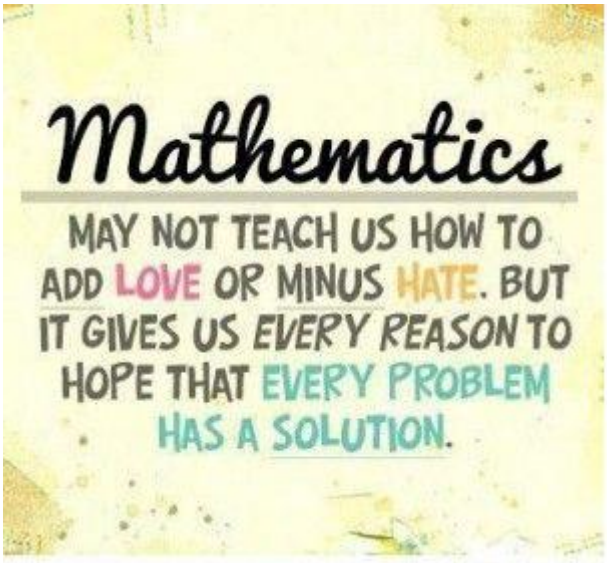
CHAPTER 4

FUNNY MATHEMATICS

We called it funny math because it contains some innovations of good practice that are suitable for extracurricular activities. Some elements of these innovations can be used in regular class of math. Chapter contains 4 innovations named:

1. Joseph's problem -How to organized a deck of ten cards, so that when shuffling and putting one on the table and one under the deck, one on the table, etc., we get cards organized into a scale from 1 to 10?
2. Participation in Kangaroo Math Competition- Live and share a day with other mathematical schools to discover the magic of mathematics.
3. Promotion of Mathematics- Creating a special classroom for studying math in the center.
4. School Month of Math: Different activities and events during March, as the month of math, and play in a mathematical games.
5. Solving math problems in fairy tales- Consolidate math knowledge in a funny and enjoyable way.

Now days is very hard to keep students motivation high. In our opinion, these educational innovations can make students motivated for learning math in a funny way. Also, we think that this is the way to make mathematics more popular subject for students that are not good at it. So, if you want to have active, attractive and interactive mathematics you can use our suggestions!



Picture 0: Introduction

4.1. Joseph's problem

Type of innovation: observation and description; relating of lessons to real life; design thinking tools and methods-problem solving; cooperative learning peer tutoring/instructions; after school programs and educational games.

Theme/Topic: Joseph's problem

Students` age: 13-19

Time (number of sessions, min): 45 minutes

Goals/aims/objectives: Spot the recursive method of problem solving. Without lessons and knowing the definitions, using brainstorming and trial and fail methods, find the solution to the problem.

Materials needed: Deck of cards from ace to ten, that is from ace to the king.

Url of the online products used:
Hot Potatoes: ONLY for loged in users
http://www.e-mathematics.eu/mod/hotpot/attempt.php?id=724
http://www.e-mathematics.eu/mod/hotpot/attempt.php?id=725
http://www.e-mathematics.eu/mod/hotpot/attempt.php?id=726

Kinds of technologies used:
PPT presentation : http://www.e-mathematics.eu/course/view.php?id=103#section-0
(www.e-mathematics.eu >Croatia > Methodological guide material>Joseph's problem- ppt)
Hot Potatoes

Procedure:
Students can be individual or in pairs, not groups because of to many different point of views. For this kind of tasks students need silence and time.
The description of Joseph's problem is on the link:

http://www.e-mathematics.eu/course/view.php?id=103#section-0

(www.e-mathematics.eu >Croatia > Methodological guide material>Joseph's problem- ppt)

The problem appeared in 37AC and it saved Joseph and his friend form certain death. Today we changed that problem in a card problem: HOW TO ORGANISE A DECK OF TEN CARDS, SO THAT WHEN SHUFFLING AND PUTTING ONE ON THE TABLE AND ONE UNDER THE DECK, ON THE TABLE WE GET CARDS ORGANISED INTO A SCALE?!

Products made: presentation:, video, quiz in Hot Potatoes for checking.

Advantages and disadvantages of the innovation:

Approach the problem individually, which is advantage and a disadvantage at the same time. Students will have problems to come up with recursive method if they don't have cards in their hands. Because of that Joseph's problem is studied with the help of ICT and practically with the cards in the hands. The advantage is high motivation of students to solve a problem because they consider that challenging: at this age they are familiar with card games.

Comments/suggestions/ pieces of advice:

It cannot be implemented into the curriculum, apart from academic program, but I definitely recommend it for the fun mathematics lessons or math festivals.

Hot Potatoes is created to check the solutions, but guests cannot use it, so the solutions are:

From Ace to ten: Aces, 6,2,10,3,7,4,9,5,8.

Other solutions are in www.e-mathematics.eu (Croatia> Methodological guide material>Joseph's problem-solutions).

Handouts

On the Evening of Mathematics at Industrial and Trade High school, a few students solved the problem with ten cards, but they didn't come up with the recursive method, but trail and fail. With the problem with 14 cards the saw that the change in approach is needed.

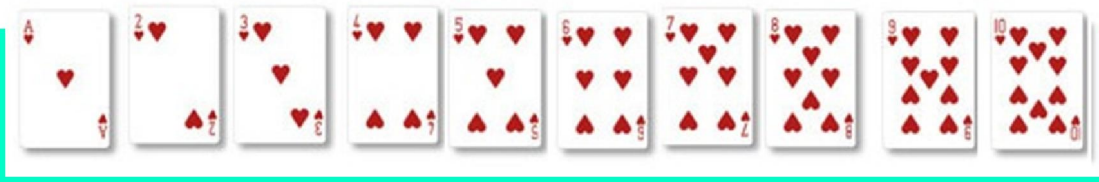
All the materials are made by Marijana Zarozinski, math teacher from Slavonski Brod, Croatia.

Josephus Flavius, Jewish scholar and historian born in 37BC, saved his laife and the life of his friend during the roman execution by the Romans. Namely, Romans placed the Jews in a circle and decided to execute every third Jew until all of them are dead.

Josephus saved them! How????

