TEACHING UNIT IX
MATHS
QUADRATIC FORMULAS

IINTERNATIONAL TEAMWORK AS A METHOD TO MAKE OUR SCHOOLS INCLUSIVE OF DIVERSITY

PROJECT N. 2014-1-ES01-KA201-003342_1

TEACHING UNIT NETHERLANDS III

MATHS: Quadratic formulas

1. **INFORMATION:**
   a. **Date:** 04-04-2016
   b. **Level:** 3 TTO (14, 15, 16 year old students)
   c. **Subject:** Mathematics
   d. **Theme:** Sustainability puzzle
   e. **Teacher:** Jeroen Bernardus

2. **AIMS/GOALS** How to create and solve a sustainability puzzle by using different type of equations

3. **COMPETENCES/SKILLS (Which competences/skills will you develop in this unit)** Calculation/algebra skills, cooperation skills, language skills

4. **METHODOLOGY**
   a. **Type of lesson:** An instructive, interactive and cooperative lesson
   b. **Type of interaction (organization in classroom):** Introduction and explanation by the teacher, instruction and group work guided by Spanish, Dutch and Italian students
   c. **Teaching aids: (like digital board, pc's ...)** Paper, pen/markers, pencils, ruler, geometrical triangles, white board, IPad/tablet/smartphone, paper copies

5. **TEACHING:**
   a. **Contents:** Working with different type of equations: Introduction 10 min, instruction 35 min, interactive 40 min, evaluation 5 min.
   b. **Activities:** How to solve different type of equations, find the solution of the word related to sustainability by solving equations and make up your own word by choosing your own equations

6. **EVALUATION:**
   a. **Individual** They managed to create their own sustainability puzzle and to solve the puzzle of someone else
   b. **Group evaluation** They managed to be cooperative and give proper explanation to the Italian and Spanish students in general

SUSTAINABILITY PUZZLE

Solving a sustainability puzzle

Introduction

Summary

Gradient and equation of a line

Linear formulas have the form $y = ax + b$, where a is the increase per step of 1 and b is the y-intercept. Another word for the fixed increase per step of 1 is gradient.
The linear formula that corresponds to a line is also called the equation of a line. The equation of a linear relation can also have the form $px + qy = r$.

Example

The equation of line l is $-7x + 4y = 12$.
The equation can be rearranged to $y = 1\frac{3}{4}x + 3$.
The gradient of l is $1\frac{3}{4}$.

Simultaneous equations

You write a set of simultaneous equations as

$$\begin{cases} y = x + 4 \\ 5y + x = 50 \end{cases}$$

You can solve simultaneous equations by rearranging the equations or by substitution.
The solution to the simultaneous equations gives the coordinates of the point of intersection of the lines, in this example the point (5, 9).

Example

Solve the simultaneous equations $\begin{cases} y = x + 4 \\ 5y + x = 50 \end{cases}$

Solution

$5y + x = 50$ can be rearranged to $y = 10 - 0.2x$
Solve:
$x + 4 = 10 - 0.2x$
$1.2x = 6$ so $x = 5$.
Solution: $x = 5$ and $y = 9$.

Fractional equation

An equation like $3 + \frac{36}{5p} = 12$, with a variable in the denominator of the fraction, is called a fractional equation.
To solve a fractional equation you usually use cards.

Example

Solve the equation $3 + \frac{36}{5p} = 12$.

Solution

$\frac{36}{5p} = 9$
$\frac{36}{4} = 9$
$5p = 4$ so $p = \frac{4}{5}$

Check $3 + \frac{36}{5 \times \frac{4}{5}} = 3 + \frac{36}{4} = 12$
which is correct.

Exponential equation

An equation like $7.2 \cdot 3^x = 64.8$, with the variable in the exponent, is called an exponential equation.
To solve an exponential equation you usually use cards.

Example

Solve the equation $7.2 \cdot 3^x = 64.8$.

Solution

$7.2 \cdot 3^x = 64.8$
$3^x = 9$ write 9 as a power of 3
$3^x = 3^2$ because $9 = 3^2$
$x = 2$ card on x

Square root equation

An equation like $\sqrt{8p + 9} = 15$, with the variable inside the square root, is called a square root equation.
To solve a square root equation you usually use cards.

