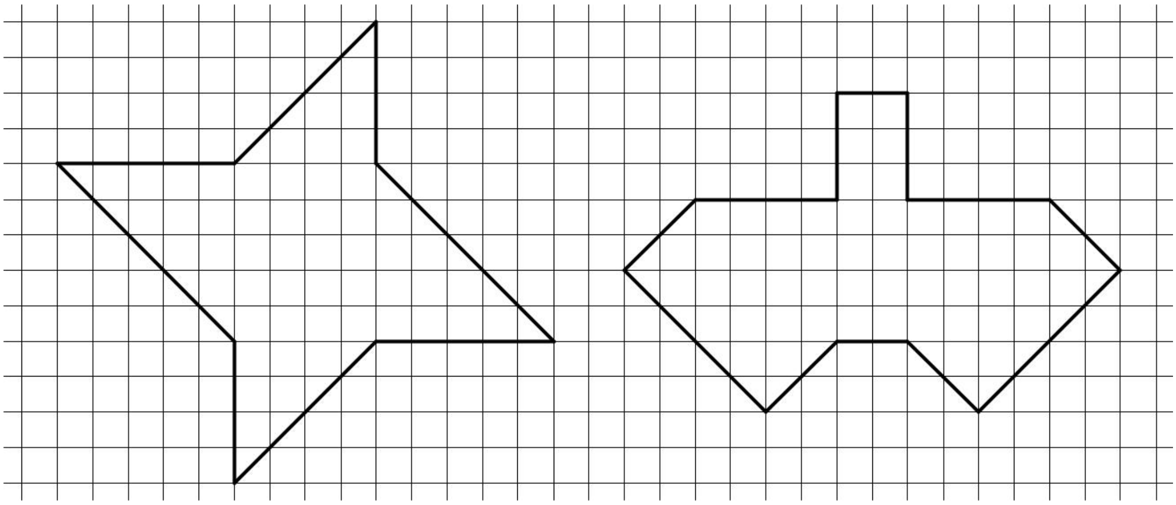
5. THE LEAVES FALL DOWN (cat. 3, 4, 5)

They decided to decorate the walls of the school gym with green and yellow cardboard leaves to celebrate autumn. Here is the model of the leaves.



Do you need more cardboard for the green or for the yellow leaf?
Explain your answer.

ANALYSIS A PRIORS

Conceptual scope

- Arithmetic: organized counting
- Geometry: square, triangle as half of a square
- Sizes and measure: "measure" of the area with appropriate units of measure or comparison by overlapping

Analysis of the task

- Understand that it is necessary to compare the areas of the two figures
- Count the squares and the half-squares of each figure one by one (method subject to counting errors), then make the necessary exchanges to count them according to a common unit of measurement: in squares (or semi-squares) , 61 (122) for the green figure and 62 (124) for the yellow. (In case the "pieces" were counted, without taking into account their respective areas, 70 pieces would be obtained for the green figure and 70 for the yellow one).
- Or: with a proper decomposition of the figures in rectangles, squares and triangles, realize that the triangles are half of squares.
- Then rebuild the "whole" squares to count the squares. For example, note that the green figure is made up of two triangles that make up a 4 x 4 square, two triangles that make up a 5 x 5 square and a 5 x 4 rectangle, for an area of 16 + 25 + 20 = 61 (in squares). In the same way, the parts of the yellow figure can be grouped into four squares of 2 x 2 and two rectangles of 4 x 10 and 2 x 3 to obtain an area of 40 + 6 + 4 x 4 = 62 (in squares).
- Or: proceed by cutting one of the two leaves to cover the other and note that a piece is missing (or advancing) and conclude that the yellow leaf has a larger area.

This problem can induce the conflict area / perimeter and the given response by measuring the length of the sides, which would mean that the green figure is the largest. But also a confusion between area and "encumbrance" (greater internal length, for example between two vertices of the figure) would lead to the same result.

Attribution of the scores

4 Correct answer (it takes more yellow cardboard or the yellow figure is the bigger one) with the details of the strategies used (calculations, cropping, equivalent parts highlighted ...)
3 Correct answer with incomplete explanations but from which The correct procedure is shown
2 Answer with errors in the count but show that the procedure is correct (for example with difficulties in the transformation of the squared means into squares)
1 Correct start of reasoning, but without the "calculation" of the areas or correct answer yellow
0 without any explanation or detail of calculations

2. THE CHICKEN POX (cat. 3, 4) ©ARMT 2014 – 22nd – final phase

In Anna's classroom there are 4 girls more than boys.
Today a half of the girls and a half of the boys are ill and did not come to school because of an epidemy of chicken pox.
In the classroom there are only 14 students.
How many boys and girls are ill?
Explain your procedure.

PRIOR ANALYSIS

mathematical task
- Find 2 numbers, whose sum is equal to 14 and the difference of double which is equal to 4.

Analysis of the task
- Understanding the arithmetical constraints of the problem (see mathematical task).
- The following strategies are faster or cheaper if the pupils realize that the total numbers of the females and males are even (so that half can be taken).
- Strategy for testing and correcting numbers that respect the constraints set out below: proof (hypothesis) that respects the 4 students of difference, calculation of half of each number (sick or present), addition of the remains and verification
to know if this sum is 14.
- Strategy for inventory of cases, for example starting the list from 2 males and then from 6 (2 + 4) females e verification of the 2nd condition (this organization may appear, but hardly in the form of a table), for example:

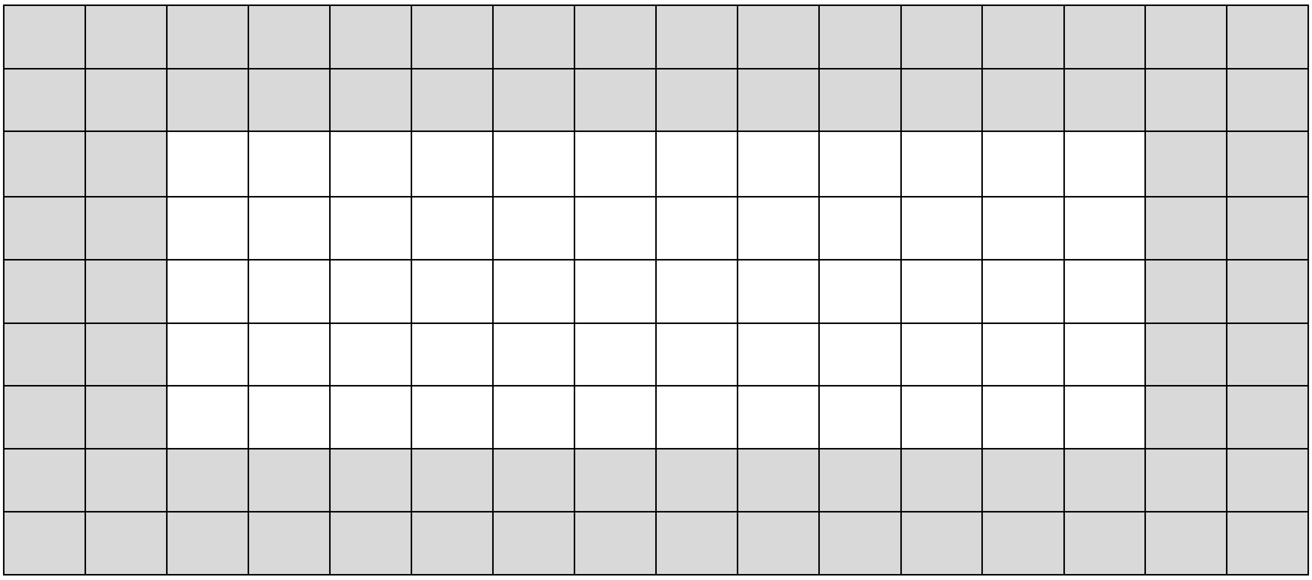
Male	Females	Half of males	Half of females	Sum of halves
2	6	1	3	4
4	8	2	4	6
6	10	3	5	8
…	…	…	...	…

