The Greek letter phi represents the golden ratio. It is an irrational number that is a solution to the quadratic equation*



$$\frac{CB}{AC} = \frac{AB}{CB}$$
$$= 1,618033988749894\dots$$

*The golden ratio has also been accepted as the order relationship that gives the best harmony and proportions in art and*

*architecture since ancient times. Let's take a look at some of the most impressive architectural works that contain the golden ratio ;*

## THE PARTHENON

*The Parthenon is a former temple on the Athenian Acropolis, Greece, dedicated to the goddess Athena in 447 B.C.E. The Greeks supposedly thought that the golden ratio was special because it repeatedly appeared in nature, and because it was pleasing to the eye. The golden ratio was even said to have been applied to the building of the Parthenon. The Greek mathematician and sculptor Phidias used the golden ratio when designing the Parthenon.*

*For example, the Parthenon is 30.8 meters wide and 69.51 meters long (101 and 228 feet, respectively). This equals a 4:9 ratio. This 4:9 ratio also is found in other parts of the building, including the width of the Parthenon's front columns, and in the height of the façade to its width.*



**"Golden ratio" on Parthenon**



# THE HAGIA SOPHIA

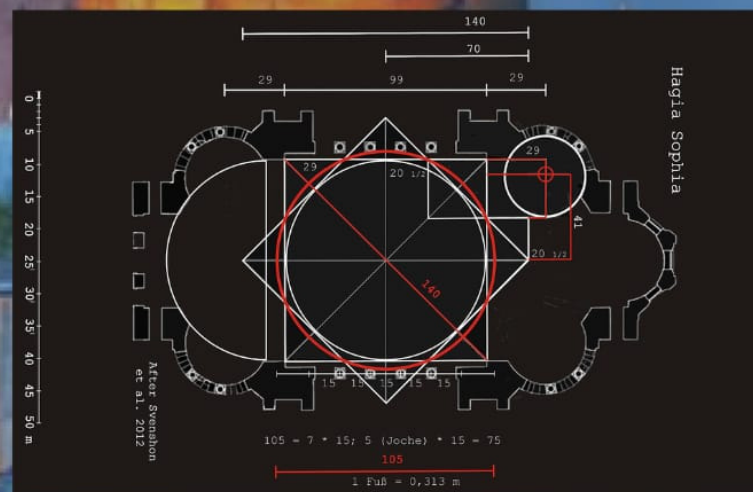
The vast interior has a complex structure. The nave is covered by a central dome which at its maximum is 55.6 m (182 ft 5 in) from floor level and rests on an arcade of 40 arched windows. Repairs to its structure have left the dome somewhat elliptical, with the diameter varying between 31.24 and 30.86 m (102 ft 6 in and 101 ft 3 in)

At the western entrance and eastern liturgical side, there are arched openings extended by half domes of identical diameter to the central dome, carried on smaller semi-domed exedrae, a hierarchy of dome-headed elements built up to create a vast oblong interior crowned by the central dome, with a clear span of 76.2 m (250 ft)

The theories of Hero of Alexandria, a Hellenistic mathematician of the 1st century AD, may have been utilized to address the challenges presented by building such an expansive dome over so large a space. Svenshon and Stiffel proposed that the architects used Hero's proposed values for constructing vaults. The square measurements were recalculated using the side-and-diagonal number progression. Which results in squares defined by the numbers 12 and 17, wherein 12 defines the side of square and 17 its diagonal, which have been used as standard values as early as in cuneiform Babylonian texts.

Each of the four sides of the great square Hagia Sophia is approximately 31 m long and it was previously thought that this was equivalent of 100 Byzantine feet. Svenshon suggests that the size of the central square of Hagia Sophia is not 100 Byzantine feet but instead 99 feet.

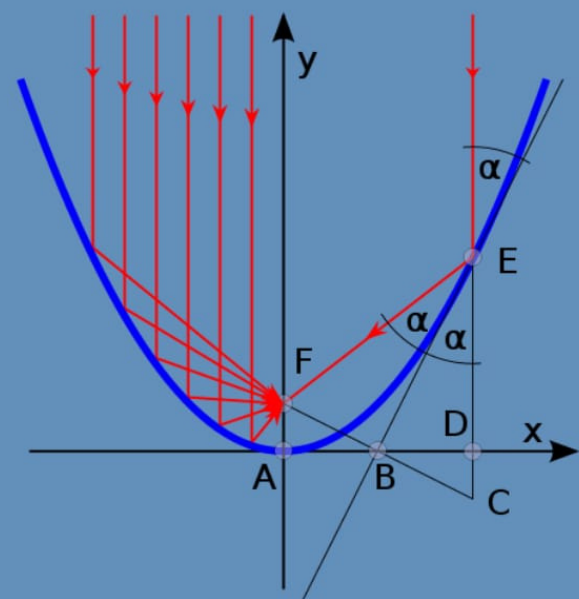
This measurement is not only rational, but it is also embedded in the system of the side-and-diagonal number progression (70/99) and mathematics of antiquity. It gives a diagonal of 140 which is manageable for constructing a huge dome like that of the Hagia Sophia.





# THE SYDNEY OPERA HOUSE

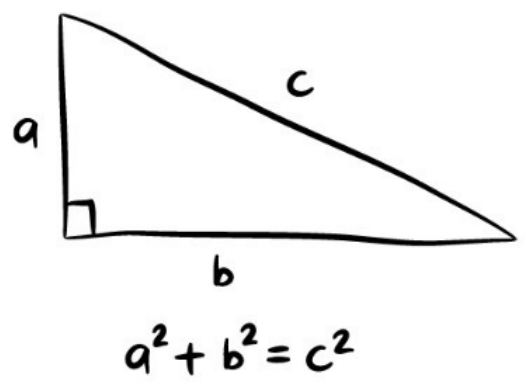
Sydney Opera House is a multi-venue performing arts center in Sydney. Located on the shore of Sydney Harbour, it is often considered one of the most famous and distinctive buildings in the world and a masterpiece of 20th century architecture. It was designed by Danish architect Jørn Utzon. The building consists of multiple performance venues attended by more than 1.2 million people.