Example

Solve the equation $\sqrt{8p + 9} = 15$.

Solution

$\sqrt{8p + 9} = 15$
$\sqrt{225} = 15$
$8p + 9 = 225$
$8p = 216$ so $p = 27$

Check:
$\sqrt{8 \times 27 + 9} = \sqrt{225} = 15$ correct.

Power equation, cube root

An equation like $250 - 6p^5 = 58$ that contains a power of the variable is called a power equation.
The solution to $x^3 = 125$ is $x = 5$ because $5^3 = 125$.
The solution to $x^3 = 80$ is $x = \sqrt[3]{80}$ because $(\sqrt[3]{80})^3 = 80$.
$\sqrt[3]{80}$ is called the cube root of 80.
A power equation of the form $x^n = c$, where the exponent n is an odd number, always has one solution. This solution is $x = \sqrt[n]{c}$.
A power equation of the form $x^n = c$, where the exponent n is an even number, has zero, one or two solutions.
When $c > 0$ the two solutions are $x = \sqrt[n]{c}$ and $x = -\sqrt[n]{c}$.

Example

Solve the equation $250 - 6p^5 = 58$.

Solution

$250 - 6p^5 = 58$
$6p^5 = 192$
$p^5 = 32$ write 32 as …^{5}
$p^5 = 2^5$ because $32 = 2^5$
$p = 2$
Check: $250 - 6 \times 2^5 = 250 - 6 \times 32 = 250 - 192 = 58$ correct.

Example

Solve the equation $-3x^4 + 92 = 62$.

Solution

$-3x^4 + 92 = 62$
$-3x^4 = -30$
$x^4 = 10$
x^4 has an even index, so there are two solutions (because $10 > 0$).
$x = \sqrt[4]{10}$ or $x = -\sqrt[4]{10}$
An approximation to two decimal places:
$x \approx 1.78$ or $x \approx -1.78$

How do you solve a fractional equation?

1. If there are variables on both sides of the equation, multiply both sides by the denominator of the fraction.
2. Remove the brackets.
3. If necessary, reduce the equation to zero and solve it.
4. Check your answer.

Example

Solve: $2x = \frac{12}{x + 1}$

Solution

❶ $2x(x + 1) = \frac{12}{x + 1} \cdot (x + 1)$

❷ $2x^2 + 2x = 12$

❸ $2x^2 + 2x - 12 = 0$
$x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$
$x = -3$ or $x = 2$

❹ Fill in $x = -3$: $2 \times -3 = \frac{12}{-3 + 1}$ so $-6 = -6$
Fill in $x = 2$: $2 \times 2 = \frac{12}{2 + 1}$ so $4 = 4$
Both answers are correct.

Solving a sustainability puzzle

Assignment

- Make up a word that's related to the topic **sustainability**.
- Choose the amount of equations from your example sheet that the word contains in letters. For example if the word contains 5 letters choose 5 equations
- Create an answer sheet for yourself. Make sure the solution of every equation is equal to a letter of your sustainability word.
- Create another sheet with your chosen equations and with boxes. In the corner of each box must be an answer of one of your equations
- Let your neighbour puzzle the word by solving the equations
- Check the calculations with your answer sheet and verify if your sustainability word is correct
- Now it's your neighbours turn.

Solving a sustainability puzzle

Example sheet

Refresh

R-1 Julian needs some sand to raise the height of his garage drive. He looks on the internet for suppliers and compares prices. Company A charges € 27 per m^3 plus € 45 for delivery. Company B charges € 22.50 per m^3 plus € 60 for delivery.

a Julian thinks he needs 8 m^3 of sand.
Calculate which company is cheapest for Julian.

b Make an equation to calculate for what number of m^3 of sand both companies charge the same amount.

c Solve the equation from question b.

d For what number of m^3 of sand is it cheaper to buy from company A?