The inventory can stop when the sum 14 is reached, observing that the continuation of the sums is increasing.
- Proceed in the same way, but starting from the sick (or present) pupils and considering that there are 2 females of more than males.
- Understand that in each of the two halves of pupils, the number of females exceeds the number of males by two, since subtracting 2 from 14 we obtain twice the number of males present.
Consequently, the calculation (14 - 2): 2 = 6 gives the number of males present and 6 + 2 = 8 the number of females present.
- Reasoning starting from the initial number of pupils (28 = 14 x 2). By subtracting the 4 females more, you get double of the number of males (24 = 28 - 4). Deduce the number of males (12 = 24: 2) and that of females (16 = 12 + 4), then the number of sick males and females (half of the previous numbers).
This reasoning can also lead to this series of calculations: (28 - 4): 2 = 12, 12 + 4 = 16, 12: 2 = 6, 16: 2 = 8.

Solution
The two correct answers (8 sick females, 6 sick males) with clear and detailed explanations.

4. THE CARPET OF MRS SHOWER (cat. 3, 4, 5) ©ARMT 2014 – 22nd – final phase

Mrs Theshower has a nice carpet in her bathroom, with two stripes of grey squares along the edge and a white central part. Here is a picture of the carpet:
Mrs Theshower would like to buy a new carpet.
The new carpet must have the same length of the old one.
But the width of the white part of the new carpet has to be double the white part of the old carpet.
Moreover, she wants that the new carpet has two stripes of grey squares along the edge, like the first carpet.



How many grey squares will be on the new carpet?
Explain your answer.

PRIOR ANALYSIS
Mathematical task
- Given a rectangle surrounded by two strips of squares, find how many squares make up the double strips that surround a rectangle of the same length as the first and double width.

Analysis of the task
- First of all understand that the white part of the new carpet doubles only in the direction of the width.
- A first strategy is to construct or imagine the new rectangle and to find a counting method of gray squares.
- To do this, after observing the drawing, determine that the white part of the new carpet must have the double the width of the other, but the same length and therefore that the white part of the new carpet is a rectangle of 12 x 10 (ie 120 square) and that the new carpet is a rectangle of 16 x 14 (ie 224 square)
- To determine the number of edge squares, you can use:
- - a procedure for counting all the gray squares on a drawing;
- a procedure for counting gray squares imagining, for example, the new carpet surrounded by one first gray strip attached to the white part composed of 12 +12 +10 +10 + 4 squares (in reference to the «Perimeter»), then by a second strip consisting of 14 +14 +12 +12 + 4 squares for a total of 104 squares;
- A second strategy is to count the number of gray squares of the first carpet (same procedures as here) above, which gives 84 squares) and in counting the gray squares added in the "transformation" from the initial carpet into the final mat, the gray squares being added only on each of the widths, ie 10 squares (2 times 5) per
every width, therefore in total 20 gray squares more than the initial mat, ie 104 squares (84 + 20 = 104).

Solution Correct answer (104 gray squares) with clear and complete explanation

7. TOM AND LULU' (cat. 5, 6) ©ARMT 2014 – 22nd – final phase

Tom plays with some red and blue coins.
There are 12 red coins more than the blue ones.
His sister Lulù takes a half of the red coins and a half of the blue ones.
Tom counts the remaining coins and find they are 78.

How many red and blue coins did Lulù take?
Explain your procedure.

PRIOR ANALYSIS
Mathematical task
- Find 2 numbers, the sum of which is equal to 78 and the difference of double which is equal to 12.

Analysis of the task
- Understand the constraints of the problem: the number of tokens taken by Lulu is equal to the number of remaining tokens,
counted by Tom, the difference of the two numbers is equal to 12, the sum of the halves of the two numbers is equal to 78.
- The following strategies are quite quick or cheaper if the pupils realize that the number of each type of tokens must be even, since Lulu takes half of it.
- Strategy for testing and correcting numbers that respect the constraints of the problem set out below: attempt (hypothesis) that respects the 12 difference tokens, calculation of half of each number (tokens taken by Lulu or that remain), addition of the remains and verification to know if this sum is 78.
- Strategy for case inventories, for example starting the count from 2 for the blue tokens and then from 14 (2 + 12) for red tokens and verification of the second condition (this organization may appear, but certainly not in the form of a table). For example :

Blue Tokens	Red Tokens	Half of the blue	Half of the reds	Sum of the halves
2	14	1	7	8
4	16	2	8	10
6	18	3	9	12
--	--	--	--	--

The inventory can stop when the sum 78 is reached, observing that the succession of the sums is growing.
- Proceed in the same way, but starting with the tokens removed (or left) and considering that there are 6 remaining tokensmore reds than the remaining blue tokens.
- Understand that in each of the two halves of the tokens, the number of red chips exceeds 6 the number of blue chips, thenby subtracting 6 from 78, you get double the blue chips. Consequently the calculation (78 - 6): 2 = 36 gives the numberof the blue chips and 36 + 6 = 42 the number of red chips.
- Reasoning starting from the initial number of tokens (156 = 78 x 2). Subtracting the 12 extra red chips, we obtain thedouble the number of blue tokens (144 = 156-12). Deduct the number of blue tokens (72 = 144: 2) and that of thered chips (84 = 72 + 12), then the number of tokens taken by Lulu (half of the previous numbers), thisreasoning can also lead to the series of calculations: (156 - 12): 2 = 72, 72 + 12 = 84, 72: 2 = 36, 84: 2 = 42
Solution
The two correct answers (36 blue, 42 red) with clear and detailed explanations