The facility features a modern expressionist design with a series of large precast concrete "shells", each consisting of a 75.2 meters (246 ft 8.6 in) radius sphere, forming the roofs of the structure. Each of the shells is made of precast concrete rib pieces that rise to a ridge beam held together by 350 kilometers of tensioned steel cable. Geometrically, each half of each shell is part of a sphere; however, the 'sails' were originally designed as parabolas for which an engineering solution could not be found. Parabola; It is the geometric placement of points equidistant from a fixed "d" line and a fixed "F" point taken in a plane. In algebra, it is known as the graph of quadratic functions  $y=ax^2+bx+c$ . Although described as reinforced concrete shells, they are actually a series of concrete ribs supporting a total of 2,194 precast concrete roof panels covered with over 1 million tiles. The building covers 1.8 hectares (4.4 acres) of land and is 183 meters (600 ft) long and 120 meters (394 ft) wide at its widest point. It is supported on 588 concrete piers that go as deep as 25 meters (82 ft) above sea level. The highest roof point is at 67 meters above sea level.

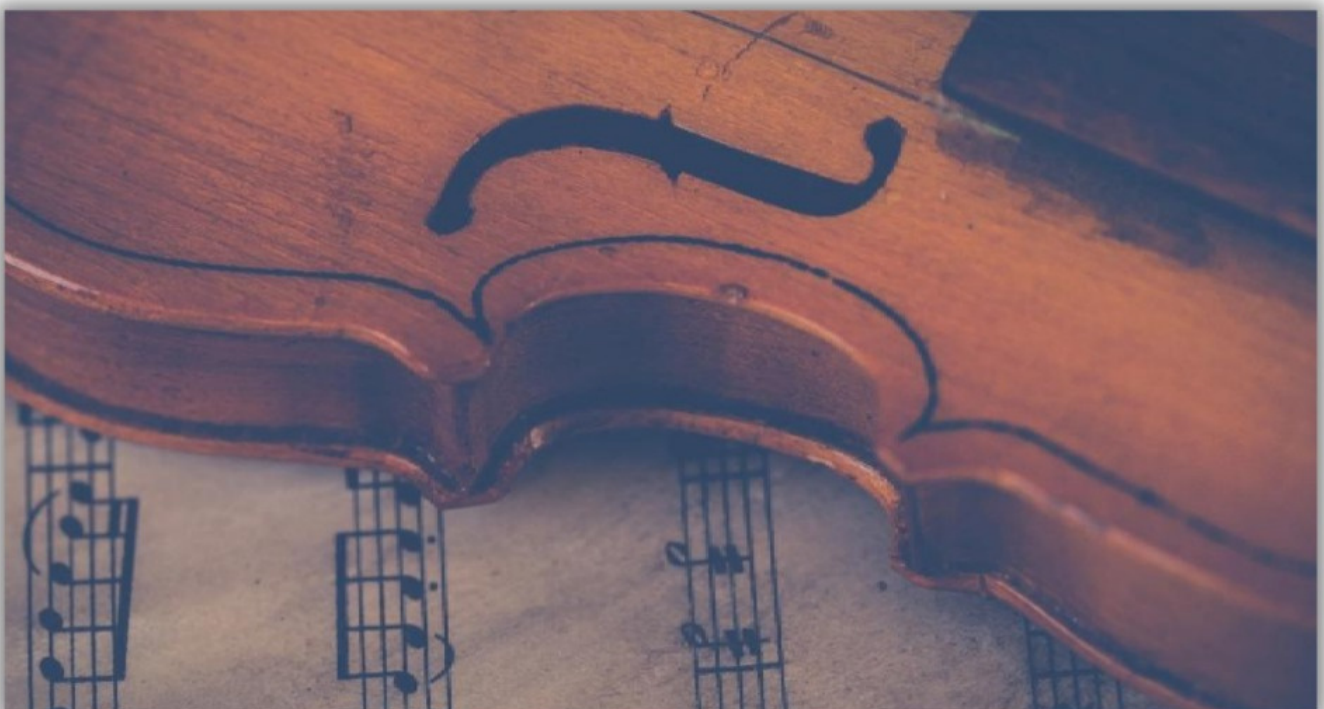


# Pythagorean musical theorem



Pythagorean musical theorem Music began to be studied with the help of mathematical tools in the sixth century BC, and not just anyone, because Pythagoras himself (known to every student from the theorem on the properties of a right triangle, which he is not really the author of). Some might consider them "mathematical fanatics" today because the Pythagoreans claimed that "everything is a number" and that every element of the universe can be expressed in fractions. This is where the concept of the harmony of spheres comes from, a concept that is a combination of music and mathematics. Pythagoras' research, however, went far beyond a lofty, unrealistic philosophy. This was due to the monochord - an instrument that had only one string. For a genius mathematician, however, he had enormous power.

Pythagoras used it to analyze how the distances between notes (i.e. intervals) change by dividing the string in various ratios. He found that the best, the so-called "most pleasant for the ear", harmonies are expressed by the simplest proportions, which correspond to fractions with consecutive natural numbers in the denominator, and one in the numerator. In this way, he discovered the so-called series of overtones (in acoustics, a component of a musical sound with a sinusoidal waveform and frequency, where  $f$  is the frequency of the fundamental tone, and  $k$  is a natural number greater than 1), which is the foundation of our musical system.





# GEOMETRY OF THE COMPOSITION

Composers like to use geometric transformations in their works. Such interventions in the musical nomenclature are called contrapuntal techniques. This includes cancer (crab canon), the symmetry to the vertical axis of the staff, which means that part of the melody is recorded from the end - in the mirror image in the further course of the composition. Symmetry to the horizontal axis in the piece can also be used.