Picture 1: Presentation for the workshop

3. Each group is given poker cards :
ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king .
4. It is necessary to fold the cards in the deck in a way when you place the cards on a table, you put one on the table and the next one at the bottom of the deck and do so repeatedly, the cards should be arranged from ace to king, like in the photo. How will you stack the cards?



Workshop 2

Picture 2:The task for students

4.2. Participation in Kangaroo Math Competition

Type of innovation: Participation in Kangaroo Math Competition

Theme/Topic: Promotion of Mathematics

Students` age: 14-18 years old

Time (number of sessions, min): One day to do some sessions before the test and prepare for it.

Goals/aims/objectives: Participate in an international mathematics competition to promote mathematics among students of the center.

Concepts: Live and share a day with other mathematical schools to discover the magic of mathematics.

Skills: mathematics competence, mathematics in everyday life competence and the learning to learn competence.

Materials needed:

- Online resources/websites: www.xtec.cat / www.educa3d.com / www.thatquiz.org
- Online activities: http://matematico.es
- Equipment: smartboard, digital board

Kinds of technologies used: Internet

Procedure:

Participation in Kangaroo is done on a particular day in the year, the month of April. The center has a number of students of 3rd and 4th ESO and Baccalaureate, which we have

previously pointed out in November. That day summon all students in schools in the municipal area of our sports people to perform the tests.

The same week, and for a few years also made Kangaroo center for 1st and 2nd ESO. The week prior to the test in math classes are working several years of testing in previous years so that students know the types of exercises that must be solved.

Advantages and disadvantages of the innovation:

Participating in such events is a way to make students understand that mathematics is not only working in the classroom, but there is also math in everyday life, our environment and that we can use to work with colleagues .

Comments/suggestions/ pieces of advice:

There are more playful activitas like kangaroo trials such as: the feast of mathematics, PBS to sprint, video maths... ever we have also participated. It should continue to work this entertaining mathematics to awaken in students a certain attraction towards this subject.

Picture 3: Kanagroo Math Competition

Picture 4: Teacher meeting for Kangaroo

4.3. Promotion of mathematics

Type of innovation: Creating a classroom mathematics laboratory in the center.

Theme/Topic: Promotion of mathematics.

Students` age: 12-18 years old

Time (number of sessions, min): School year

Goals/aims/objectives: Having a specific space to do some math activities (especially manipulated) and an allocation of specific material for the course.

Concepts: Working in a physical space mathematics motivator for the area.

Skills: All the skills inherent in mathematics than their own education.

Materials needed: All that is in the mathematics classroom and more than we can go shopping to complete the endowment.

Kinds of technologies used: They incorporate new technological tools for sizing knowledge and funding for the purchase.

Procedure:

The classroom mathematics laboratory is located on the first floor of the center, identified as such. It was created root participation in Erasmus + center around mathematics. The mathematics department is responsible for its maintenance, provision of equipment and management use. Every year we make a purchase of material and change what is broken.

Advantages and disadvantages of the innovation

The fact teach classes in a different space and a very mathematical means that students are more motivated to work more focused on the subject and get better results.

Comments/suggestions/ pieces of advice:

It is not possible to do the math for all groups of the center in this class, but it is intended that all groups spend at least once a week for the classroom laboratory.

We attach some pictures of the classroom laboratory.

Picture.5: Perform a line in a coordinate plane

4.4. School Month of Math

Type of innovation: online learning, after school programs, educational games.

Theme/Topic: Month of Math

Students` age:13-19

Time (number of sessions, min): about 10 hours with students, all together 30 hours

Goals/aims/objectives: to make math lessons and mathematics as a science more popular, to motivate students to solve problem tasks and participate in projects outside the regular lessons, to promote new teaching methods and usage of IKT tools, invite students to participate in different events during Math month and play in a mathematical way, decoration of the classroom

Materials needed: smart phone, projector, internet, color printer, posters

Kinds of technologies used: gamming with Kahoot!, eTwinning, smart phones, virtual classroom

Procedure:

At the end of February we announced the dates in March when will certain events take place and who are they for-

March 1st–World Math day – visit of the math counselor

March 3rd – County meeting for the math teachers

March 9th – eTwinning workshop

March 14th – World π Day

March 23rd – Kangaroo without borders

March 30th – Evaluation – photos and conclusions

Since it is always about the students, it was their task to make posters and decorate the hallway for Math month.

The events were announced on eTwinning as physical events, but about 50 e-Twinners joined in.

The counselor was given all the information about the project “Active, Attractive And Interactive eU Mathematics”.

World Math Day was celebrated with riddles in “Kahoot” and “Jumble”. The riddles were from Kangaroo without borders. 20 students and 2 teachers participated.

https://create.kahoot.it/#jumble/e2395f67-9a30-436c-ad0f-88658ccdcf4e

https://create.kahoot.it/#quiz/b49c5924-f97d-4502-a0d9-cda87c0950de

Picture 6: Students play Kahoot!

County meeting for the math teachers is not related to the project but it does relate to teaching mathematics. There were 20 high school (students aged 14 to19) math teachers at the meeting and the topics were related to advancement, including math into all activities in schools, math festival and making of a digital course book in the program “Mathematica”.

World π Day was celebrated with mathematical and logical riddles that can be found in IQ tests: https://create.kahoot.it/#quiz/49654a70-f3c0-44e0-91e6-73c6052f1c4a. 30 students and 3 teachers participated. But we had a problem when we lost the internet connection so we repeated the event next week and awarded the best team of students with eTwinning promotional materials and classroom materials. The students were very pleased.

Picture 7: Students received prize for winning the competition

Photography teacher worked with her students on a method of taking photos on π symbol, so we made a small exhibition.

And finally, on 23rd March, 23 students participated in the biggest school math competition "Kangaroo without borders", that took place for the first time at our school. We found out about it at one of the project team meetings and we loved it: students from all countries in this project participate in this competition. It is paid for from the project, but the school management confirms participation each year.