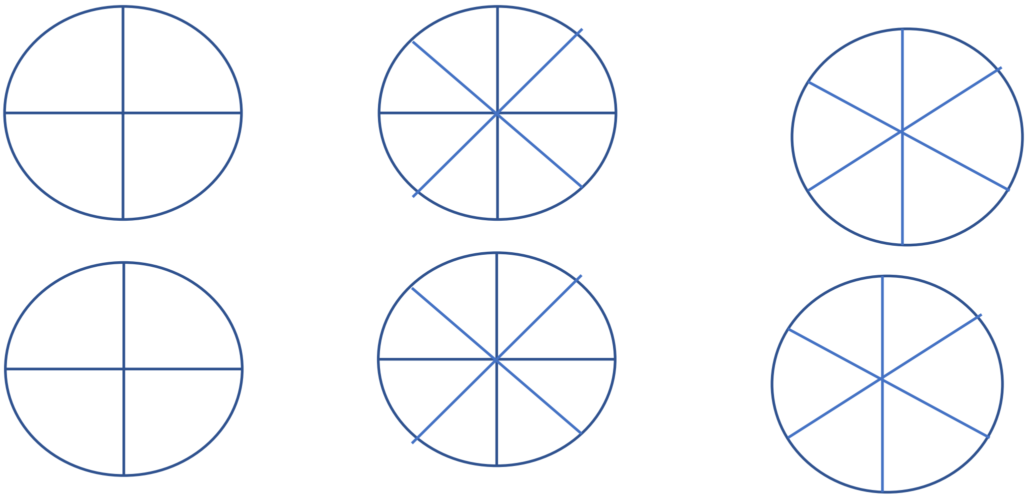
10. SLICES OF CAKE (cat. 6, 7, 8) ©ARMT 2014 – 22nd – final phase

Eight friends order six cakes for their break. The baker gives them two strawberry cakes, two apple cakes and two kiwi cakes. All the pies have the same size but the strawberry pies are already divided into four equal parts, the apple pies are divided into six equal parts and the kiwi ones are divided into eight equal parts.

The friends decide to eat the same quantity of cake each, without cutting other slices. Each guy wants to eat two different types of cake. Since they are gluttonous, not a single slice of cake remains.

How could the eight friends share the slices of the cakes?

Indicate all the possibilities you obtain and explain your procedure.



PRIOR ANALYSIS

Mathematical task

Find all the ways to divide 8 quarters, 12 sixths and 16 eighths into eight equivalent parts, each consisting of two types of fractions.

Analysis of the task

- Understand that the pies must be divided into 8 equal portions, taking slices of two cakes for each portion.

Reasoning that uses the calculation on fractions:

- Since the 6 cakes have been entirely consumed, everyone will have eaten 6/8 of cake.
- It is therefore necessary to obtain 6/8 adding both the quarters and the sixths, and the quarters and the eighths, both of the sixths and the eighths.
- The simplest reasoning consists in trying to complete one or more parts of each cake to get some portions of 6/8 (or 3/4) of cake.
- Complete 1/4 (equal to 2/8) with 4/8 (or 1/2). Then you can have: $1/4 + 3 \times 1/6$ or $1/4 + 4 \times 1/8$.
- Complete $2 \times 1 / 4$ (equal to 1/2 or 4/8) with $2 \times 1 / 8$. Then $2 \times 1/4 + 2 \times 1/8$.
- Complete 1/6 or 2/6 with the eighths to get 6/8 is not possible.
- Complete 3/6 (equal to 1/2) with the eighths: $3 \times 1/6 + 2 \times 1/8$.
- You can then get 6/8 of cake in four different ways, taking into account that each friend takes only two types of cake

11. NOT SO EASY… (cat. 6, 7, 8) ©ARMT 2014 – 22^{nd} – final phase

The Maths teacher proposes a riddle to his students:
Using three times number 5 and once number 1 you must obtain 24 making additions, subtractions, multiplications or divisions.
For example:
(5 + 1) x (5 – 1) = 24 does not work because there are two number 5 and two number 1.
(5 x 5) – 1^5 = 24 again does not work because 1^5 is not among the authorized operations.
But you can be sure that a solution exists.
What is the solution of the riddle proposed by the teacher?
Explain your procedure.

PRIOR ANALYSIS
Mathematical task
- Obtain 24 starting from four numbers 5, 5, 5 and 1 with the arithmetic operations: addition, subtraction, multiplication and division.

Analysis of the task
- Make attempts, possibly using a calculator, and convince yourself that there is no solution remaining in the set of natural numbers.
- Try, starting from 24, to add, remove or divide by 5 and reason on the numbers obtained using again twice the number 5 and once the number 1.

Or
- realize that the only way to obtain a decimal number with a single operation starting from numbers 5 and 1 is to divide 1 by 5, which gives 0.2.

Continue trying to get 24 starting at 0.2 using the number 5 twice. Find that the only way to to get there is to make (5 - 0.2) × 5 = 4.8 × 5 = 24.

Solution
The correct expression, (5 - 1/5) × 5, with explanations on the search procedure

15. THE SPRING SCALE (cat. 8, 9, 10) ©ARMT 2014 – 22nd – final phase

A spring scale is formed by a spring with a hook at one end and fixed on a support on the other end. When you hang a weight on the hook the spring lengthens.
The lengthening of the spring is proportional to the weight of the object hung.
In the scheme the scale is first represented without any weight and its length is *l*, then with a weight and its length is *a*, then with two weights and its length is *b*, which is double of *a*.
The spring of a scale A, without any weight, is 10 cm long. When you hang on object weighting 3 kgs, its length is 16 cm.
The spring of a scale B, without any weight, is 5 cm long. When you hang on object weighting 2 kgs, its length is 11 cm.
When you hang a new object to the scale A or the scale B, the springs of the two scales have the same length.
Find out the weight of an object so that the springs of the scales A and B have the same length when the object is hung to one scale or the other.

The scale without a weight	**with a weight**	**with two weights**

Which length will the springs have with the weight hung?
Explain your answers.

PRIOR ANALYSIS

Mathematical task

- In a context of spring balances, whose elongation is a linear function of the weight, determine the weight for which the springs of two scales in extension, with different stretching ratios, reach the same length.