Then we speak of inversion, that is the reversal of the direction of the melodic line. The contrapuntal techniques also include augmentation and diminution, which correspond to the multiplication of the rhythmic values with the scalar, lengthening and shortening them, respectively.

Transposition, on the other hand, can be equated with the shift of the melody by a positive vector, the length of which determines a certain interval.

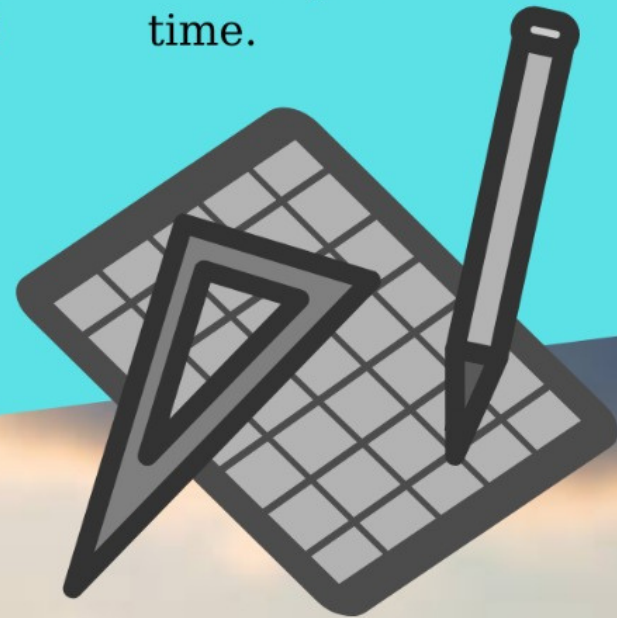


# Rythm, time, signature, tempo

The note values are strictly defined: there are two half notes in the whole note, one half note contains 2 quarter notes, one quarter note contains 2 eighth notes, and one eighth note contains 2 sixteenth notes. In measures with, for example, a 4/4 time signature, one whole note must be present, that is, for example, 16 sixteenth notes.

There must be equal values in each bar and there cannot be too much or too little of them. Tempo is how fast the piece is to be performed. For example, tempo ♩ = 80, means that in one minute of the song there should be 80 quarter notes, all should be equal.

The notes and their performance have a lot to do with music, because when creating or performing a piece of music, we have to count the value of the notes in measures and the tempo all the time.







## FREQUENCY AND HARMONY

A music scale is an unobtrusive set of tones used to create or describe music.

Each step corresponds to a specific frequency expressed in hertz (Hz), sometimes referred to as cycles per second (cps). The scale has a repetition interval, usually an octave. An octave of any pitch refers to a frequency exactly twice the pitch.

The next super taves are the tones at frequencies four, eight, sixteen, etc., of the fundamental frequency. Tones with frequencies equal to half, quarter, eighth, etc. of the fundamental tone are called sub-octaves.

Expressed as a frequency bandwidth, the octave A 2 - A 3 covers the range from 110 Hz to 220 Hz (range = 110 Hz). The next octave will cover the range from 220 Hz to 440 Hz (span = 220 Hz). The third octave is from 440 Hz to 880 Hz (range = 440 Hz), and so on. Each subsequent octave covers twice the frequency range of the previous octave.

Common term	Example name	Hz	Multiple of fundamental	Ratio of within octave	Cents within octave
Fundamental	A <sub>2</sub>	110	1x	1/1 = 1x	0
Octave	A <sub>3</sub>	220	2x	2/1 = 2x	1200
				2/2 = 1x	0
Perfect Fifth	E <sub>4</sub>	330	3x	3/2 = 1.5x	702
Octave	A <sub>4</sub>	440	4x	4/2 = 2x	1200
				4/4 = 1x	0
Major Third	C# <sub>5</sub>	550	5x	5/4 = 1.25x	386
Perfect Fifth	E <sub>5</sub>	660	6x	6/4 = 1.5x	702
Harmonic seventh	G <sub>5</sub>	770	7x	7/4 = 1.75x	969
Octave	A <sub>5</sub>	880	8x	8/4 = 2x	1200
				8/8 = 1x	0



# MATHEMATICS AND NATURE

The purpose of mathematics; it is to improve the thinking ability that people bring innately. In order to achieve this development, it provides us with some information and makes reviews, researches and comparisons of the events and problems we will encounter, allowing us to be regular and careful, think rationally and find the truth about everything. The scientific explanation of many seemingly supernatural events, such as natural phenomena, can be made by mathematics. It is understood that there is mathematics in the perfect order of the universe. Known since prehistoric times, this fact is reflected in our age with a more advanced technology. The most basic mathematical concepts are in nature. The deepest, most abstract concepts of mathematics arise as a result of a necessity from the most basic concepts that nature offers us. In the bosom of each concept, other concepts are included. Mathematics exists independently of mathematicians and people. Pythagoras did not create steep triangles, he discovered them. Galois did not create or explore groups. Noether didn't create the rings, he discovered them. Hilbert did not create hilbert spaces, he discovered... In short, mathematics exists in nature. Now let's examine the hidden mathematics that exists in nature together.



## SYMMETRY AND BUTTERFLIES

We say an object is symmetric if it is invariant to any various transformation (including reflection, rotation or scaling: enlarging or reducing). If we want to be very rigorous (mathematically), we can say "A mathematical object is symmetric with respect to a given mathematical operation, if, when applied to the object, this operation preserves some property of the object." For mathematics, maintaining the same property after some sort of operation is key to understanding many advanced concepts.

Looking at the beautiful patterns of a butterfly's wings is the first step at understanding concepts from calculus (even vs odd functions), linear algebra abstract algebra, statistics and way more.



## THE BIRDS

Many bird species fly in regular groups and especially in the form of 'V'. According to research in the journal 'NATURE', this flight form is the most aerodynamically useful form for birds. The researchers, who did this research, showed that birds make the most beneficial use of airflow by adjusting their wing movements in flight to the movement of the nearest bird in the flock. During 'V' flight, the regular movement of the wings of the birds allows them to make the most of the upward airflow. As they fly one after the other, they make the least use of downward air movement, as the harmony between their wings is reduced. As a result, 23% energy efficiency is achieved for each bird, and 60%-70% in general.