Picture 8: Kangaroo Mathematical Competition

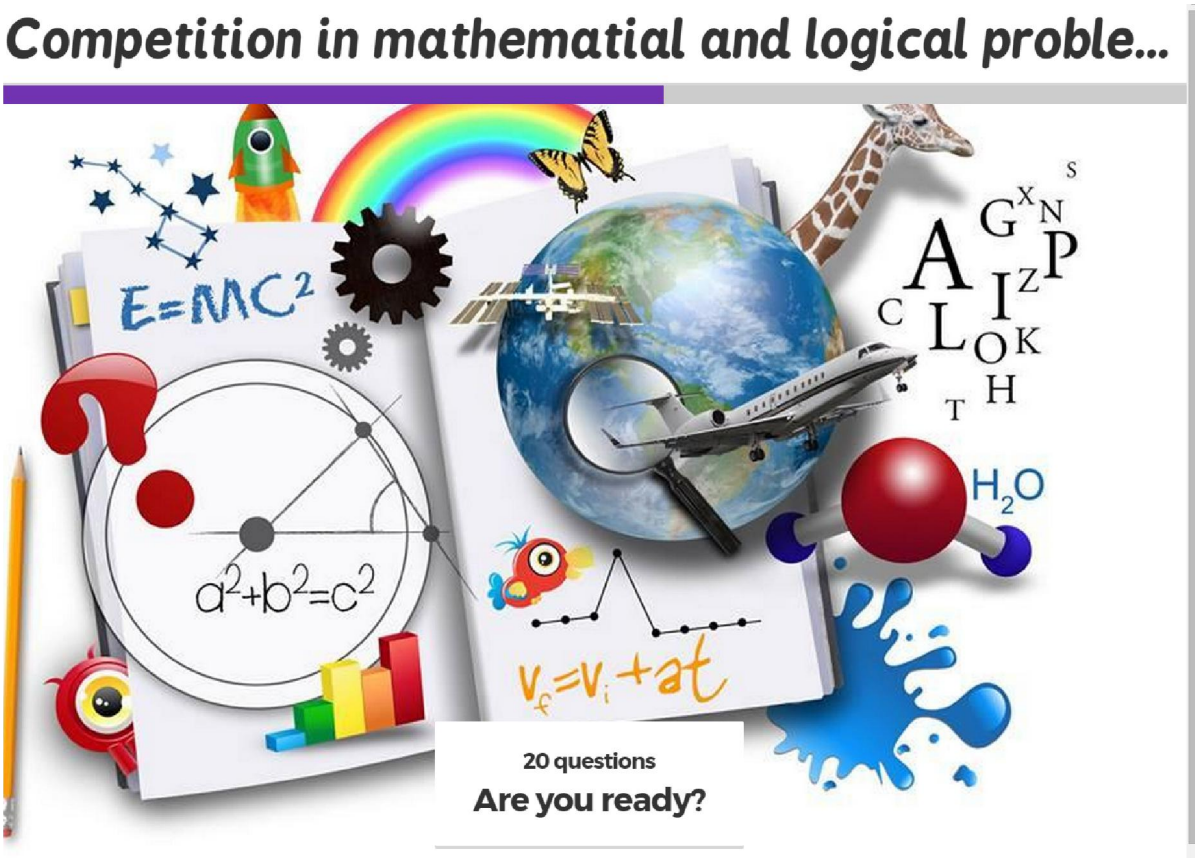
Evaluation was done in a form of exhibition. And an article was published on the school web site. All the participants are very satisfied.

Products made: https://create.kahoot.it/#quiz/49654a70-f3c0-44e0-91e6-73c6052f1c4a

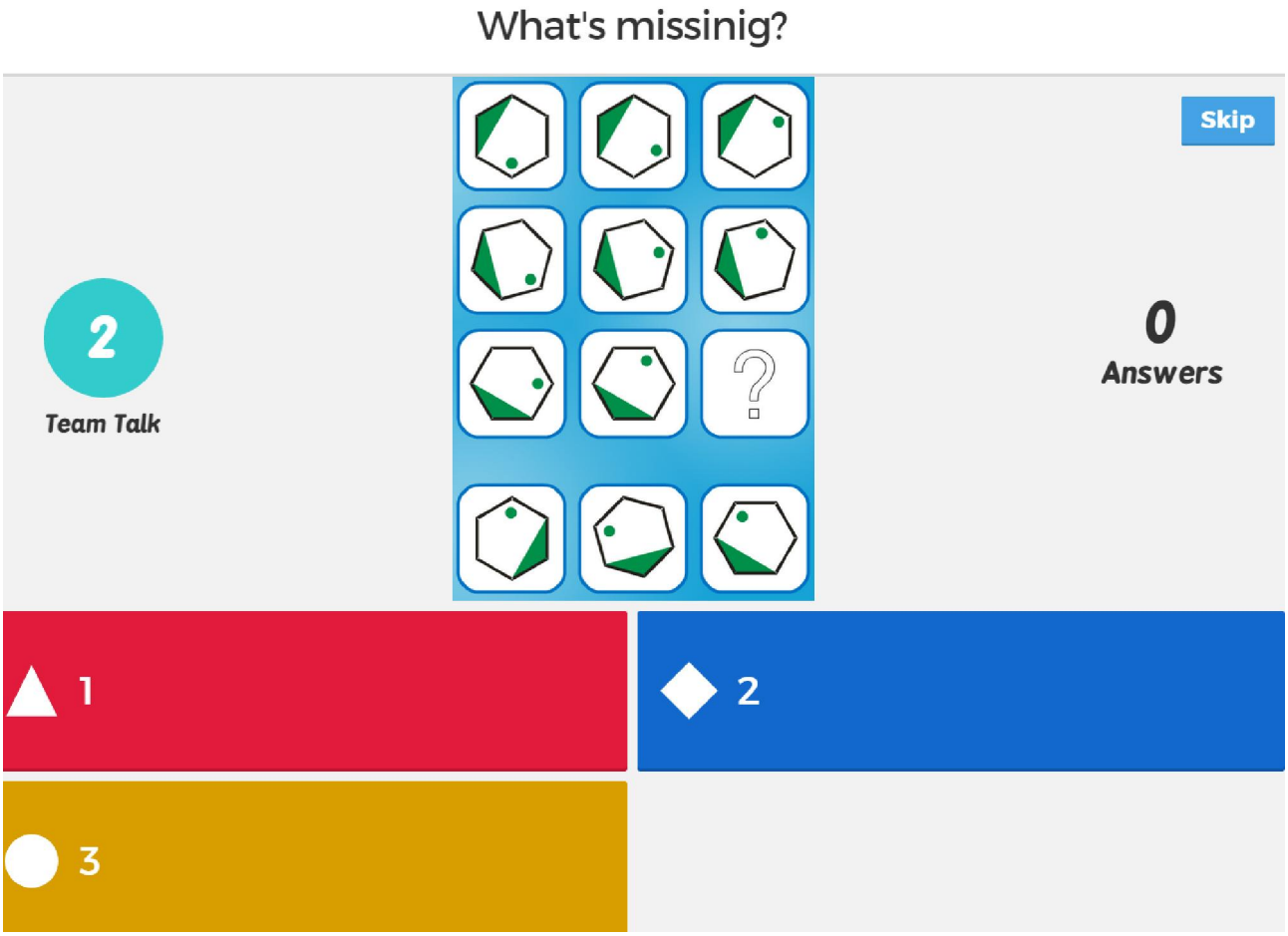
Advantages and disadvantages of the innovation:

Advantages: high student motivation, change of the student's attitude towards the subject, school and the teacher, students who take part in extracurricular activities have less problems and more motivation during regular lessons, we have noticed a significant progress with students who have always struggled to pass math, colleagues' and parents' support

Disadvantages: a lot of free time was used for preparation of the events during the whole moth, no support from a few parents and colleagues



Picture 9: Kahoot!



Picture 10: Task from Mathematical Competition

All the materials are made by Marijana Zarozinski and Igor Vidovic, math teachers from Slavonski Brod, Croatia.

4.5. Solving Math problems in fairy tales

Type of innovation: Educational games-Solving Math problems in fairy tales

Theme/Topic: 8th grade General Revision (Exit Test)

Students` age: 13-14 years

Time (number of sessions, min): 120 min

Goals/aims/objectives:

- Solving the quadratic equations
- Finding the square root of a fraction
- Finding the median of a trapezoid
- Solving of linear equations
- Finding the angles of a triangle inscribed in a circle
- Finding the tangents in a triangle circumscribed about a circle
- Finding the angels of a quadrilateral inscribed in a circle
- Finding the radius of an isosceles trapezoid circumscribed about a circle

Materials needed:

- Laptops/ Tablets
- Internet
- Projector
- Papers and pens/ Whiteboard and markers

Kinds of technologies used: Microsoft PowerPoint

Procedure: PPT

Advantages and disadvantages of the innovation

- Advantages:
- Motivation
- Meaningful situations

- Increased learning
- Positive attitude
- Different levels
- Independence

Disadvantages:

- Takes more than 2 hours
- The teacher needs a lot of time to prepare the lesson and presentation
- Students can guess the answers

Picture 11: Fairy Tales

4.6. Board game “Maths and money”

Type of innovation: team work; relating of lessons to real life; multi-disciplinary and cross-curricular learning; educational games

Theme/Topic: Board game “Maths and money”

Students` age: 14-15 years

Time (number of sessions, min): 30-40 min

Goals/aims/objectives:
Understanding of money management
Practical money skills
Using Maths knowledge for personal and family budgeting in real-life situations
Maths knowledge consolidation and application
Taking financial responsibility

Materials needed: board game, dice, pools; list/cards of questions -ractical math problems and brainteasers about money

Some possible questions:

1.

A farmer bought 3 cows for $3000 and sold them for $4000.

He bought them back for $6000 and sold again for $6600.

How much did he gain or lose on these transactions?

- 0
- $1600
- $600
- $400 lost

2.

A team has $90.00 to buy balls.
One ball costs $7. The team gets every fifth ball for free.

What is the largest number of balls the team can get?

- 12 balls
- 15 balls
- 16 balls
- 20 balls

3.

Anna has two $100 notes.
She also has eleven $10 notes and nine $1 notes.

How much money does she have in total?

- $329
- $319
- $219
- $913

4.

Our school will have to pay $24 for 15 dozen pencils.

How much will the school have to pay for 50 dozen pencils?

- $60
- $80
- $72
- $96

5.

The drama club is selling tickets to a play for $8 per ticket.
The cost to rent the theater and costumes is $500.
In addition, the printers are charging $1 per ticket to print the tickets.

What is the minimum number of tickets the drama club must sell to make a profit?

- 71
- 72
- 70
- 73

6.

A stadium contains 20,000 seats. At a match, 25% of the seats remain empty. One ticket costs $20.

Find the cost of the total tickets sold.

- $400 000
- $300 000
- $120 000
- $200 000

7.

Three boys put $52, $60, and $70 in a piggy bank.

Alex is 100 cm tall.
Bill is 20% taller than Alex.
Craig is 20% taller than Bill.

How should the money be split if it is distributed as a ratio that is proportional to the heights of the children?

- $52: $60 : $70
- $50: $60 : $72
- $50: $61 : $71
- $51: $60 : $71

8.

If a real estate agent received $60,000 as a 5% commission on the selling price of a house, how much was the house sold for?

- $12,000,000
- $1,200,000
- $3,000,000
- $30,000,000

9.

Mrs. Smith puts $10,000 into a saving account that earns 2% interest compounded annually.

Which expression can be used to find the value of her money in the account at the end of the third year?

- $10000\ (1 + 0.02^3)$
- $10000\ (1 + 0.02)^3$
- $10000\ (1 + 0.03)^2$
- $10000\ (1 + 0.2)^3$

10.

John and Mary borrow $3,000 to pay for new furniture.
They will pay back the loan by making 12 monthly payments of $333.

How much does the loan cost?

Compare the loan with the original price of the furniture.

13.2%
33.2%
23.2%
43.2%

BOARD GAME "MATHS AND MONEY"

CHAPTER 5

EXAMPLES OF LESSON PLANS LOCATED IN E-PLATFORM

Within our partnership, each partner school has created six lesson plans: two web-based, two practical (or field lessons) and two cross-curricular projects. These lesson plans have been created by the involved in the project Mathematics teachers. The plans are a contribution to the other intellectual outputs.

All Mathematics lesson plans are uploaded in the project`s e-learning platform. They are offered to be used in the partner schools, by Mathematics teachers and other interested stakeholders.

In this chapter you can find six of the lesson plans. The choice has been made by our Mathematics teachers.