Analysis of the task

- Understand the operation of a spring balance (or dynamometer): the spring, its mechanical characteristics, its length, elongation and proportionality relation: elongation = force × "coefficient of elongation ", between the lengthening of the spring and the weight suspended thereon. – Determine how the length of each scale changes according to the weight: initial length + elongation o initial length + force × "elongation coefficient".
- For scale A, the elongation is 16 - 10 = 6 (cm) for 3 (kilograms force), so the coefficient is 2 (cm / kilogram-force), while for balance B, we have: 11 - 5 = 6 (cm) for 2 kg, so the coefficient is 3 (cm / kilogram-force).
- Deduct the lengths of the two springs (in cm) according to a weight P (in kilogram-force): L = 10 + 2P for the balance A, L = 5 + 3P for the scale B.
- To find the weight that allows you to get the same length for the two scales, there are different ways of proceed:
    - With algebra: consider equal the two lengths and obtain the equation in P: 10 + 2P = 5 + 3P, then deduce that the weight P is equal to 5 kg and that the length of the scales A and B is then 20 cm;
    - Graphically in representing the two linear functions with two lines, identified by two points (0, 10) and (3, 16) for A and (0, 5) and (2, 11) for B, whose intersection has for coordinates: L = 20 and P = 5

With a table writing the lengths of the springs according to the weights using the proportionality of the elongation:

	0	1	2	3	4	5	6	7	8
Elongation of the spring A	0	2	4	6	8	10	12	14	16
Length of the springA	10	12	14	16	18	20	22	24	26
Elongation of the spring B	0	3	6	9	12	15	18	21	24
Length of the spring B	5	8	11	14	17	20	23	26	29

With a table writing the lengths of the springs according to the weights using the proportionality of the
elongation:

Observe that for a weight of 5 kg, the two springs measure 20 cm. Solution Correct answers (5 kg and 20 cm) with clear and complete explanations

16. A NEW CAR (cat. 8, 9, 10) ©ARMT 2014 – 22nd – final phase

The new car RMT22 has the same price in all the countries where it has to be sold.
A rich American man decides to buy three of them to give his nephews as a present. His nephews live in different countries. The American man buy a car in Italy where he pays 21% VAT over the price; he buys another car in France where the VAT is 20%.
He spends 22,413 euros for the two cars.
He buys the third car in a Transalpine country, where he just spends 10,044 euros, VAT included.
What is the VAT in the Transalpine country?
Explain your procedure.

PRIOR ANALYSIS

Mathematical task

- Determine the basic price of a car without VAT, which is the same in the three countries, starting from the information on the its price with VAT in Italy and France and deduct the percentage of VAT in Transalpino.

Analysis of the task

- Proceed through attempts at a plausible basic cost for the car (for example € 10,000). The application of the 21% in the first case and 20% in the second case, gives a total of € 24 100.
- Noting that the base price is lower, make other attempts in order to reduce the price of 50 € in 50 €, for example, then finish to make the calculation with a base price of € 9,300.
- deduct that VAT in the country of Transalpino reaches € 744 and that this corresponds to 8% of the basic cost of the car (744/9 300 = 0.08)

Or, use for Italy and France the average value of 20.5% VAT, divide the price paid 22 413 € for 2 (average price of a car) and divide by 1,205 to get its base price: € 9300.

Or, use an equation of the form x + 0.21 x + x + 0.20 x = 22 413 or 1.21 x + 1.20 x = 22 413 and deduce that x = € 9 300, then, to find the VAT of Transalpine (y), it solves the equation 9 300 (1 + y) = 10 044 from which y = 0.08 or 8%.

Solution

Correct answer (8%) with clear and complete explanation

18. AFTER 2013! (cat. 9, 10) ©ARMT 2014 – 22nd – final phase

Here are the first four terms of a long series, formed this way:

$$\frac{1}{1} \leftrightarrow \frac{1}{2}; \frac{1}{2} \leftrightarrow \frac{1}{3}; \frac{1}{3} \leftrightarrow \frac{1}{4}; \frac{1}{4} \leftrightarrow \frac{1}{5} \ldots.$$

Calculate the sum of these terms until the 2013th:

$$\frac{1}{1} \leftrightarrow \frac{1}{2} + \frac{1}{2} \leftrightarrow \frac{1}{3} + \frac{1}{3} \leftrightarrow \frac{1}{4} + \frac{1}{4} \leftrightarrow \frac{1}{5} + \frac{1}{5} \leftrightarrow \frac{1}{6} + \cdots + \frac{1}{2013} \leftrightarrow \frac{1}{2014}.$$

Then multiply the results by 2014.

Which number did you find?

Explain your procedure.

PRIOR ANALYSIS

Mathematical task

Calculate the sum of the first 2013 terms of the succession $\frac{1}{1} \leftrightarrow \frac{1}{2}; \frac{1}{2} \leftrightarrow \frac{1}{3}; \frac{1}{3} \leftrightarrow \frac{1}{4}; \frac{1}{4} \leftrightarrow \frac{1}{5} \ldots.$ .. and multiply it by 2014

Analysis of the task

- Observe the first terms of the succession, understand the rule of construction and generalize to find out the 2013th term: $\frac{\mathbf{1}}{\mathbf{2013}} \frac{\mathbf{1}}{\mathbf{2014}}$ understand that each term is the product of two fractions with numerator 1 and the denominators are two numbers one after the other and that the term order n is given by the product

- Then switch to the sum: $\frac{1}{1} \leftrightarrow \frac{1}{2} + \frac{1}{2} \leftrightarrow \frac{1}{3} + \frac{1}{3} \leftrightarrow \frac{1}{4} + \frac{1}{4} \leftrightarrow \frac{1}{5} + \frac{1}{5} \leftrightarrow \frac{1}{6} +$ ... and realize that it is not possible explicitly write the sum of the 2013 terms of the succession and that a rule must be found generalizable: it is observed that

the first term is $\frac{1}{1} . \frac{1}{2} = \frac{1}{2}$

The second one $\frac{1}{2} . \frac{1}{3} = \frac{1}{6}$

, the third is $\frac{1}{2} . \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots$

… and the sum of others of these terms is: $\frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5}; \frac{5}{6}; \frac{6}{7} \ldots \ldots ..$ ...

Then calculate the succession of the partial sums at least of the first terms and simplify:
and then get the sequence:

1 that this new succession consists of fractions where the denominator exceeds 1 numerator
- So the 2013rd term of this succession will be

$$\frac{2013}{2014}.$$

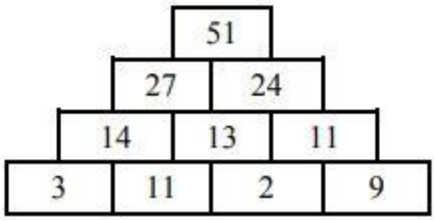
and its product for 2014 will be 2013

Solution

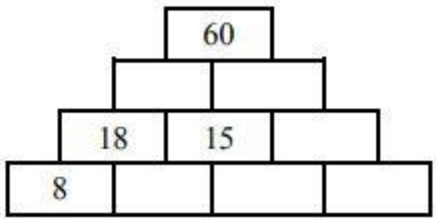
Correct answer (2013) with clear explanation or explicit calculations of the partial sums

3. PYRAMID OF BRICKS (I) (Cat. 3, 4, 5) ©ARMT 2013 – 21st - 1st phase

You can write a number on each brick of this pyramid, according to the following rule: the number on each brick is the sum of the numbers written on the two bricks below it.
For example: 14 is the number written on the brick out on the bricks 3 and 11, since 14 = 3 + 11.
51 is the number on the top brick, and it is the sum of 27 and 24.