## BEES AND MATHS

Honeycombs are built in a hexagonal shape to fully utilize the space available and with the least amount of materials. Honeycombs are three-dimensional shapes in the form of hexagonal prisms. The honeycombs in the form of hexagonal prisms are in two layers, with one open end and the other closed ends placed back to back. When the frame is placed perpendicular to the ground, the prisms are constructed to make an inclination angle of  $130^\circ$  with the horizontal, and this angle is the smallest sufficient angle for the honey not to flow. Bees visit flowers in many different regions everyday and use a lot of energy. Therefore, they immediately learn to fly on the best route to save energy and time.



## THE BEAVERS

The beaver's nest in the form of a very wide dam. The dam built by the beaver blocks the water at an angle of exactly  $45^\circ$ . In other words, the beaver builds the dam not randomly, but in a completely planned way. All of today's hydroelectric power plants are built in this way. Beavers also do not completely block the water. They build the dam to hold water at the height they want, and leave special channels for excess water to flow out.





## WAVES AND MATHS

Waves are periodic disturbances in some medium, like water waves in water, vibrations of a string or wire (e.g. guitar), sound waves in air, or electromagnetic waves in the electromagnetic field. To describe wave motion mathematically, we refer to the concept of a wave function that describes the position of a particle in the medium at any given time. The most basic of wave functions is the sine wave or the sinusoidal wave, which is a periodic wave. These are the benefits of mathematics in terms of understanding wave signals.



## WAYFINDING METHOD OF ANTS

The desert ants that live in the Sahara, always manage to return to their nest by walking hundreds of meters in the barren desert land. A team that consists of German and Swiss biologist and zoologist, liken this feature to the speedometer in cars. The ants come and go the same distance in an infallible way. The ants of the Sahara use road integration system to find their way. In this system the ant uses its walking and rotations total to calculate its distance to its nest. A series of mathematical operations are performed during this time period. The ant divides the distance to its nest into small segments; each segment carries the appropriate vector of direction and distance. With the sum of these vectors, the 'homing' vector, which gives the distance and direction of the nest, is obtained. It is not yet clear how the Sahara ant measures its forward movements and turns and how it makes these calculations.



## CICADAS

There is a type of cicadas, who have weird lifestyle, in the North America forests. These cicadas have been hiding underground for 17 years. Then, in May of the seventeenth year, they rise to the surface. We generally call them 'periodical cicadas'. These cicadas only live on earth for 4-6 weeks. They mate during that time, they try to lay as many eggs as possible with the hope that who come after them will survive. Then they die singing their song. The most interesting thing for a mathematician, choosing of the prime number 17.

There are other types of cicadas that stay underground for 7 and 13 years. These numbers are also prime numbers. If these periodical cicadas decided to rise early from the ground, they do it in a way that puts them in a subgroup, not a year. So they are rise from the ground in the 13 years instead of 17 years. The reason is why they instinctively organize their life cycles into prime numbers, 13 and 17 are prime numbers and these numbers. It is also possible for two different groups of cicadas to appear simultaneously in nature every 221 years, which is a multiple of 13 and 17. As a result, prime numbers are directly related to the life of cicadas.





## TREES AND MATH

A 'Social Network' was discovered where trees and plants were connected underground.

A study conducted by the Crowther Laboratory in Zurich, Switzerland, and Stanford University in the US found that under every forest and grove, it was discovered that there is a complex web structure consisting of roots, fungi and bacteria that connect trees and plants underneath. The research, published in the journal Nature, used the Global Forest Initiative's database covering 1.2 million forests and 28,000 species in more than 70 countries

The way the spruce tree and climbing angle are shown. Trees and plants can alert each other to danger by sending electrical signals underground via a network called the Wood Wide Web. Some trees also feed on a sugary solution to keep the roots of other trees alive. Each forest creates an aesthetic order in itself, and the trees are positioned at certain distances so that enough light goes to all plants. According to researches, in an adult forest, the average distance between trees of the same mass is proportional to the body diameters.

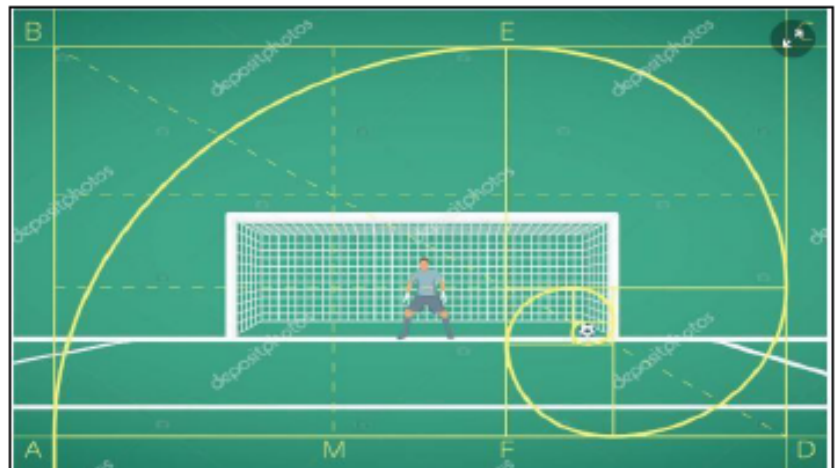






# MATHEMATICS AND Sport

Football / Soccer Goalkeeper Stand at Goal on Field. Mathematical Calculation of Flight of Ball. Principle of The Golden Ratio.



## DIMENSIONS OF THE COURT

- **WIDTH:** 6.1 m (20 ft).
- **LENGTH:** 13.4 m (44 ft).
- **SHORT SERVICE LINE:** 1.98 m (6 ft. 6 inch) from the net.
- **HEIGHT OF NET:** 1.55 m (5 ft. 1 inch) high at the edges and 1.524 m (5 ft.) high in the center.





