THE DROP-TOWER

Mirabilandia Park houses, since 1997, two towers built by the American attraction company S&S Power. Today they're called Oil Towers and they are situated in the 'Far West Valley', a Far West- themed area. They're the seconds highest towers in Italy, about 60 meters, and the firsts to be built in our country; they are dark-green metal squared towers connected together in their highest point.

Oil Tower 1 is a space shot tower: that means it's composed by a pneumatic lift which suddenly acceletates from the bottom to the top, launching the passengers in the air to a maximum speed, about 80 km/h. When it reaches the top, it stops rising and the lift starts to swing up and down before returning to the ground.

The passengers sit on the 4 sides of the tower, staying with their legs dangling, 3 people per side: the maximum capacity is 12 passengers.
One ride takes about 35s.
Even if the tower is about 60 meters high, the highest point the lift reaches is about 47 meters.

The Screaming Eagle is a drop tower attraction from the type Shot 'n drop, it's situated in the Flemish amusement park Bellewaerde. The tower was built in 1999 by the German attraction company Huss.

It's a blue, metal square tower that's about 52 meters high. One ride takes around one minute. The orange seats can take 24 people in a ride.

Instead of classic drop towers this one first launches the passengers in the air to a maximum speed. Then it drops the seats down to earth, one to two times in a ride. The tower has four sides with six seats on each side, so it has a maximum capacity of 24 people. One ride takes between 25-50 seconds, depending on how busy it is. The maximum speed is between 65 and 70km/h.

It takes between 25 and 40 seconds to fill all the seats with passengers, this depends on how much passengers there are. The tower is about 52 meters high but the highest point on which the passengers will get is 38 meters. The distance between the highest point and the lowest point is 36 meter. When the seats are going up for the first time the acceleration is 12 m/s. The acceleration is 16 m/s when the seats are going up for the second time. The maximum g-force during the whole process is between three and four.

OIL TOWER 1

SCREAMING EAGLE

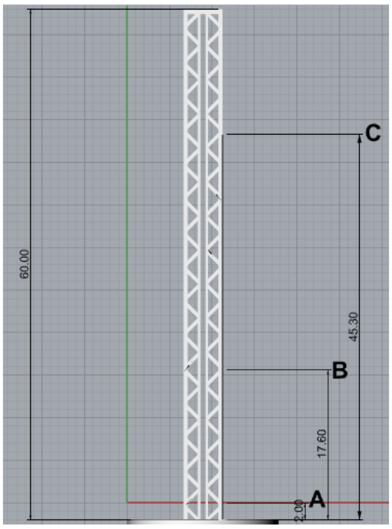
Do you dare to take a ride?

Oil Tower 1	Vs	**Screaming Eagle**
60m	Height	52m
45m	Max height	38m
80 km/h	Max speed	65 -75 km/h
2g	Acceleration	$12 m/s^2$ - $16 m/s^2$
12	n° passengers	24
35s - 40s	Time for a ride	25-50s

Oil Tower 1 gives you more adrenaline than the Screaming Eagle, because it reaches higher height, speed and the acceleration is bigger.

Speed and Acceleration

FIRST SHOT



We've chosen a coordinate system in the start point height, about 2 meters from the ground (A).

(B) is the point where the lift has the maximum speed

(C) is the point where the lift has reached the top of the tower (maximum height).

A. $h_A = 0m$
$v_A = 0m/s$

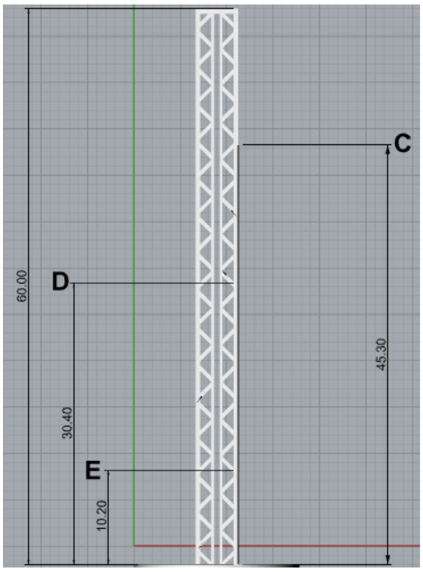
B. $v_{max\ B} = 80\ km/h \approx 22{,}2\ m/s$
$\Delta t_{A\text{-}B} \approx 1{,}4s$
$a_{m\ A\text{-}B} = \dfrac{22{,}2m/s}{1{,}4s} \approx 15{,}9\ m/s^2$
$a_{max\ A\text{-}B} \approx 2g$
$h_B = ½ \cdot a_{m\ A\text{-}B} \cdot (\Delta t_{A\text{-}B})^2 = ½\ 15{,}9m/s^2 \cdot (1{,}4s)^2 \approx 15{,}6\ m$ $+2m \approx 17{,}6m$

The lift reaches the maximum acceleration in $\Delta t \approx 0{,}25s$, then the acceleration decreases in $\Delta t \approx$ 1,2s.