Write the missing numbers to complete the pyramid below, following the same rule.

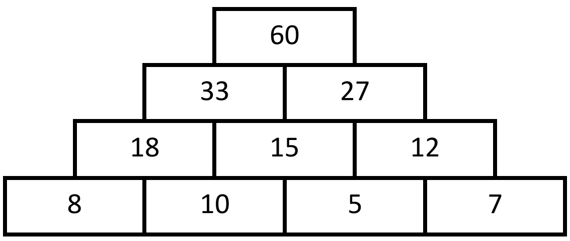


Write the calculation you performed to find the missing numbers.

PRIOR ANALYSIS
Conceptual scope
- Arithmetic: addition and subtraction with natural numbers

Analysis of the task

- Understanding the operation of the pyramid.
- Find the two numbers that can be completed directly using the numbers in the pyramid: for 10 with a subtraction (18 - 8 = 10) or a lacunar addition (8 +? = 18); for 33 with an addition (18 + 15 = 33).
- Continue in the same way for the other four bricks, with a series of lacunar or subtraction additions: for example: 27 = 60 - 33; 10 +? = 15; 15 +? = 27; 12 - 5 = 7. (For the first stages you have the choice between two bricks from complete, but the solution is unique)



Solution

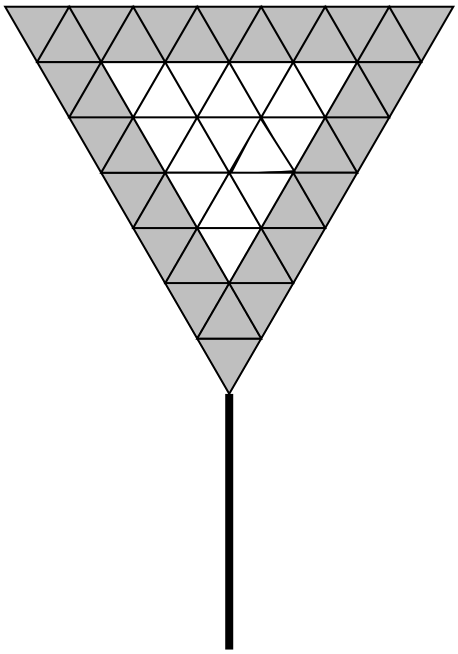
Complete solution (the six missing numbers, 33, 27, 10, 5, 12 and 7) with the six operations written correctly: addition 18 + 15 = 33, then subtractions 60 - 33 = 27 ... or completed lacunar additions: 33 + 27 = 60, etc.

14. WHAT A STRANGE SIGN! (cat. 7, 8, 9) ©ARMT 2005 – 13TH – 1ST phase

This triangular sign "give the precedence" is formed by little equilateral isometric triangles.
16 of them form an internal triangle, the other 33 form the external edge.

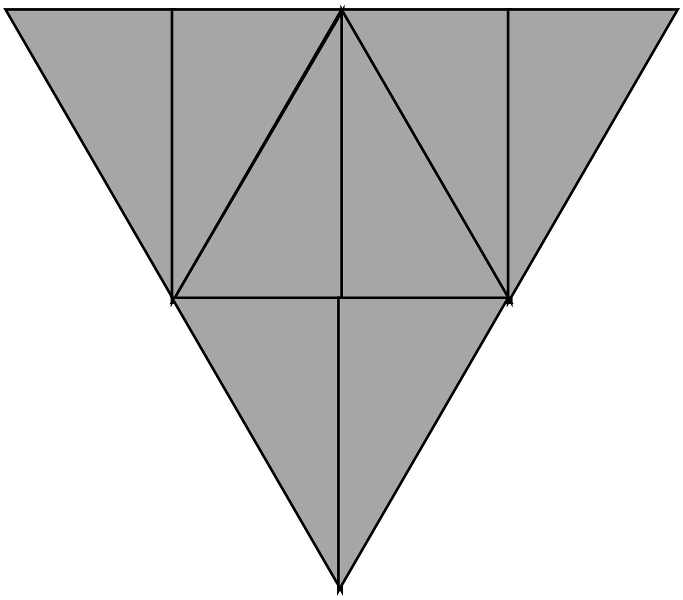
Can you build a new triangular panel with a different size, having the same width of the edge and the same number of internal and external triangles?

Explain the procedure you follow, and justify your answer.



5. THE SHAWL OF GRANDMOTHER (cat. 3, 4, 5) ©ARMT 2005 – 13TH – 1ST phase

Grandmother Piera realised a shawl with this pattern.
Her granddaughter Camilla says it is very beautiful because it has a lot of triangles.
She tries to count them but she is in trouble and never certain of the results.
How many triangles can you see in this picture?
Draw them so that it is possible to understand the way you counted them.



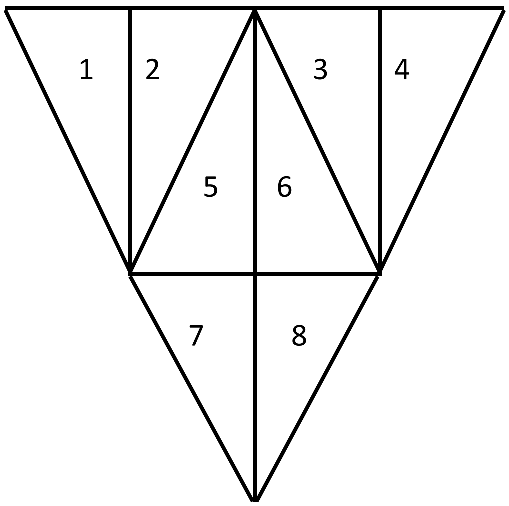
PRIOR ANALYSIS

Conceptual scope

- Geometry: identify triangles inside a complex figure
- Logic: proceed with a certain order in the counting of the figures

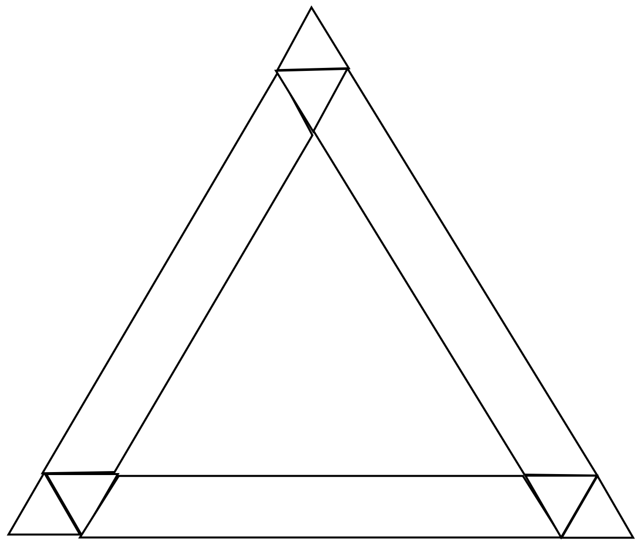
Analysis of the task

- Realize that you can see other triangles over the 8 "small" rectangle triangles that make up the figure.
- Identify and count triangles formed by two «small» triangles. There are 6 in all, of two types: 4 equilateral triangles (3 in the upper part of the figure, one - overturned - in the center)
2 isosceles triangles in the center (symmetrically positioned, with the long vertical side)
- Identify the two rectangle triangles formed by 4 «small» triangles.
- Consider the "big triangle" that delimits the figure.
- Calculate the total 8 + 6 + 2 + 1 = 17.
- Indicate the 17 triangles: using different colors on different figures
(in fact with this procedure it is not possible to distinguish them on one
single design), through letters placed at the top, through numbers, etc.