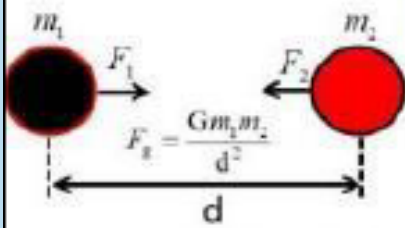
$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{9}{5}C + 32$$

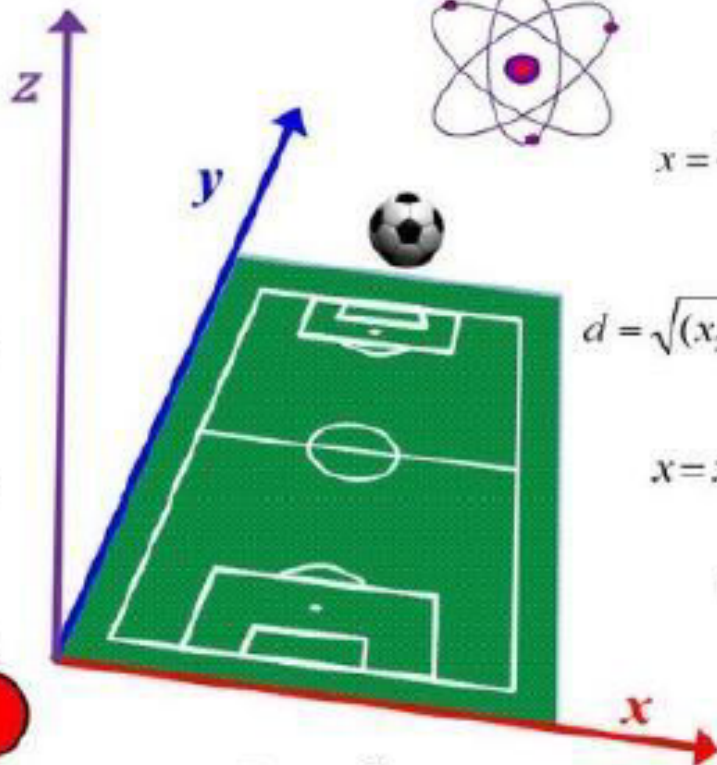
$$K = C + 273.15$$

$$z = \sqrt{x^2 + y^2}$$

$$s = vt + \frac{1}{2}at^2$$



$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$x = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

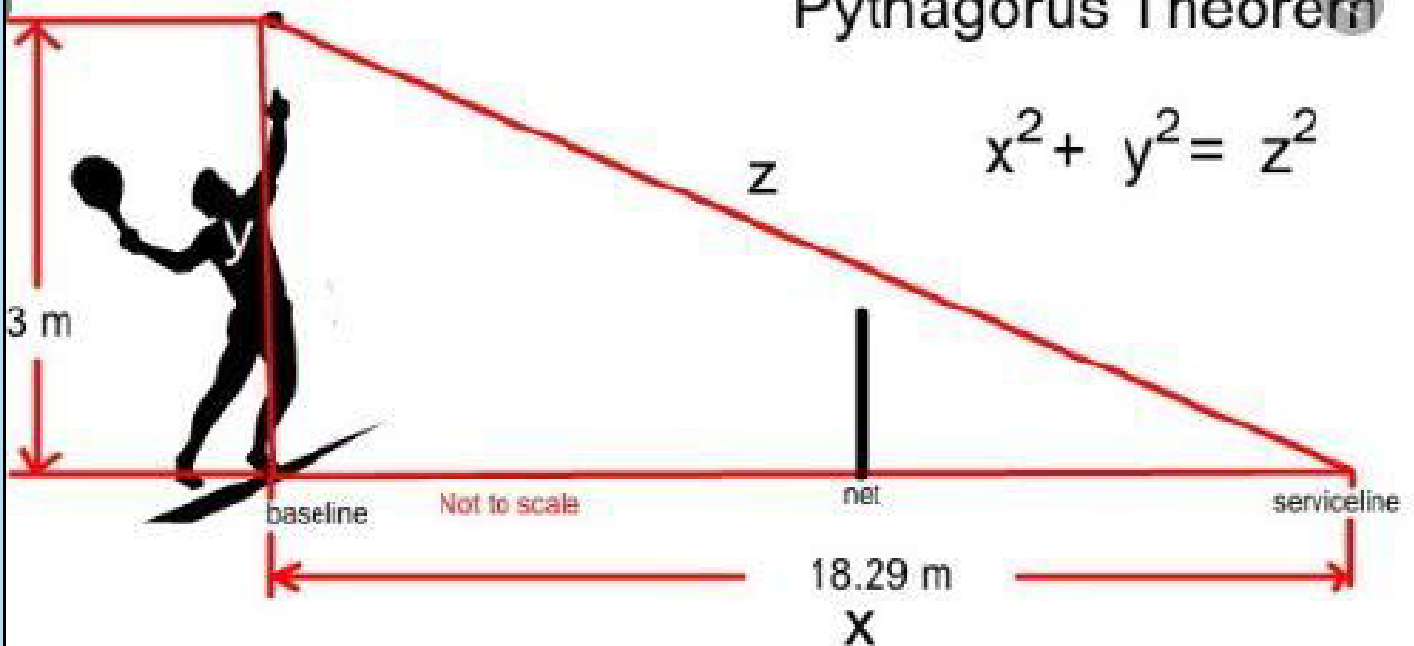
$$\cos^2 \theta + \sin^2 \theta = 1$$

$$E = mc^2$$

$$v = \frac{1}{2}(v + v_0)$$

$$W = mg, \quad g = 32 \text{ ft/s}^2 = 9.8 \text{ m/s}^2$$

## Pythagoras Theorem



Distance tennis ball travels to service line:

$$z^2 = 18.29^2 + 3^2 = 343.5 + 9 = 343.5$$

$$z = 18.5 \text{ m}$$

$$v = \frac{z}{t}$$

But  $x = 18.5$

$$t = \frac{z}{v}$$

$$t = \frac{18.5}{v}$$

Samuel Groth ( $v_1$ )

$$z = 18.5 \text{ m}$$

$$v_1 = 263.4 \text{ km/h}$$

Convert to m/sec

$$v_1 = 73.2 \text{ m/sec}$$

$$t = \frac{18.5}{73.2} \text{ sec}$$

$$t = 0.25 \text{ sec}$$

**WHAT?** It's the same time. That extra distance  $18.5 - 18.29 = 0.21 \text{ m}$  is not significant when dealing with these very high speed tennis balls.