Lesson Plan 1

Subject: Mathematics

Grade/Students` age: 14-15 years old

Topic/Theme: Algebra

Lesson type: Web based

Time: 1 week

Aims/Goals: Calculation of a proportion

Objectives:

- Concepts: Mathematical proportions
- Skills: Mathematic competence and the competence of learning to learn

Materials/Resources/Equipment:

- Online resources/websites: See introductive actitivities
- Online activities :Exercises
- Worksheets:Yes
- Equipment: Digitalboard
- Other: Notebooks

Lesson Procedure

№	Lesson stage	Activities/tasks description	Time	Notes
1.	Introduction/ Motivation/ Warm-up/ Review	The lesson begins with some introductive exercises SeeAnnex 1	1 session	
2.	Presentation	Explanation of thetheory: SeeAnnex 2	2 sessions	
3.	Practice/ Activities ➢ Guided ➢ Independent	The mathematical formula storage: SeeAnnex2 and Annex3.1 Exercises on-line: http://www.mathplayground.com/index_ratio_proportion_percent.html See Annex 3.3	2 sessions	

4.	Checking for understanding/ Assessment/ Evaluation	Exercises on-line: https://www.ixl.com/math/grade-8/solve-proportions See Annex 3.3	3 session	
5.	Closure/ Homework	Activities that students will do on their own. See Annex 3.2	4 session	
6.	Follow-up/ Extension	Property and conceptual map: See Annex 4 and Annex 5 Connections with other subjects: See Annex Chemistry and proportion The mole	5 session	To do the written test
7.	Reflection	Online exercises in form of game will stimulate the students to apply the concept of proportion.	2 session	

Lesson Plan 2

Subject: Mathematics

Grade/Students` age: 14-16 years

Topic/Theme: Trigonometry on a right-angled triangle

Lesson type: New knowledge

Time: 90 minutes

Aims/Goals:

Students will be introduced into trigonometric ratios in right-angled triangle and how to apply them to solve right-angled triangle.

Objectives:

Students should be able to:

- Concepts:
  - explain the trigonometric ratios.
  - make a difference between adjacent and opposite leg.
  - calculate sine, cosine, tangent and cotangent.
- Skills:
  - name the sides of a right-angled triangle in relation to an identified angle.
  - derive general trigonometric statements showing the relationship between the angles and sides of the right-angled triangle.
  - determine the trigonometric ratios of given angles using calculators.

Materials/Resources/Equipment:

- Online resources/websites:
  - https://prezi.com/awigialcx3qt/present/?auth_key=cjpe6b6&follow=b8adn0rhmzwi&kw=present-awigialcx3qt&rc=ref-184480623
  - https://tube.geogebra.org/b/3138875#
  - https://create.kahoot.it/?_ga=1.235765697.1770291522.1460227990&deviceId=157169ab-0fac-4c7e-8105-97411e6e3406#quiz/0b0988ff-a597-4f3f-88cc-c45f1a2b78f6
  - http://www.e-mathematics.eu
- Online activities:

- Presentation
- Watch videos
- Geogebra
- Kahoot
- Hot Potatoes
- Online test

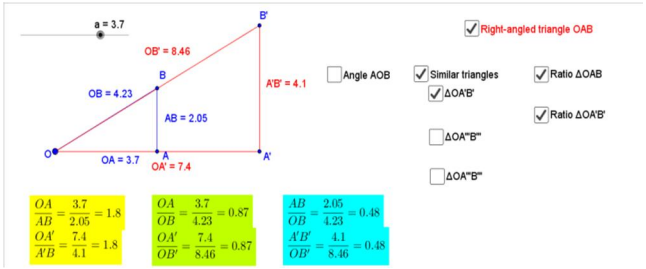
➢ Worksheets: -

➢ Equipment:

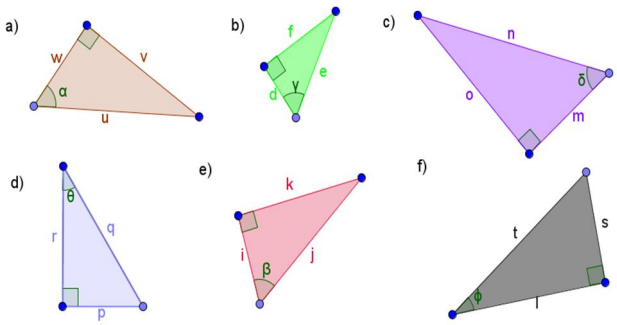
- Whiteboard
- Computer/tablets
- Projector
- Calculators

➢ Other: -

Lesson Procedure

№	Lesson stage	Activities/tasks description	Time	Notes
1.	Introduction/ Motivation/ Warm-up/ Review	Using Geogebra show students about ratios in similar right-angled triangles.  https://tube.geogebra.org/material/simple/id/3151767	5	A short discussion about ratios of similar right-angled triangle. Teacher asks Student answer

2.	Presentation	https://prezi.com/awigialcx3qt/present/?auth_key=cjpe6b6&follow=b8adn0rhmzwi&kw=present-awigialcx3qt&rc=ref-184480623 1. Trigonometry on a right-angled triangle • What is right-angled triangle? • How do we call sides in right-angled triangle? • What theorem can we use in right-angled triangle? • What is the sum of angles in triangle? 2. Trigonometric functions • Name trigonometry functions • Define trigonometric ratios for angle α Task: Define trigonometric ratios for angle β • Solving right-angled triangles using trigonometry 3. Using trigonometry in real life "Trigon" is Greek for triangle, and "metric" is Greek for measurement. The **trigonometric ratios** are special measurements of a **right-angled triangle.** There are four basic trigonometric ratios: **sine, cosine, tangent and cotangent.** The ratio of opposite leg and hypotenuse is called the sine. The ratio of adjacent leg and hypotenuse is called the cosine. The ratio of opposite leg and adjacent leg is called the tangent. The ratio of adjacent leg and opposite leg is called the cotangent.	2 7 3 2 1 1 3	Teacher explains using presentation Discussion Students write in their notebook