C. After the lift reaches h≈15,6m, it starts to decelerate.
$v_C = 0m/s$
$\Delta t_{B\text{-}C} \approx 2{,}5s$
$a_{max\ B\text{-}C} \approx -15m/s^2$
$a_{m\ B\text{-}C} = \dfrac{v_C - v_{max\ B}}{\Delta t_{B\text{-}C}} = \dfrac{(0 - 22{,}2)m/s}{2{,}5s} \approx -8{,}9m/s^2$
$h_C = h_B + v_{max\ B} \cdot \Delta t_{B\text{-}C} + ½ \cdot a_{m\ A\text{-}B} \cdot (\Delta t_{A\text{-}B})^2 = 15{,}6m + 22{,}2m/s \cdot 2{,}5s - ½\ 8{,}9 \cdot (2{,}5s)^2 \approx 43{,}3m$

$h_{max} = h_C = 43{,}3m$ $+2m \approx 45{,}3m$

FIRST FALL



We've chosen a coordinate system in the start point height, about 2 meters from the ground (A).

(D) is the point where the lift starts to slow down.

(E) is the lowest point reached by the lift in the first fall.

C. $h_C = 43{,}3m$ $+2m \approx 45{,}3m$
$v_C = 0m/s$

D. $\Delta t_{C\text{-}D} \approx 2{,}5s$
$h_D \approx 28{,}4m$ $+2m \approx 30{,}4m$
$a_{max\ C\text{-}D} = -15m/s^2$
$a_{m\ C\text{-}D} = \dfrac{2 \cdot \Delta h_{D\text{-}C}}{\Delta t_{C\text{-}D}^2} = \dfrac{2 \cdot (28{,}4 - 43{,}3)m}{(2{,}5s)^2} \approx -4{,}8\ m/s^2$

$v_D = a_{m\ C\text{-}D} \cdot \Delta t_{C\text{-}D} = -4{,}8\ m/s \cdot 2{,}5s \approx -11{,}9m/s$

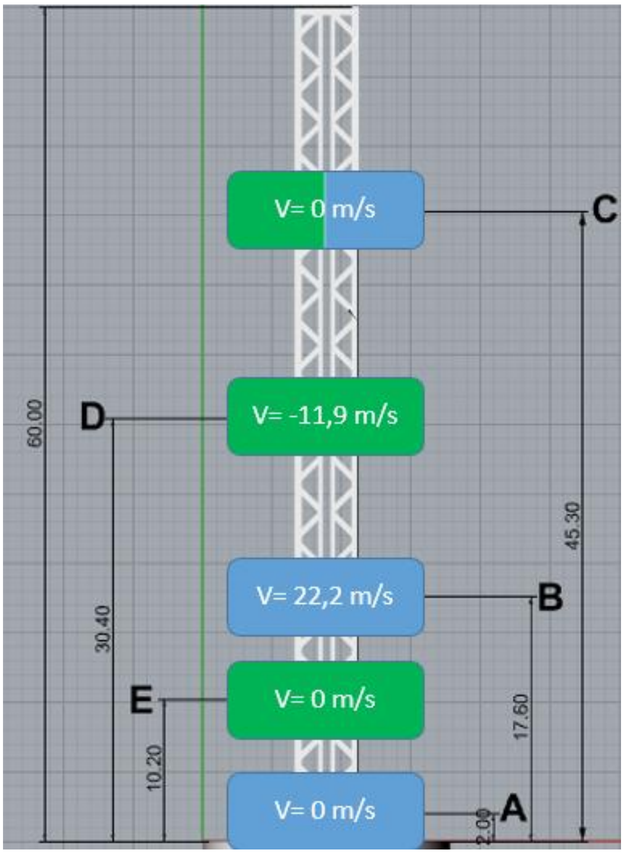
E. $\Delta t_{D\text{-}E} \approx 1{,}4s$
$a_{max} \approx 2g$
$h_E \approx 8{,}5m$ $+2m \approx 10{,}2m$
$v_E \approx 0m/s$
$a_{m\ D\text{-}E} = \dfrac{v_E - v_D}{\Delta t_{D\text{-}E}} = \dfrac{(0 + 11{,}9)m/s}{1{,}4s} \approx 8{,}5m/s^2$