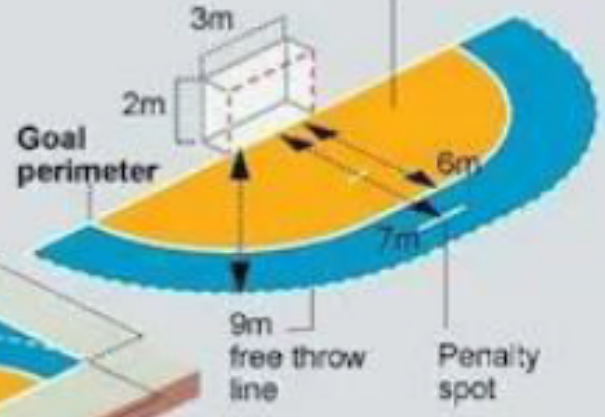
**HANDBALL** Play advances towards the goal, with the red side on the attack, during an Olympic handball match.

**THE PITCH**

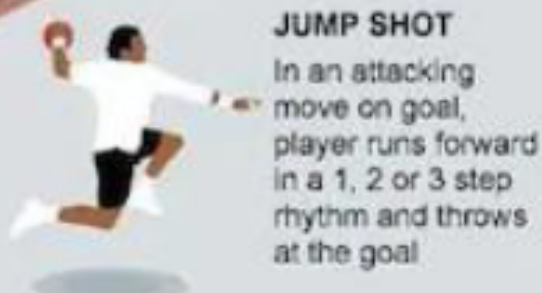
Each team: 6 outfield players



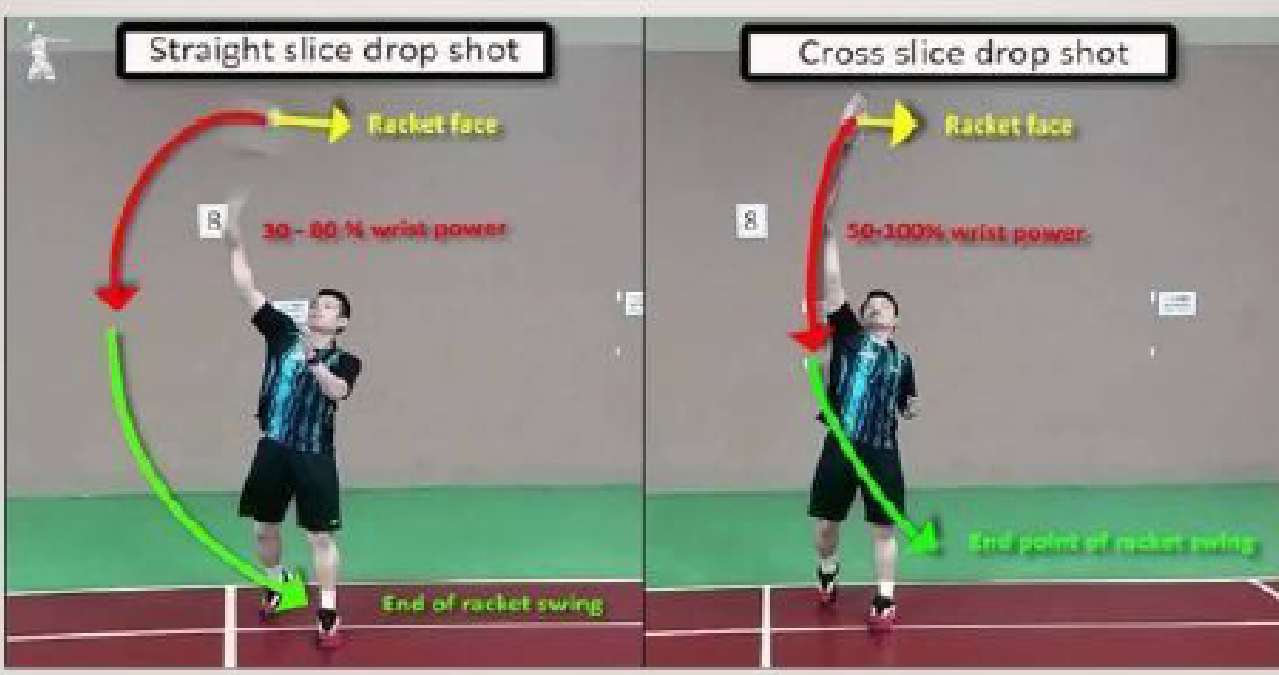
**GOAL AREA**



**THE BALL**



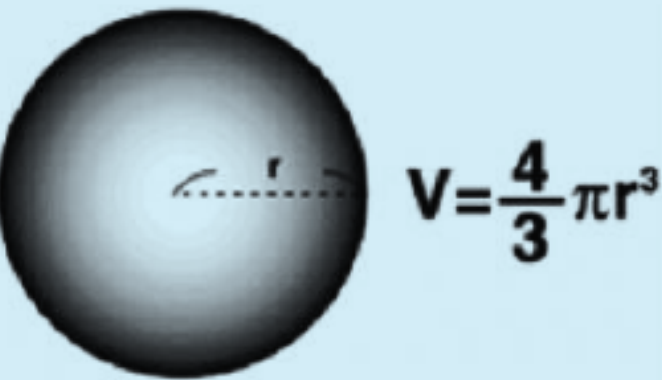
A CROSSCOURT DROP SHOT NEAR THE NET USING AN UNDERHAND GRIP LOOKS LIKE A SEMI-CIRCLE:



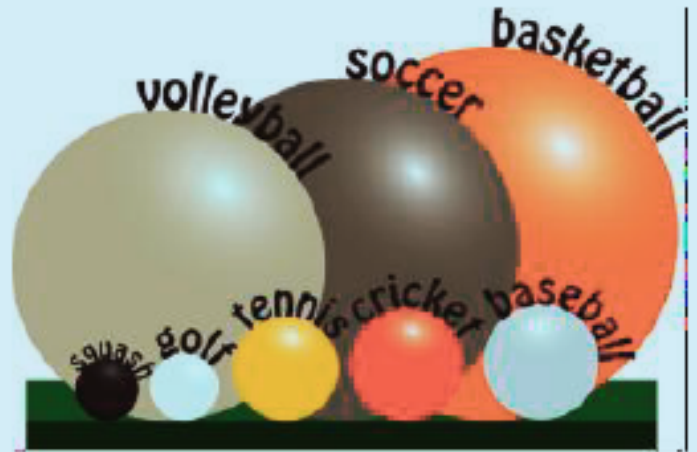


## Volume in Sports

Mens Basketball diameter 238.8mm  
 Soccer ball 110mm  
 Volleyball 105 mm  
 Baseball 37mm  
 Cricket 36mm  
 Tennis 33mm  
 Golf 12.35mm  
 Squash 20mm