		$\sin\alpha = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{a}{c}$ $\cos\alpha = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{b}{c}$ $\tan\alpha = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{a}{b}$ $\text{cotan}\,\alpha = \frac{\text{adjacent leg}}{\text{opposite leg}} = \frac{b}{a}$		
3.	Practice/ Activities ➢ Guided	**Example 1.** Find trigonometric ratios for given triangles:  https://tube.geogebra.org/material/simple/id/3161673 *Solution:* a) $sin\alpha = \frac{v}{u}$ $cos\alpha = \frac{w}{u}$ $tan\alpha = \frac{v}{w}$ $cot\alpha = \frac{w}{v}$ b) $sin\gamma = \frac{f}{e}$ $cos\gamma = \frac{d}{e}$ $tan\gamma = \frac{f}{d}$ $cot\gamma = \frac{d}{f}$ c) $sin\delta = \frac{o}{n}$ $cos\delta = \frac{m}{n}$ $tan\delta = \frac{o}{m}$ $cot\delta = \frac{m}{o}$ d) $sin\theta = \frac{p}{q}$ $cos\theta = \frac{r}{q}$ $tan\theta = \frac{p}{r}$ $cot\theta = \frac{r}{p}$ e) $sin\beta = \frac{k}{j}$ $cos\beta = \frac{i}{j}$ $tan\beta = \frac{k}{i}$ $cot\beta = \frac{i}{k}$ f) $sin\Phi = \frac{s}{t}$ $cos\Phi = \frac{l}{t}$ $tan\Phi = \frac{s}{l}$ $cot\Phi = \frac{l}{s}$ **Example 2.** Find trigonometric ratios for angles $\alpha$and βin right-angled triangle if $a = 3$and $b = 4$. https://tube.geogebra.org/material/simple/id/3186209	5	Students do guided example in their notebook Teacher shows solution using Geobgebra

		Solution: $a = 3$ $b = 4$❑ $sin\alpha, cos\alpha, tan\alpha, cot\alpha = ? sin\beta, cos\beta, tan\beta, cot\beta = ?$ Using Pythagorean Theorem we can find hypotenuse c. $c^2 = a^2 + b^2$ $c^2 = 3^2 + 4^2 c^2 = 9 + 16$ $c^2 = 25$ $\sqrt{}$❑ $c = 5$ $sin\alpha = \frac{a}{c} = \frac{3}{5} = 0.6$ $sin\beta = \frac{b}{c} = \frac{4}{5} = 0.8$ $cos\alpha = \frac{b}{c} = \frac{4}{5} = 0.8$ $cos\beta = \frac{a}{c} = \frac{3}{5} = 0.6$ $tan\alpha = \frac{a}{b} = \frac{3}{4} = 0.75$ $tan\beta = \frac{b}{a} = \frac{4}{3} = 1.333$ $cot\alpha = \frac{b}{a} = \frac{4}{3} = 1.333$ $cot\beta = \frac{a}{b} = \frac{3}{4} = 0.75$ **Example 3.** Measure leg a in right-angled triangle if hypotenuse is 15 and $sin\alpha = \frac{3}{5}$. *Solution:* $sin\alpha = \frac{3}{5}$ $c = 15$ $a = ?$ $sin\alpha = \frac{a}{c}$ $\frac{3}{5} = \frac{a}{c} \quad \cdot 5c$ $3c = 5c$ $3 \cdot 15 = 5a$ $45 = 5a \quad :5$ $9 = a$	4 3	Students do guided example in their notebook Teacher shows solution using Geobgebra Teacher explains solution Students do guided

		$a = 9$ **Example 4.** Measure hypotenuse in right-angled triangle if $b = 24$and $sin\beta = \frac{12}{13}$. *Solution:* $sin\beta = \frac{12}{13}$ $b = 24$ $c =?$ $sin\beta = \frac{b}{c}$ $\frac{12}{13} = \frac{b}{c} \quad \cdot 13c$ $12c = 13c$ $12c = 13 \cdot 24$ $12c = 312 \quad :12$ $c = 26$ **Example 5.** If in right-angled triangle $cos\alpha = \frac{3}{5}$, find the other trigonometric ratios of α. *Solution:* $cos\alpha = \frac{3}{5}$ $sin\alpha, tan\alpha, cot\alpha =?$ $cos\alpha = \frac{b}{c}$ $\frac{b}{c} = \frac{3}{5} \quad \cdot c$ $b = \frac{3}{5}c$ Using Pythagorean Theorem we can find leg a conditioned by hypotenuse c.	3 6	example in their notebook Teacher explains solution Students do guided example in their notebook Teacher explains solution

$c^2 = a^2 + b^2 a^2 = c^2 - b^2 \quad a^2 = c^2 - (\frac{3}{5}c)^2 \quad a^2 =$

$c^2 - \frac{9}{25}c^2$

$a^2 = \frac{16}{25}c^2 \qquad \sqrt{\square}$

$a = \frac{4}{5}c$

$$sin\alpha = \frac{a}{c} = \frac{\frac{4}{5}c}{c} = \frac{4}{5}$$

$$tan\alpha = \frac{a}{b} = \frac{\frac{4}{5}c}{\frac{3}{5}c} = \frac{4}{3}$$

$$cot\alpha = \frac{b}{a} = \frac{\frac{3}{5}c}{\frac{4}{5}c} = \frac{3}{4}$$

Example 6. Fill the table

a	b	c	$sin\alpha$	$cos\alpha$	$tan\alpha$
8	15				
6		10			
	5	13			
11		61			
	16	20			

Solution:

Missing sides sre measured by Pythagorean Theorem:

$c^2 = a^2 + b^2 a^2 = c^2 - b^2 b^2 = c^2 - a^2$

Trigonometric ratios:

$$sin\alpha = \frac{a}{c} \qquad cos\alpha = \frac{b}{c} \qquad tan\alpha = \frac{a}{b}$$

a	b	c	$sin\alpha$	$cos\alpha$	$tan\alpha$

Students do guided example in their notebook

➢ Independent

10

Students do individual example in their notebook

		8	15	17	$\frac{8}{17} = 0.471$	$\frac{15}{17} = 0.882$	$\frac{8}{15} = 0.533$	
		6	8	10	$\frac{6}{10} = 0.6$	$\frac{8}{10} = 0.8$	$\frac{6}{8} = 0.75$	
		12	5	13	$\frac{12}{13} = 0.923$	$\frac{5}{13} = 0.385$	$\frac{12}{5} = 2.4$	
		11	60	61	$\frac{11}{61} = 0.180$	$\frac{60}{61} = 0.984$	$\frac{11}{60} = 0.183$	
		12	16	20	$\frac{12}{20} = 0.6$	$\frac{16}{20} = 0.8$	$\frac{12}{16} = 0.75$	
		Quiz in Hot Potatoes http://www.e-mathematics.eu Test from Kahoot! https://create.kahoot.it/?_ga=1.235765697.1770291522.1460227990&deviceId=157169ab-0fac-4c7e-8105-97411e6e3406#quiz/0b0988ff-a597-4f3f-88cc-c45f1a2b78f6						15 15	Students do individual quiz in Hot potatoes Students do individual test from Kahoot!
4.	Checking for understanding Assessment/ Evaluation	The results from quiz and test						2	Agreement, feedback, self-correction.

5.	Closure/ Homework	**Task 1.** Find trigonometric ratios for angles $\alpha$and βin right-angled triangle if b5 and c 13. **Task 2.** Measure leg b in right-angled triangle if hypotenuse is 36 and $cos\alpha = \frac{1}{6}$. **Task 3.** Measure leg b in right-angled triangle if c169 and $cos\beta = \frac{12}{13}$.	2	
6.	Follow-up/ Extension	Trigonometry has a very wide use in real life. We have now learned how to make trigonometric ratios on a right-angled triangle. We will study how to use calculator for trigonometry functions and how to use trigonometric ratios to find missing angle.	1	Teacher explains
7.	Reflection			

Lesson Plan 3

Subject: Mathematics

Grade/Students` age: age 15

Topic/Theme: Axial symmetry in Mathematics and in the world around us

Lesson type: web-based

Time: 45 min

Aims/Goals:

Objectives:

- Concepts: Educational – new knowledge: Introducing axial symmetry as a type of symmetry to students – definition and characteristics
- Skills: Developing skills for drawing images of shapes of axial symmetry and identifying the axes of symmetry of different shapes

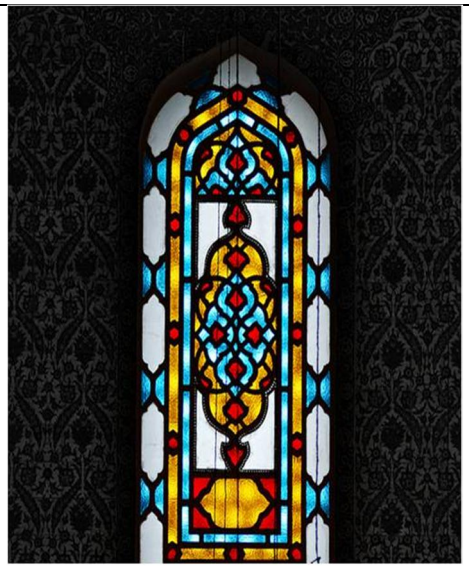
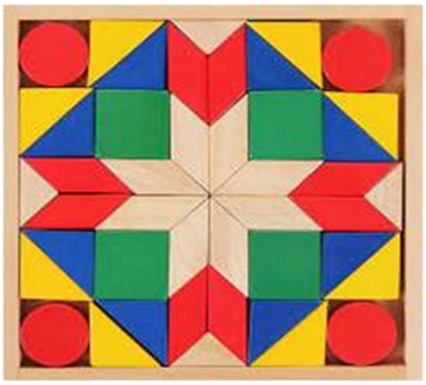
Materials/Resources/Equipment:

- Online resources/websites: GeoGebra, e-textbooks
- Online activities: A presentation, searching for information on the Internet
- Worksheets:
- Equipment: A computer, instruments for drawing

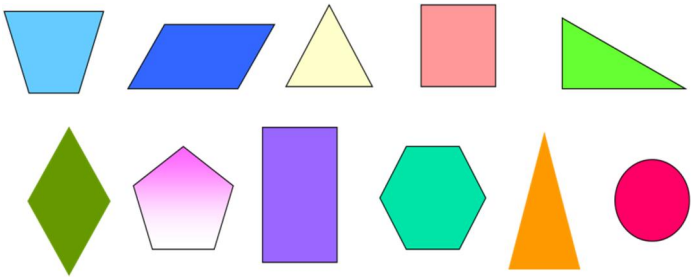
Lesson Procedure

№	Lesson stage	Activities/tasks description	Time	Notes
1.	Introduction/ Motivation/ Warm-up/ Review	Students are given photos of different objects – butterflies, leaves, buildings, folklore patterns, folded along the axis of symmetry.	5 min	

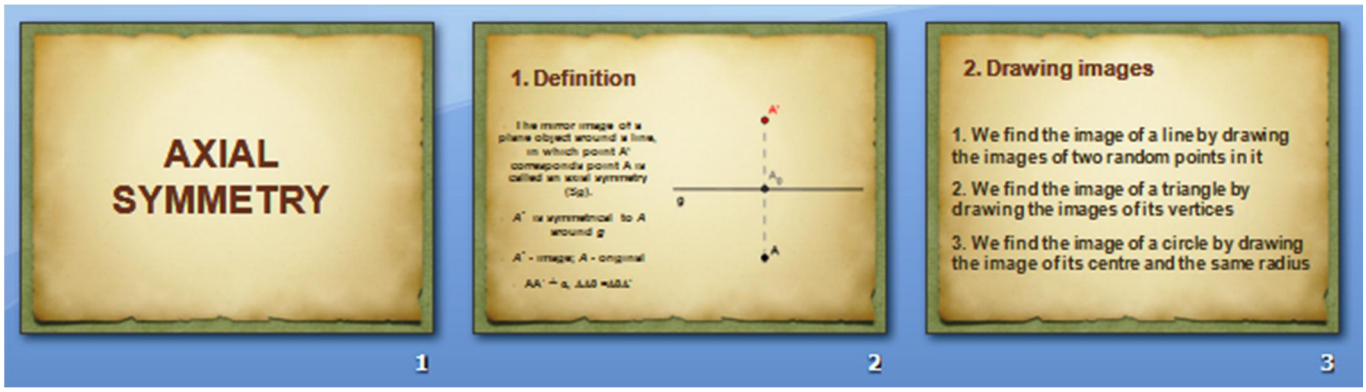
		  		

		   Teacher's introductory questions: "What do you notice when folding and unfolding the pictures?", "What happens to both halves?". Teacher's comment: "Shapes whose halves match up when folded are called symmetrical and the line that divides them is called the axis of symmetry." If we fold the sheet along this line the shapes match up perfectly (we demonstrate this with different shapes). In this lesson you will be introduced the amazing notion of "axial symmetry" both from mathematical		

