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EUCLIDIAN GEOMETRY

Design Geometry with Computer Aid

GEOCAD

SLOVENIA-ESTONIA-GREECE-PORTUGAL-TURKEY

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CONTENTS

1) Bisection-Proposition 9

(To cut a given rectilinear angle in half.)

Bisection is dividing line segments or angles into two equal parts by a line, which is called a *bisector.* Bisecting an angle means drawing a ray in the interior of the angle, with its initial point at the vertex of the angle such that it divides the angle into two equal parts.

How to Bisect an Angle?

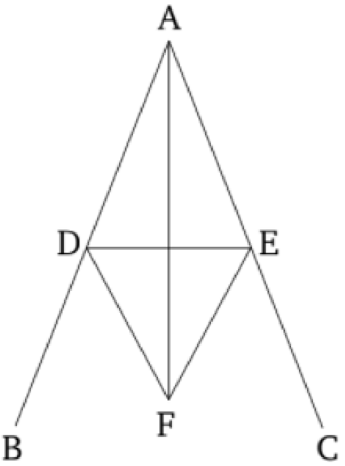
To bisect an angle without protractor you will need:

- a paper sheet;
- a pencil;
- a ruler;
- a compass.

Let's do this!

1. Draw a random angle BAC.
2. Place the compass on the vertex of the angle (point A). Draw an arc across each arm of the angle.
3. Place the compass on the point where one arc crosses an arm and draw an arc inside the angle. Without changing the compass width, repeat for the other arm so that the two arcs cross.
4. Use a ruler to join the vertex to the point where the arcs intersect (F).
5. AF is the bisector of BAC.

To cut a given rectilinear angle in half.



Let BAC be the given rectilinear angle. So it is required to cut it in half.

Let the point D have been taken at random on AB, and let AE, equal to AD, have been cut off from AC [Prop. 1.3], and let DE have been joined. And let the equilateral triangle DEF have been constructed upon DE [Prop. 1.1], and let AF have been joined. I say that the angle BAC has been cut in half by the straight-line AF.

For since AD is equal to AE, and AF is common, the two (straight-lines) DA, AF are equal to the two (straight-lines) EA, AF, respectively. And the base DF is equal to the base EF. Thus, angle DAF is equal to angle EAF [Prop. 1.8].

Thus, the given rectilinear angle BAC has been cut in half by the straight-line AF. (Which is) the very thing it was required to do.

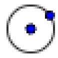
Euclid's Elements Book 1: Proposition 9, Bisecting An Angle

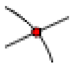
Let's proof, that AF is the bisector of BAC. If we connect point D with B and C, then we get two triangles ADC and ADB, which have a common side AD; side AB is equal to side AC, and BD is equal to CD. On three sides, the triangles are equal, which means that the angles BAD and DAC are also equal, lying opposite the equal sides BD and CD. Therefore, line AD will halve the angle BAC.

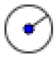
You can also see explanation in Euler's Academy video: https://youtu.be/cbCvpx1Am9M

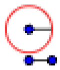
How to Bisect an Angle in Geogebra?

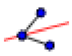
1. Draw two rays AB and AC, from the same point, using the *Ray tool*. 

2. Then proceed in the same way as when drawing with a compass and a ruler. Draw a *Circle with Center at Point* A passing through some point D. 

3. Mark the points of intersection of this circle with two rays with the letters E and F using *Intersect tool* Click on the circle and the side of the corner. 

4. Now we have to draw two *Circles of the same radius*. Remember which points need to be taken as the centers of these circles. After constructing one circle, use the *Compass tool* to draw last one. 

5. Mark the points of their intersection with the letter H. Now it remains to draw the ray AH, which will be the bisector of the angle BAC. 

In Geogebra there is also a special tool for dividing an angle into two equal parts - *Angle Bisector*. Draw some angle and bisect it using this tool.Geogebra activities 

https://www.geogebra.org/m/nkr88adr

2) Proposition 10

"How to split a given finite straight line in half"

Follow the instructions given in the right column to make the geometrical construction of the middle of a straight segment

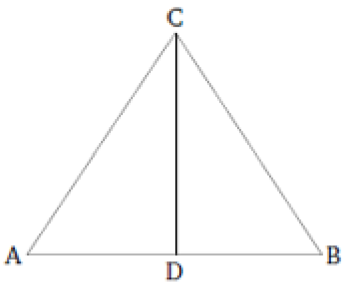
Proposition 10

To cut a given finite straight-line in half.