For example:
the 8 «small»: 1, 2, 3, 4, 5, 6, 7, 8,
the 6 triangles formed by two «small»: 1-2, 3-4, 5-6, 7-8, then 5-7, 6-8
the 2 triangles made up of four «small»: 1-2-5-7, 3-4-6-8,
the "big" triangle: 1-2-3-4-5-6-7-8

9. SO MANY PARALLELOGRAMS! (cat. 5, 6, 7) ©ARMT 2014 – 22nd – final phase

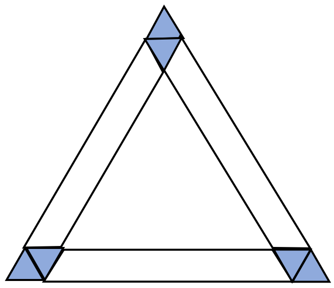


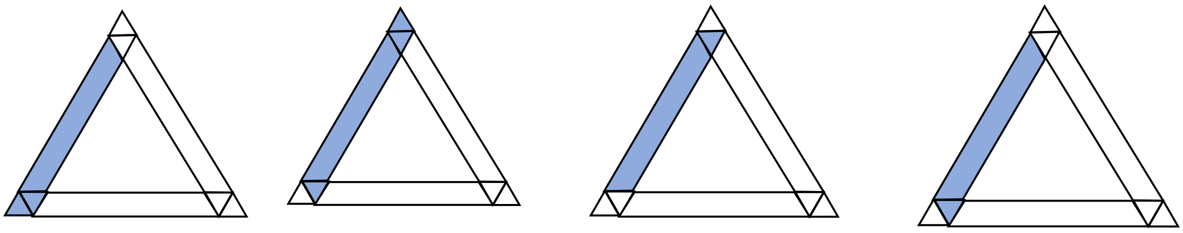
All the triangles you can see in this picture are equilateral.

How many parallelograms can you see in the picture?

Show and describe them.

PRIOR ANALYSIS
Mathematical task
- Recognition and counting of parallelograms in a figure.
Analysis of the task
- Knowing how to recognize parallelograms in a figure, taking into account the regular character of the figure for identify equal parallelograms;
- organize themselves so as not to forget any parallelograms and not to count twice the same.
- Take advantage of the fact that in a parallelogram the opposite sides are parallel.
- Choose two pairs of parallel sides in order to form a quadrilateral that will necessarily be a parallelogram.
- Consider that the rhombus is a parallelogram.
There are evidently many ways of organizing the inventory, with numerous risks of confusion or forgetfulness:
- assign a letter to all the vertices of the figure (or to the segments) and designate the parallelograms with these vertices (or these segments), which leads to a heavy and long notation, difficult to control,
- use colors to revise the contours, which does not allow to distinguish the various segments,
- work for types of parallelograms in a different way from those described so far, taking into account, for example, the transformations of the equilateral triangle ...
The main task is precisely that of choosing the most effective representation.
You get to:
- 3 parallelograms that are also rhombs (1 parallelogram family)





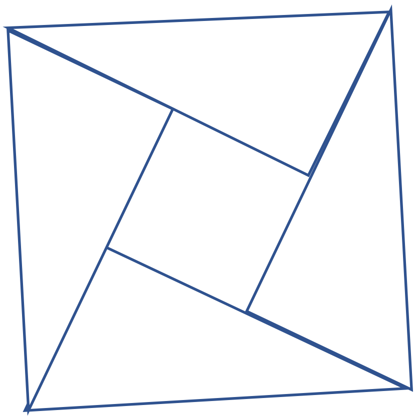
12 parallelograms (which are not rhombs),
3 families of 4 parallelograms, once on the left as in the figure
una a destra e una in basso.

Ci sono in tutto 3 + 12 = 15 parallelogrammi.

Solution
Complete answer (15) with complete description (drawing or other descriptions)

12. THE GRANDFATHER'S FIELD (cat. 6, 7, 8) ©ARMT 2009 – 1st phase

Grandfather gives to his five grandchildren a squared field divided in five parts: one square and four triangles; the side of the internal square must be equal to the length of the lesser catheti of the triangles (see the picture).



Do you think that the five parts have the same area?

Justify your answer.

PRIOR ANALYSIS

Conceptual scope

- Geometry: decomposition and recomposition of flat surfaces; area of the square and the triangle rectangle

Analysis of the task

- Observe the figure and note that, by construction, each of the four triangles is a rectangle (one of the angles, the whose vertex is common to that of the inner square, is rectum, like that of the square).
- Also note that the length of the major cathetus of each triangle is constituted by the side of the inner square and that, consequently, the greater cathete of the triangle is double the lower or the side of the square indoor. Understand then that the four triangles are isometric and, consequently, have the same area.
- Understand that the problem consists in comparing the area of the central square with that of one of the triangles rectangles:

• with the calculation of areas: if c is the measure of the side of the internal square, its area is c2, the measures of the cathets of one of the triangles are c and 2c and the area is valid (c2c) / 2 = c2. This justification by means of a writing literal can also be made with verbal explanations, or by attributing a numerical value to the side of the square internal (for example, 1) and the values corresponding to the sides of the triangle, or still taking the necessary measures on the figure (on condition of explicitly observing the relationship of the sides of the triangle);

• with a decomposition into elementary area units, for example the paving with the triangles of figure 1 (below).

(In this regard, it must be noted that the triangle at the bottom of the figure is divided into four triangles isometric, of which three are the image of each other for translation and the one at the center for central symmetry. However, a more formal demonstration is not required).

• with decompositions and recompositions, for example: translation of the triangle at the bottom to give rise to a rectangle having the width of the side of the square and twice the length of that side. From which 2 triangles equivalents (as area) to 2 squares and 1 triangle is therefore equivalent to the square (figure 2); or rotation of a 180-degree triangle to reconstitute a square with the remaining rectangle trapezoid (figure 3) or transformation of the big square into five cross-shaped squares repeating the previous transformation (figure 4) etc.