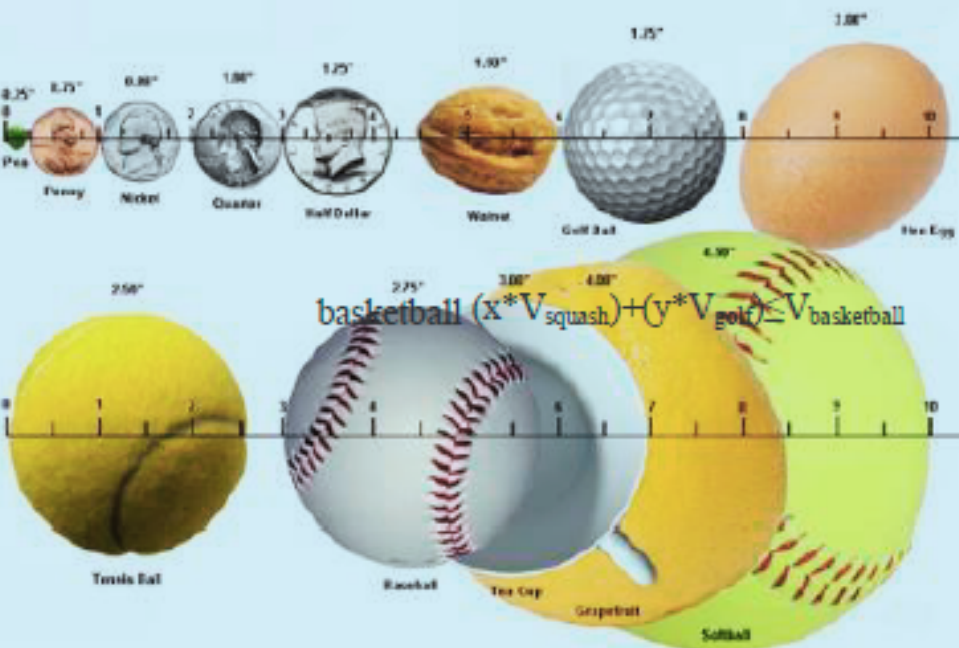


Cubic volume of a sphere



## HAIL SCALE

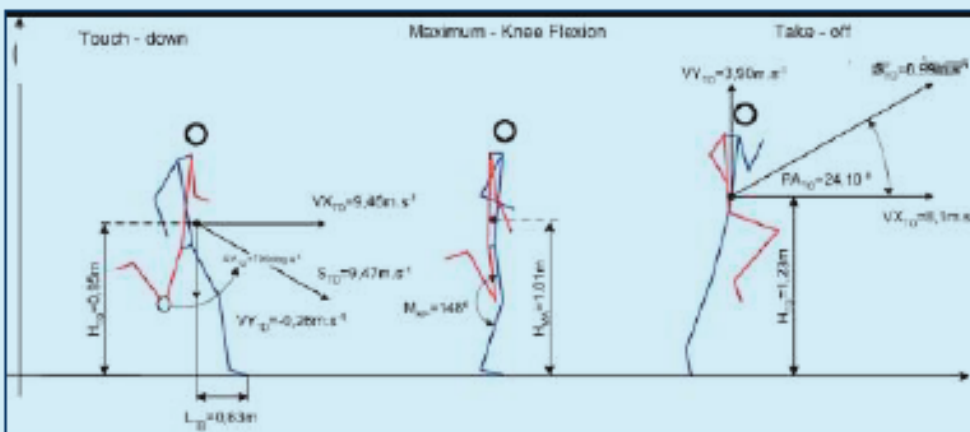
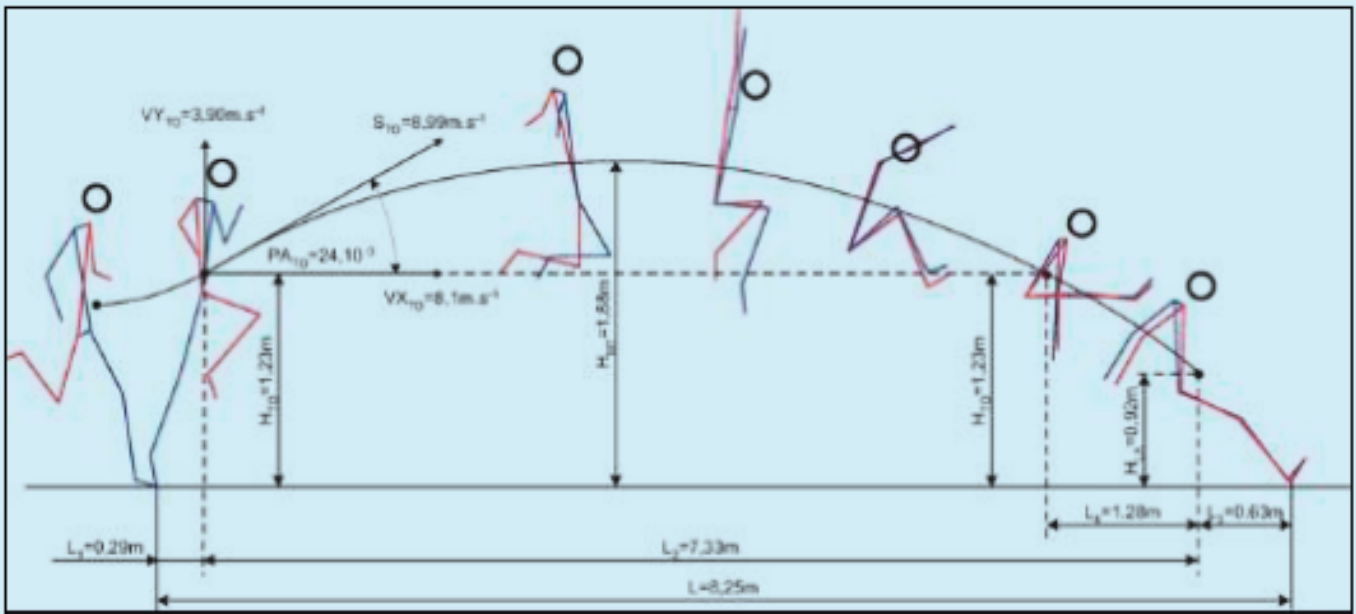
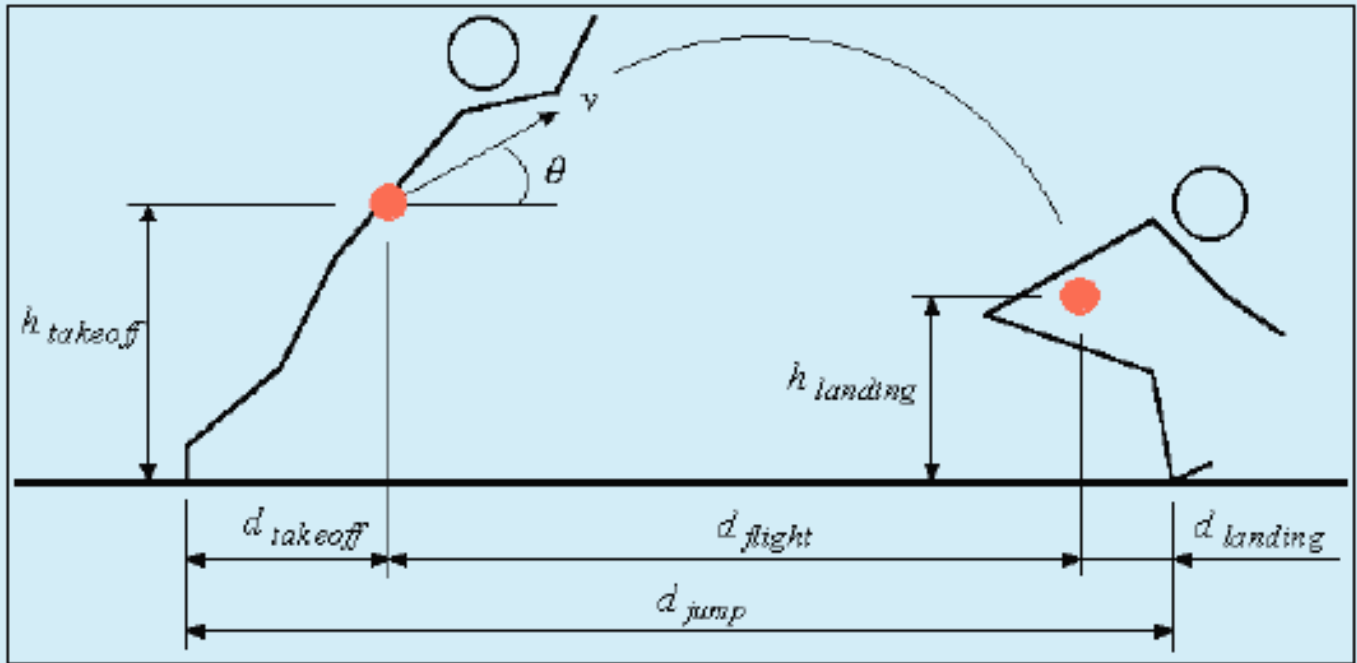
By Apollon



Most sports throughout history have been played with some manipulated object, and most popular of all, is the ball. The object of this activity would be to compare the volumes of a variety of different sports balls. We can find the ratios between the different sized sports balls and find equations to represent the relationships between their volumes.

For example, the volume of a soccer ball is  $5575279.67\text{mm}^3$ , and the volume of a squash ball is  $33510.3\text{mm}^3$ , and the ratio of squash balls to soccer balls is  $116.375:1$ , or perhaps that a soccer ball could hold 116 squash balls (theoretically).





**Romanian Team :**  
**Olaru Denis**  
**Popovici Marian**  
**Anton David**  
**Binga Raluca**  
**Zlotea Rares**



# Resources

<http://meakultura.pl/artykul/matematyka-muzyki-muzyka-matematyki-873>

[https://en.m.wikipedia.org/wiki/Origins\\_and\\_architecture\\_of\\_the\\_Taj\\_Mahal](https://en.m.wikipedia.org/wiki/Origins_and_architecture_of_the_Taj_Mahal)

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<http://www.ams.org/publicoutreach/feature-column/fcarc-sports>