R-2 For each pair of linear formulas, calculate the coordinates of the point of intersection of the corresponding graphs.

a $y = 2x + 8$ and $y = x - 10$

b $y = 2x + 12$ and $y = -x + 6$

c $y = 11x - 28$ and $y = -4x + 2$

R-3 Solve the following equations.

a $3(2x - 7) + 4 = 5x - 12$

b $18 - 4(3 - x) = 6(2x - 1) + 28$

c $4x - 21 = 3(2x - 2) - 8x$

d $7(8 - 3x) - 2\frac{1}{2} = 10x + \frac{1}{2}\left(5x + 6\frac{1}{2}\right)$

R-4 Sven and Arman are going to solve the equation $3(5x - 8) = 96$.

a Sven first removes the brackets and then solves the equation using the balance method.
Write down Sven's calculation.

b Arman places a card on $5x - 8$ and writes down $5x - 8 = 32$.
Where does Arman get the number 32 from?

c Finish solving the equation using Arman's method.

Method

How do you solve an equation using cards?

1. Think carefully where the card should be placed.
2. Work out what number should be written on the card.
3. Calculate the solution.
4. Check your solution by filling it in in the equation.

Example

Solve the equation $-2(4x + 1) = 18$ using cards.

Solution

❶ $-2(4x + 1) = 18$

❷ $-2 \times \boxed{-9} = 18$

❸ $4x + 1 = -9$
$4x = -10$
$x = -2\frac{1}{2}$

❹ Check $-2 \times (4 \times -2\frac{1}{2} + 1) =$
$-2 \times (-10 + 1) = -2 \times -9 = 18$, correct.

R-5 Solve the following equations using cards.

a $6k + 12 = 30$

b $0.5(4t - 12) = 2$

c $-4(d + 2.5) = 20$

d $20 - 5v = 100$

e $\frac{1}{3}(15 - 3c) = 10$

f $100 - 2(3f - 2) = 50$

R-6a What two solutions does the equation $x^2 = 49$ have?

b Why does the equation $2x^2 = -8$ have no solutions?

c Solve the following quadratic equations using cards.

A $2x^2 = 162$

B $5x^2 + 1 = 21$

C $120 + 4x^2 = 20$

D $-3x^2 + 108 = 0$

R-7 Write the following formulas without brackets and as short as possible.

a $k = x(5x - 3)$

b $g = 3(12p - 5)$

c $m = -5(4g + 3) - 3g$

d $h = 7u \times u^2 + 5u^3$

e $q = 4r(4 - r) - 4r^2$

f $y = 7x(x - 4) + 2x^2$

R-8 Write down the gradient and y-intercept of each linear formula.

a $y = -9x + 3$

b $k = -4r + 1$

c $d = -23 - g$

d $b = 3(a - 5)$

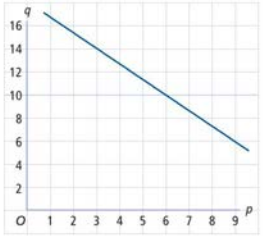
e $y = -5u$

f $v = -3.5(2 - 6m)$

Theory

This graph corresponds to the formula $3q + 4p = 54$.
Because p is on the horizontal axis and q is on the vertical axis it is useful to rewrite the formula as a formula you can use to directly calculate q when you know p.

In $q = -1\frac{1}{3}p + 18$, q is expressed in terms of p.



Example

Take the formula $4q - 5p = 40$. Express q in terms of p.

Solution

	$4q - 5p = 40$
left and right + $5p$	$4q = 40 + 5p$
left and right ÷ 4	$q = 10 + 1.25p$
or	$q = 1.25p + 10$

R-9 Rearrange the following formulas to express q in terms of p.

a $6q + 12p = -18$

b $2 + 12p = 2q$

c $-2p - 4q = 100$

d $21 - 3q + 2p = 0$

R-10 Take the formulas $b = 6t + 12$ and $t = 3p + 1$.
On the right you can see how Elma used substitution to express b in terms of p.

a Remove the brackets in this formula and write the formula as short as possible.

b Substitute formula B into formula A to express b in terms of p.
Simplify your answer.

A: $b = 2t - 4$ and B: $t = 4p$
A: $b = 7 - t$ and B: $t = 5p - 3$
A: $b = 8 + 4t$ and B: $t = 3 - 3p$
A: $b = -2 - 10t$ and B: $t = 0{,}5p$

Elma:
$b = 6t + 12$
$t = 3p + 1$
$b = 6(3p + 1) + 12$

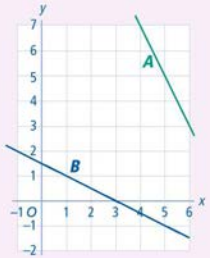
Solving a sustainability puzzle

Example sheet

Test yourself

Do these exercises to check if you have understood the material.