(Concerning Figure 3, it must be noted that the prolongation of the major cathetus of a cut triangle the side of the initial square at its midpoint, which could be justified by Thales' theorem or with the clipping of figure 1.)

Solution

Correct answer (yes, the five parts have the same area) with a clear and complete justification (acknowledgment explicit of the properties of the triangles and their isometries, equivalence of the triangles and the square with one of the methods provided in the a priori analysis)

1. **FOUR NUMBERS TO WRITE** (cat. 3, 4)

Write one of the following numbers in the four boxes below:

1, 2, 3, 4, 5, 6.

Respect these three conditions:

- the four numbers must be different from each other;
- if you add all of them, the result must be 15;
- if you multiply the number in the box *d* by 3, the result is the number in the box *a*.

box *a*	box *b*	box *c*	box *d*

Write all the possible solutions.

Explain how you could find them.

ANALYSIS A PRIORS

Conceptual scope

- Arithmetic: addition, subtraction, multiplication
- Logic - combinatory

Analysis of the task

- In a random approach, write one or more combinations of four numbers and check if the conditions are met.
- Realize that the combinations of four numbers are many (360)! and that verifying which ones respect the conditions is too long a task. Consequently, choose, among the conditions, the one that allows to limit the search (condition of the triple or the sum) and to construct in a deductive way the possible solutions, taking into account the fact that the numbers must all be different from each other.

The condition of "triple" gives only two possibilities for the first and last number: 3 and 1 or 6 and 2; for subsequent attempts or taking into account the sum «15», the pupils can arrive at solutions 3651 and 3561 in the first case, 6432 or 6342 in the second case.

The condition of the sum "15" chosen at the beginning would lead to the four numbers (6; 5; 3; 1) (6; 4; 3; 2) and to the inventory of their permutations to determine the four solutions.

Or: find the sum of the six numbers (21) and understand that 6 (21-15 = 6) represents the sum of the two unselected numbers, which are: both 1 and 5, both 2 and 4. Deducting that the four numbers that can be chosen are 2, 3, 4, 6 or 1, 3, 5, 6.

- Understand that, in the first and in the last position, in the first case there will be 6 and 2 and in the second there will be 3 and the 1, being the only possibility to respect the condition of the "triple". Understand that the two "central" numbers can still be exchanged between them.
- Express the four solutions and give some explanatory elements (related to one of the procedures illustrated above)

Attribution of the scores

4 The four correct solutions (3561, 3651, 6342, 6432), with a beginning of explanation of the procedure used

3 Only the four correct solutions, without explanation

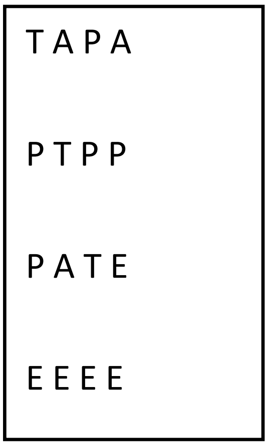
2 Three or four correct solutions, with an incorrect solution or two correct solutions because one of the two pairs (3; 1), (6; 2) has not been taken into consideration or because the permutations of the two central numbers have not been considered, without incorrect solutions

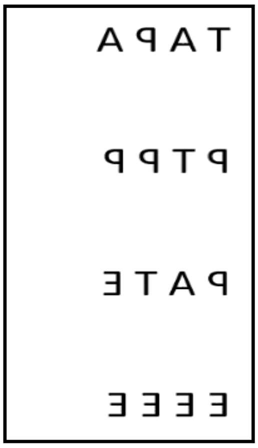
1 A single correct solution or two correct solutions with other incorrect or solution with confusion between the aed box (1563: 1563; 2346; 2436)

0 Misunderstanding of the problem

2. PLANET TAEP (cat. 3, 4)

The alphabet only has 4 letters on Planet TAEP: A, E, P, T.
Each word has 4 capital letters.
Four children TAPA, PTPP, PATE and EEEE write their names on a piece of transparent paper (picture 1).
When they turn the paper upside down they can't read their names as they wrote them (picture 2).





PTPP says: when my sister write her name and turns the paper upside down she can exactly read her name as she wrote it at the beginning!

What can be the PTPP's sister's name?

Show all the names which do not change on the planet when you turn the paper upside down.

ANALYSIS A PRIORS
Conceptual field
- Geometry: axial symmetry
- Combinatorics

Analysis of the task –
Understanding the writing rules of the TAEP planet based on the deliveries and examples given: there are only 4 letters available, each word has 4 letters, there may be more sometimes the same letter in the same word, ...
- Note that two of the four letters, A and T, remain identical when the sheet is turned upside down because they have an axis of vertical symmetry. The name searched must consist of these two letters.
- Bear in mind that the word must also be symmetrical: it must be read either from right to left or from left to right or both on one side and on the other side of the sheet. - Compile the inventory of the words composed of A and T, which have this property, in a systematic way so as not to forget any of them (for example starting with the word composed of four A, then three A and one T and so on .. .)
- Write the list of the four possible names: AAAA; TTTT; ATTA; TAAT.

Attribution of scores
4. The four names, without errors: AAAA; TTTT; ATTA; TAAT.
3. Discovering the two letters A and T, three names are written correctly, but a name is missing or there is one that is not symmetrical (of the AAAT type).
2. Discovery of the two letters A and T, but with only two correct names and at most one wrong or three correct but with other incorrect names for the presence of one of the non-symmetrical N or P letters.
1. One correct solution or one only the two conditions are taken into consideration (examples: PEEP, PAAP, or TATA ...)
0. Misunderstanding of the problem

3. QUESTIONS AND ANSWERS (cat. 3, 4)

Nicola got a new game, where the gamer has to answer to some questions and move his piece on a track with the numbers from 0 to 50.

At the beginning of a new match, the piece is on the box number 25.

Every time a gamer answers correctly he moves three steps (boxes) forward.

Every time he gets wrong he moves three steps behind.

At the end of the match Nicola in on the box 40.

During the match Nicola answered correctly 7 questions and got all the others wrong.

How many wrong answers did Nicola give during the match?

Explain your answer.

ANALYSIS A PRIORS

Conceptual scope

- Arithmetic: progression of natural numbers, the four operations

Analysis of the task

- Understanding the displacement rules: make some test shots
- Establish that, if he had never replied wrong, Nicola would have gone ahead, for his seven correct answers, of 21 boxes (3 x 7) and which would have reached box 46 (21 + 25); understand that there are six too many boxes (46 - 40) that must be compensated by three false answers (6: 2 or 6 - 2 - 2 - 2).