Let AB be the given finite straight-line. So it is required to cut the finite straight-line AB in half.

Let the equilateral triangle ABC have been constructed upon (AB) [Prop. 1.1], and let the angle ACB have been cut in half by the straight-line CD [Prop. 1.9]. I say that the straight-line AB has been cut in half at point D.

For since AC is equal to CB, and CD (is) common, the two (straight-lines) AC, CD are equal to the two (straight-lines) BC, CD, respectively. And the angle ACD is equal to the angle BCD. Thus, the base AD is equal to the base BD [Prop. 1.4].



Thus, the given finite straight-line AB has been cut in half at (point) D. (Which is) the very thing it was required to do.

• Draw a finite line , that is, a segment AB with constant length .
• Draw two equilateral triangles ABC and ABE in both half-planes defined by the section.
• Draw the line CE that intersects the AB segment at point D. • The AD segment is the bisector of the ACB angle
• The D point is the middle of the segment therefore the mean of the finite straight line AB.

Can you explain why this is true?
(write a brief proof)

3) Straight-line-Proposition 11

You will learn how to draw straight-line at right-angles. ☺

You will need:

- a compass
- a ruler
- a pencil
- a white paper.

You can find this drawings and more from Euqlid's Elements of Geometry Book1.

Let's do this !

1. Draw a straight line and mark to point A and B ;
2. İn the middle of the AB segment draw the C point;
3. In between AC points draw D point ;
4. The equilaeral triangle FDE is made by points DE where FC join;

4.1. with the help of a compass put the dry end of the compass in point C and the other end in D point turning the compass without move the dry end to find E point ;

4.2. Put the dry end of the compass in D point and the other end in E point and draw a trail.

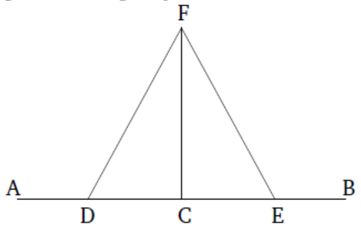
4.3. Then put the dry end of the compass in E point and draw another trail.

4.4. Where the two trails intersect we have point F

5. At the end of the previous steps draw a straight line from D to F and F to E.

Proposition 11

To draw a straight-line at right-angles to a given straight-line from a given point on it.



Let AB be the given straight-line, and C the given point on it. So it is required to draw a straight-line from the point C at right-angles to the straight-line AB.

Let the point D be have been taken at random on AC, and let CE be made equal to CD [Prop. 1.3], and let the equilateral triangle FDE have been constructed on DE [Prop. 1.1], and let FC have been joined. I say that the straight-line FC has been drawn at right-angles to the given straight-line AB from the given point C on it.

For since DC is equal to CE, and CF is common, the two (straight-lines) DC, CF are equal to the two (straight-lines), EC, CF, respectively. And the base DF is equal to the base FE. Thus, the angle DCF is equal to the angle ECF [Prop. 1.8], and they are adjacent. But when a straight-line stood on a(nother) straight-line makes the adjacent angles equal to one another, each of the equal angles is a right-angle [Def. 1.10]. Thus, each of the (angles) DCF and FCE is a right-angle.

Thus, the straight-line CF has been drawn at right-angles to the given straight-line AB from the given point C on it. (Which is) the very thing it was required to do.

4) Straight-line-Proposition 12

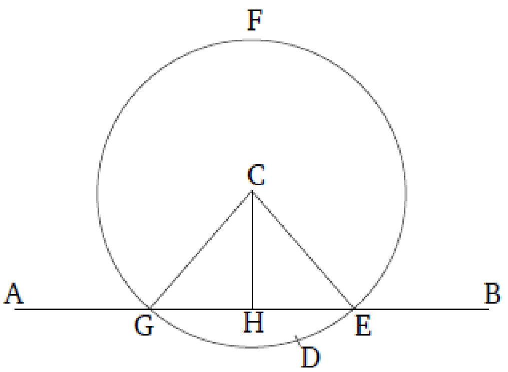
You will learn how to draw a straight-line perpendicular to a given infinite straight-line from a given point which is not on it in old fashioned way. ☺
You will need a compass,a ruler,a pencil, a protractor, and a white paper.

You can find this drawings and more from Euqlid's Elements of Geometry Book1.

Let's do this !

Proposition 12

To draw a straight-line perpendicular to a given infinite straight-line from a given point which is not on it.



Let AB be the given infinite straight-line and C the given point, which is not on (AB). So it is required to draw a straight-line perpendicular to the given infinite straight-line AB from the given point C, which is not on (AB).