T-1 The graphs of $y = 15 - 2x$ and $2y + x = 3$ are shown.



a Make a calculation to show which graph corresponds to $2y + x = 3$.
b Rearrange the formula $2y + x = 3$ into the form $y = ax + b$.
c Give the gradients of both graphs.
d Calculate the coordinates of the point of intersection of the graphs.
e Use substitution to solve the simultaneous equations

$$\begin{cases} 2y + x = 3 \\ y = 15 - 2x \end{cases}$$

and show that you find the same solution as in question d.

This exercise corresponds to section 9-1.

T-2 Solve the following equations.

a $\frac{24}{5b - 3} = 4$
b $1 + \frac{30}{7h} = 6$
c $\frac{10}{5y + 1} + 2\frac{1}{2} = 0$
d $\frac{11}{x + 1} = x + 11$
e $\frac{2}{x} - 3 = -2x + 1$
f $7x + 5 = \frac{20}{x} + 1$

This exercise corresponds to section 9-2.

T-3a Here is a table with powers of 7.

power	7^1	7^2	7^3	7^4	7^5	7^6
result	7	49	343	2401	17807	117649

Use the table to solve the following exponential equations.

A $7^{2a} = 343$
B $7^{x+4} = 117\,649$
C $7^{6-x} = 49$
D $7^{-2+5a} = 7$
E $2 \cdot 7^p = 686$
F $7^{3k-1} - 1 = 2400$

b A number of exponential equations containing powers of 2 are shown below.
Solve these equations.

A $2^{3a} = 64$
B $2^{2x+4} = 32$
C $2^{4-2x} = 16$
D $2^{2+3a} = 2$
E $256 - 2 \cdot 2^q = 0$
F $0.2 \cdot 2^x = 12.8$

This exercise corresponds to section 9-3.

T-4 Take the function $k(t) = 4\sqrt{t + 1}$.

a For which values of t does function k have results?
b How many solutions does the equation $4\sqrt{t + 1} = 3$ have?
And $4\sqrt{t + 1} = -7$?
c Calculate $k(8)$ and $k(15)$.
d Solve the equation $k(t) = 20$.

This exercise corresponds to section 9-3.

T-5 Solve the following power equations.

a $w^7 = 128$
b $2k^3 = -250$
c $(z - 6)^4 = 81$
d $0.25y^5 = 25\,000$
e $t^6 + 3 = 67$
f $2x^4 + 1 = 3$

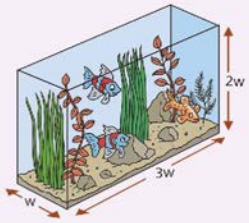
This exercise corresponds to section 9-4.

T-6 Solve the following power equations. Give an exact answer and an approximation to two decimal places.

a $70 - h^5 = -30$
b $10 - 3g^6 = 10$
c $\frac{1}{4}d^4 + 12 = 112$
d $71 + 0.5p^3 = 31$
e $25 + 3x^2 = 16$
f $-0.3c^7 + 6 = 12$

This exercise corresponds to section 9-4.

T-7 You can calculate the volume of a cuboid-shaped aquarium using the formula $V = 6w^3$. Here V is the volume in litres and w is the width of the aquarium in dm.



a Calculate the volume of the aquarium when $w = 3$.
b A similarly shaped aquarium has a volume of 384 litres.
Calculate the dimensions of this aquarium.
c 80% of the aquarium is filled with water. It contains 240 litres of water.
Calculate the height of this aquarium in whole cm.

T-8 Solve the following equations.

a $(20q - 15)^3 = 125$
b $27 - 3\sqrt{3t} = 0$
c $\frac{150}{9 - 4b} + 10 = 35$
d $5t^3 + 140 = 5$
e $20 + 6 \cdot 2^m = 68$
f $4r^4 - 40 = 44$
g $g + 1 = \frac{25 + g}{g - 3}$
h $\frac{32}{4\sqrt{2u}} + 2 = 4$

Solving a sustainability puzzle

Answers and solutions

Solving a sustainability puzzle

Equations

Boxes

Solving a sustainability puzzle

Extra notes