Summary of speed and acceleration part

	A	B	C	D	E
V	0 m/s	22,2 m/s	0 m/s	11,9 m/s	0m/s
H	2m	17,6 m	45,3 m	30,4 m	10,2 m
A_{med}	15,9 m/s^2	-8,9 m/s^2	-4,1 m/s^2	8,5 m/s^2	

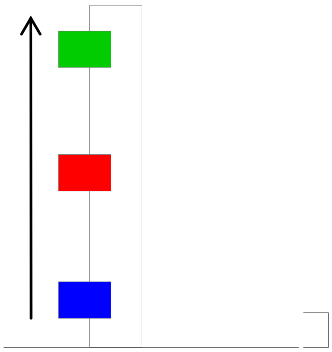
First Shot

First Fall



Energy and Work

Now we have to discuss the different forms of the energy throughout the rollercoaster.



2 m

Thanks to a graphic we've found about the rollercoaster, we've calculated the rollercoaster reaches the highest speed (22,2 m/s) when it's 17,6 m for from the reference system (that is 2m upper than the land).

Energy in B and C

$Etot_C = Etot_B$

$Ep_C = Ep_B + Ecin_B$

$\gamma mh_{AC} = \gamma mh_{AB} + ½\, mV_B^2$

$9{,}81 \cdot 43{,}3 = 9{,}81 \cdot 15{,}6 + ½ \cdot 22{,}2^2$

$424{,}78 \cdot m = 399{,}95 \cdot m$

We can overlook the FRICTION as we've demonstrated that. The mechanic energy doesn't change (424,78 ≈ 419,08). That doesn't mean the FRICTION is absent; quite the opposite there is, but, as vry big forces work, we can say it's almost 0.

A - B = CONSERVATIVE FORCE = $Em_f – Em_i = Em_B – Em_B$

From A to B there is a force who works, as $Em_A = 0$ and $Em_B = 399{,}95$ J/Kg · m

So $LF_{AB} = 399{,}95$ J/Kg · 2000 Kg = 799900 J = $8{,}0 \cdot 10^5$ J L/x = F
$F_{AB} = LF_{AB} / AB$ = 799900 J / 17,6 m = 45 KN (overage force that makes the rollercoaster speed up from m/s to 22,2 m/s)

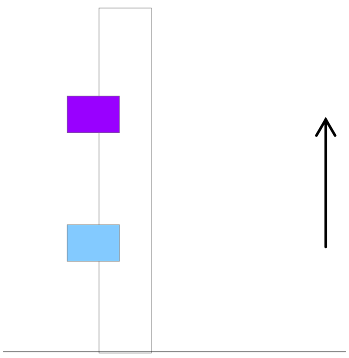
Em_D = 349,41 J/Kg · m = 349,41 J/Kg · 2000 Kg = 698820 J

$Em_E = Ep_E$ because $V_E = 0$ m/s $Em_E = \gamma mh_E$ = 9,81 J/Kg m · 2000 Kg · 8.2 m = 160884 J

D - E = NON CONSERVATIVE FORCE = $Em_i - Em_f = Em_D – Em_E$

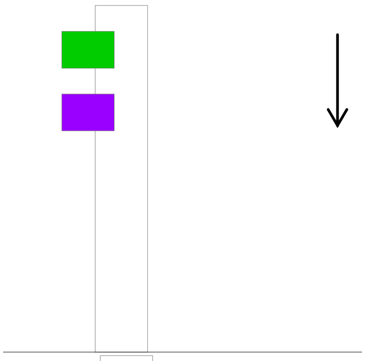
$LF_{DE} = Em_D – Em_E$ = (698820 – 160884) J = 537936 J = $5{,}4 \cdot 10^5$ J

$F_{DE} = LF_{DE} / DE$ = 537936 J / (28,4 – 8,2)m = 2 KN FRICTION FORCE
(+ Because it stands opposite o motion with negative speed)