Or: calculate that Nicola went ahead with his pawn of 15 boxes in all (from box 25 to box 40 or 40 - 25), and that he arrived there with five correct answers; we must consider that he has answered just two other questions that would make him advance by another six boxes. To stand still on the 40 must therefore have answered wrong three questions, so as to go back six steps.

Or: proceed step by step with a sequence of seven additions and a few subtractions to get to 40, with the necessary adjustments (eg 25 + 3 + 3 - 2 + 3 + 3 - 2 + 3 + 3 + 3 = 42; 42 - 2 = 40) and subtraction counting.

Or: draw the track and make seven moves of three in three from 25 to get to 46 and return to 40 with three moves of two in two.

Score allocation

4 Correct answer (3 wrong answers) with a clear explanation (operations or indications on the line of numbers or graphic solution ...

) 3 Correct answer, without explanation or with partial explanation or little comprehensible

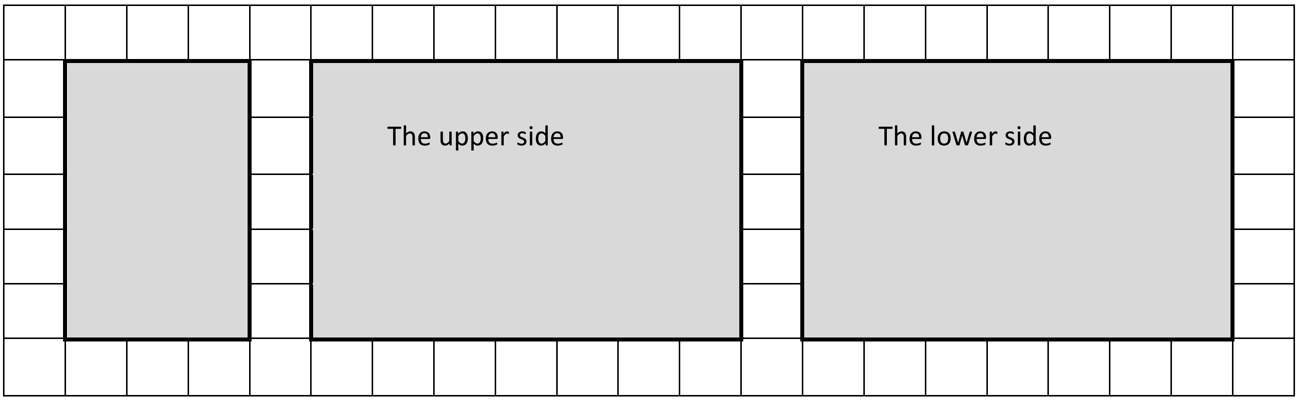
2 Response obtained starting from a correct reasoning, but with a miscalculation or an oversight

1 Beginning of consistent search

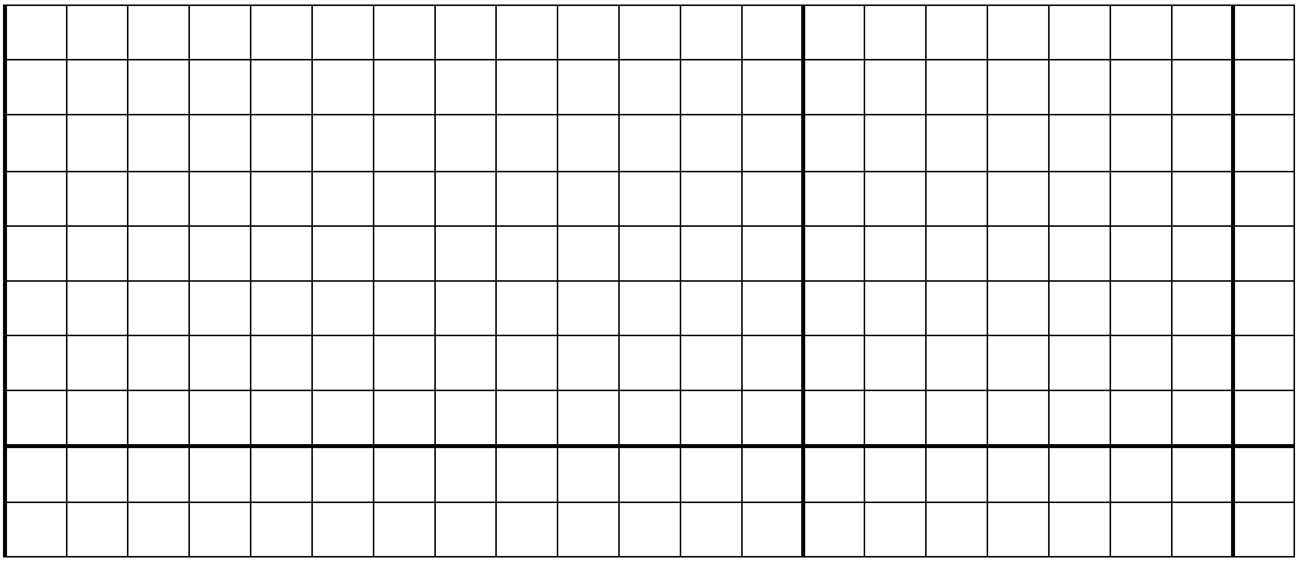
0 Misunderstanding of the problem

4. THE BOX TO WRAP UP (cat. 3, 4, 5)

Graziella wants to cover a box with some paper rectangles.
She already drew these three triangle to cover the upper part of the box, its bottom part and one of the other three faces.



Draw below the other three rectangles needed to cover the other sides of the box.



ANALYSIS A PRIORI
Conceptual field - Flat geometry and geometry of space: rectangle and parallelepiped rectangle

Analysis of the task

- Understand that it is a "familiar" box (rectangular rectangle) since the sentence speaks of rectangles.
- Imagine the box and its six faces: the one below and the one above (imagine horizontal) and the four faces (imagine vertical)
- Realize that the six faces can be divided into three pairs of equal faces (the opposite faces) and deduce that since the above and below are the two equal rectangles given, the third given rectangle is one of the vertical faces.
- To realize that it will be necessary to draw a fourth rectangle equal to the one given.
- Understand that the last two rectangles must adapt to the former. They must have the same length as the two bases (the above and the below) and their width must correspond to the "height" of the box, given by one of the sides of the vertical face already drawn.
- Draw the three faces reporting the measurements either by counting the squares or by trial and error.

Score allocation

4 Correct and accurate drawing of the three missing faces (one face 3×5, two faces 7×3) 3 Correct drawing of the three missing faces, but with imprecise lines (freehand, lines that do not match exactly to those of the quadrature, ...)

2 Correct drawing of the fourth face (3×5) and an error in the 5th and / or 6th face (sides not corresponding to the data)

1 Correct drawing of one of the two faces or three faces drawn, but with of the errors (but they do understand that the pupils realized that it took six faces in all)

0 Misunderstanding of the problem.