For let point D have been taken at random on the other side (to C) of the straight-line AB, and let the circle EFG have been drawn with center C and radius CD [Post. 3], and let the straight-line EG have been cut in half at (point) H [Prop. 1.10], and let the straight-lines CG, CH, and CE have been joined. I say that the (straight-line) CH has been drawn perpendicular to the given infinite straight-line AB from the given point C, which is not on (AB).

For since GH is equal to HE, and HC (is) common, the two (straight-lines) GH, HC are equal to the two (straight-lines) EH, HC, respectively, and the base CG is equal to the base CE. Thus, the angle CHG is equal to the angle EHC [Prop. 1.8], and they are adjacent. But when a straight-line stood on a(nother) straight-line makes the adjacent angles equal to one another, each of the equal angles is a right-angle, and the former straight-line is called a perpendicular to that upon which it stands [Def. 1.10].

Thus, the (straight-line) CH has been drawn perpendicular to the given infinite straight-line AB from the given point C, which is not on (AB). (Which is) the very thing it was required to do.

- Draw an AB segment using the ruler.(10 cm length)
- Mark a point above AB segment,name it C point.
- Mark a point under AB segment, name it D point.
- Open the compass CD lenght and draw a circle (center is C point).
- Mark G and H points on AB segment.
- Draw two segments CG and CE.
- Divide GE into 2 segments and mark H point on it. (Ps: We have learnt in in Proposition 9, you can use that rules or you can divide your own way using ruler.)
- Draw CH segment and measure with the protractor.

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5) Stright Line -Proposition 29

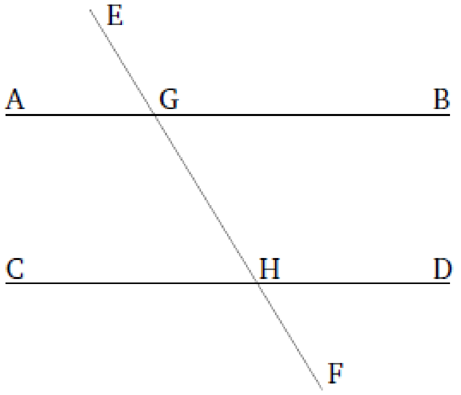
You will learn how to draw - in an old fashioned way - a straight-line falling across parallel straight-lines, which makes the alternate angles equal to one another. ☺

You will need: a ruler, a pencil, white paper.

You can find this drawings and more from Euqlid's Elements of Geometry Book1.

Proposition 29

A straight-line falling across parallel straight-lines makes the alternate angles equal to one another, the external (angle) equal to the internal and opposite (angle), and the (sum of the) internal (angles) on the same side equal to two right-angles.



For let the straight-line EF fall across the parallel straight-lines AB and CD. I say that it makes the alternate angles, AGH and GHD, equal, the external angle EGB equal to the internal and opposite (angle) GHD, and the (sum of the) internal (angles) on the same side, BGH and GHD, equal to two right-angles.

For if AGH is unequal to GHD then one of them is greater. Let AGH be greater. Let BGH have been added to both. Thus, (the sum of) AGH and BGH is greater than (the sum of) BGH and GHD. But, (the sum of) AGH and BGH is equal to two right-angles [Prop 1.13]. Thus, (the sum of) BGH and GHD is [also] less than two right-angles. But (straight-lines) being produced to infinity from (internal angles whose sum is) less than two right-angles meet together [Post. 5]. Thus, AB and CD, being produced to infinity, will meet together. But they do not meet, on account of them (initially) being assumed parallel (to one another) [Def. 1.23]. Thus, AGH is not unequal to GHD. Thus, (it is) equal. But, AGH is equal to EGB [Prop. 1.15]. And EGB is thus also equal to GHD. Let BGH be added to both. Thus, (the sum of) EGB and BGH is equal to (the sum of) BGH and GHD. But, (the sum of) EGB and BGH is equal to two right-

Let's do this !

∇ Draw an AB segment using the ruler. (10cm length)

∇ Draw a CD segment (parallel to AB) using the ruler. (10cm length)

∇ Draw a straight-line EF fall across parallel straight-line AB and CD using the ruler.

∇ Name the intersection between AB and EF line G point.

∇ Name the intersection between CD and EF line H point.

∇ Alternate angles AGH and GHD are equal to one another.

∇ External angle EGB and internal and opposite angle GHD are equal as well.

∇ The sum of internal angles on the same side BGH and GHD is always equal to two right-angles.