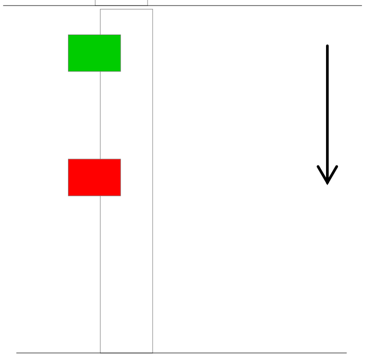
D-E

Non conservative force
(Friction force that makes the rollercoaster stop from D to E)



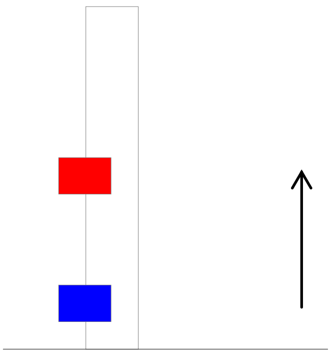
C-D

Total
(force that pushes down the r.c.)



B-C

Non conservative force
(FRICTION / BRAKING)



Conservative force
(Force applied by the rollercoaster)

B – C → non conservative force → $Em_I - Em_f = Em_B - Em_C$

$Em_B = 399{,}95 \cdot m$

$Em_C = 424{,}78 \cdot m$

$LF_{BC} = Em_B - Em_C = (399{,}95 - 424{,}78)\ m = -24{,}83\ J/Kg \cdot m$

Let's esteen the mass: it's approximately 2000 Kg

So $LF_{BC} = -24{,}83\ J/Kg \cdot 2000\ Kg = -49600\ J \approx -50\ KJ$

$F_{BC} = LF_{BC} / BC = -49660\ J / (43{,}3 - 15{,}6)\ m \approx -1{,}8\ KN$ (FRICTON FORCE* from B to C)
it makes the rollercoaster stop

C – D → conservative force → $Em_f - Em_I = Em_D - Em_C$

$Em_C = 424{,}78 \cdot m$

$Em_D = \gamma m h_D + ½\ m\ |V_D|^2 = m\ (9{,}81\ J/Kg\ m \cdot 28{,}4\ m + ½ \cdot 11{,}9^2\ m^2/s^2) = 349{,}1\ J/Kg \cdot m$

$LF_{CD} = Em_D - Em_C = (349{,}41 - 424{,}78)\ m = - 75{,}37\ J/Kg \cdot m$

$m = 2000\ Kg$

So $LF_{CD} = - 75{,}37\ J/Kg \cdot 2000\ Kg = - 150740\ J \approx - 1{,}5 \cdot 10^5\ J$

$F_{CD} = LF_{CD} / CD = - 150740\ J / (43{,}3 - 28{,}4)\ m \approx - 10\ KN$ (conservative force that makes the rollercoaster have negative** speed in point D)

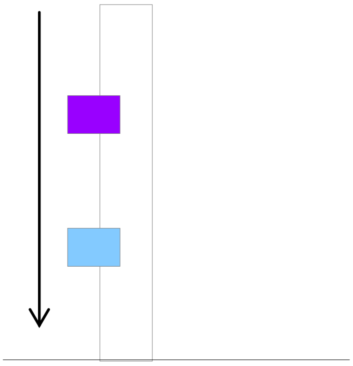
$A\ m = F_{CD} / m = - 10116{,}77852\ N / 2000\ Kg = - 5{,}06\ Kg\ m/s^2 \cdot 1/Kg = - 5{,}06\ m/s^2$

$A\ gravity = - 9{,}81\ m/s^2 \rightarrow (9{,}81 + 5{,}06)\ m/s^2 = - 4{,}75\ m/s^2\ (\Delta_{ACC})$

$Fatt_{CD} = a \cdot m = - 4{,}75\ m/s^2 \cdot 2000\ Kg = 9500\ N$ (is positive because it stands opposite a motion that has negative speed)

$Latt_{CD} = Fatt_{CD} \cdot CD = 9500\ N \cdot (43{,}3 - 28{,}4)\ m = 141550\ J$ (is positive because it makes the speed in D be less negative).

*is a non conservative force
**because it works close to s.d.r.



D is a point the rollercoaster passes through when it goes down after having reached point C $\rightarrow h_D = 28{,}4\ m$
$V_D = - 11{,}9\ m/s$

E is the lowest point the rollercoaster reaches after having reached the highest one (C) $\rightarrow h_E = 8{,}2\ m$
$V_E = 0\ m/s$