6) Constructing Parallel Lines-Proposition 31

Parallel lines are lines that are equidistant at all points and would never touch if they went on forever. You will learn how to draw parallel straight-line throught a given point using angle copy method.

You will need:

- a ruler,
- a pencil,
- a compass;
- paper sheet.

Let's do this!

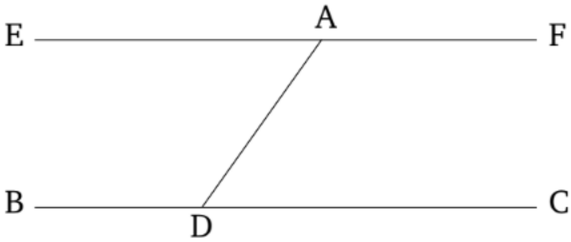
1. Using a ruler draw a straight line BC.
2. Put some random point A, that does not belong to a straight line. It can be above or below it.
3. Put some point to line BC and label it D.
4. Using a ruler connect the points A and D with a line.
5. Prepare the compass. Set the compass to a width that is less than half of the line segment you constructed. Draw the angle ADC.

Proposition 31

To draw a straight-line parallel to a given straight-line, through a given point.

Let A be the given point, and BC the given straight-line. So it is required to draw a straight-line parallel to the straight-line BC, through the point A.

Let the point D have been taken a random on BC, and let AD have been joined. And let (angle) DAE, equal to angle ADC, have been constructed on the straight-line DA at the point A on it [Prop. 1.23]. And let the straight-line AF have been produced in a straight-line with EA.



And since the straight-line AD, (in) falling across the two straight-lines BC and EF, has made the alternate angles EAD and ADC equal to one another, EAF is thus parallel to BC [Prop. 1.27].

Thus, the straight-line EAF has been drawn parallel to the given straight-line BC, through the given point A. (Which is) the very thing it was required to do.

Euclid's Elements Book 1: Proposition 31, Constructing Parallel Lines

6. Using the same compass width, place the tip of the compass on the point A. Draw an arc that intersects the transverse.
7. Draw the corresponding angle. Using the width of the first angle, set the tip of the compass at the point on the transverse line above the given point, and draw an arc that intersects the arc you created before.
8. Draw a line through the given point and the point created by the two intersecting arcs. This line is parallel to the given line through the given point.

You can also see detailed explanation in Euler's Academy video: https://youtu.be/qh-RabJvEfc

7) Square-Proposition 46

"How to describe a square on a given straight line"

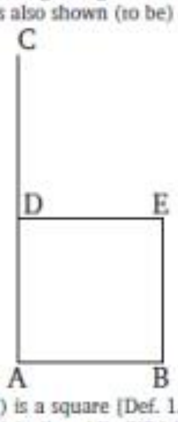
Follow the instructions given in the right column to make the geometrical construction of the description of a square on a straight line

Proposition 46

To describe a square on a given straight-line.

Let AB be the given straight-line. So it is required to describe a square on the straight-line AB.

Let AC have been drawn at right-angles to the straight-line AB from the point A on it [Prop. 1.11], and let AD have been made equal to AB [Prop. 1.3]. And let DE have been drawn through point D parallel to AB [Prop. 1.31], and let BE have been drawn through point B parallel to AD [Prop. 1.31]. Thus, $ADEB$ is a parallelogram. Therefore, AB is equal to DE, and AD to BE [Prop. 1.34]. But, AB is equal to AD. Thus, the four (sides) BA, AD, DE, and EB are equal to one another. Thus, the parallelogram $ADEB$ is equilateral. So I say that (it is) also right-angled. For since the straight-line AD falls across the parallels AB and DE, the (sum of the) angles BAD and ADE is equal to two right-angles [Prop. 1.29]. But BAD (is a) right-angle. Thus, ADE (is) also a right-angle. And for parallelogrammic figures, the opposite sides and angles are equal to one another [Prop. 1.34]. Thus, each of the opposite angles ABE and BED (are) also right-angles. Thus, $ADEB$ is right-angled. And it was also shown (to be) equilateral.



Thus, $(ADEB)$ is a square [Def. 1.22]. And it is described on the straight-line AB. (Which is) the very thing it was required to do.

• Draw a segment AB.
• Draw the AC line at right-angles to the AB segment at A point.
• On AC line take the AD segment equal to AB.
• Draw the DE segment parallel to the AB. • Repeat the previous step from B to E point.
• The ADEB is a square and it is described on a straight line AB.

Can you explain why this is true?
(write a brief proof)