		point of view and with examples from nature,architecture and everyday life. In ancient times the word “symmetry”stood for “beauty”, “harmony”. In Greek “harmony”means “proportionality”, identity in the disposition of parts.		
2.	Presentation	Using a presentation, the teacher teaches on the board how to draw the image of a point when it is subject of axial symmetry. Then students draw the images of a line,a circle and a triangle, supervised by the teacher.	25 min	
3.	Practice/ Activities ➢ Guided ➢ Independent	Problems: 1.ABC is an isosceles triangle (AC=BC). Prove that the angle bisector at C is the axis of symmetry. 2.G is the axis of the axial symmetry. Draw the image of a ractangular trapezium whose long base lies on the axis of symmetry.	5 min	
4.	Checking for understanding/ Assessment/ Evaluation	Students work in groups and are given sets of a worksheet and a sheet with geometrical shapes. They have to mark the shapes as: having no axis of symmetry, having just one axis of symmetry and having more than one axis of symmetry. 	10 min	

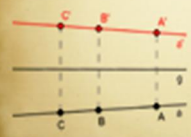
<table>
<tr>
<td></td>
<td></td>
<td>Worksheet
<table><tr><td>Shapes, which have no axis of symmetry</td><td>Shapes, which have one axis of symmetry</td><td>Shapes, which have more than one axis of symmetry</td></tr></table>
After solving the problem each group elicits a representative to explain the solution of the group.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Closure/ Homework</td>
<td>Individually or in groups, students have to prepare presentations, reports or posters to demonstrate different occurrences of axial symmetry.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Follow-up/ Extension</td>
<td>After teaching central symmetry, the exercise with different shapes can be developed further by looking for shapes with central symmetry and shapes with both axial and central symmetry.</td>
<td></td>
<td></td>
</tr>
</table>

There is a PPT which can be found in the e-learning platform.

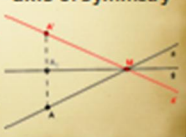


3.Images of basic shapes

- A line



A line through the axis of symmetry



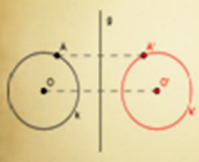
4

A line, parallel to the axis of symmetry

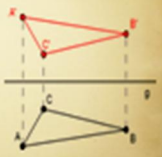
A line, perpendicular to the axis of symmetry

5

A circle



A triangle



6

4. Characteristics

A. The image of a segment is a segment, identical to it
B. The image of a line is a line and of a ray – a ray
C. The image of an angle is an angle, equal to it
D. The image of a triangle is a triangle, identical to it
E. The image of a circle is a circle, identical to it

7

5. Symmetry in some flags

8

6. Symmetry in art

9

7. Symmetry in the human body

Leonardo da Vinci's Vetruvian Man is a fine example of the idea of art, symmetry and science during the Renaissance.

10

8. Symmetry in nature and animals

11

9. Symmetry in architecture

12

Lesson Plan 4

TOPIC: Area of triangles

AIMS AND GOALS

- to generalize the knowledge of the area of a triangle

ABILITY TEACHING

- students are able to use the formulae of the area of a triangle using visual materials and students' knowledge for doing tasks

LESSON TYPE: LESSON OF KNOWLEDGE GENERALIZATION (THE USE OF KNOWLEDGE AND ABILITIES)

METHODS OF THE LESSON: demonstration, group/ team work, explanatory discussion

PROCEDURE OF THE LESSON

No.	Time	Activity	Teacher's activity	Students' activities	Materials and tools
1.	4 min.	Introduction	Teacher introduces the topic, aims and goals of the lesson, talks about expectations, enables students to do self-assessement and make groups.	• Students familiarize with the the topic, aims and goals of the lesson, expectations; • They contemplate the level of the topic knowledge and abilities, assess their knowledge using the Internet tool *Kahoot* ; • Make groups of three using cards in different colours.	• Slides with the topic, aims and goals of the lesson and expectations; • The Internet tool *Kahoot* for self-assessment; • Cards for making students' groups.
2.	9 min.	Revision	• Teacher demonstrates the	• Listen to the materials demonstrated;	https://prezi.com/f53t7l2ge1nc/trikampio-rusys-pagal-

			revision material of the topic; • gives questions for topic revision; • reminds of the formula for calculating of the area of a triangle.	• Answer the questions; • Revise the formulae for calculating of the area of a triangle.	kampus-ir-krastines-pitagoro-ir-atvir/ Slides with questions. http://www.kontroliniai.lt/trikampio-ploto-formules.php
3.	20 min.	Group work	• Teacher gives copies with tasks in groups; • gives advice.	Students do tasks in groups, check, discuss; when the whole group does not know something, students ask for teacher‘s advice.	Slides for demonstrating tasks; Copies with tasks.
4.	8 min.	Generalization	Teacher asks students to use a calculator on the Internet in order to assure if they have calculated correctly; he/she demonstrates how to use the online calculator for calculating of the area of a triangle;	Students listen to the explanation and try to use the possibilities of the online calculator; Each group comments about the result of one task to the class – one student of each group gives the opinion of that group members: 1. Did they reach the goal of the lesson?	http://www.mat.lt/matematikos-formules/figuru-plotai.html Slides with questions.

				2. Were they successful in collaboration and help with each other? 4. What mistakes did they make most? 5. What was the best way to work: in a group or individually?	
5.	1 min.	Homework	Copies with homework tasks individually prepared for every student.	Take copies with tasks.	Tasks can also be given using e-mails or a class Facebook group where students can find their homework.
6.	2 min.	Self-assessement		Students fill in self-assessment sheets or assess themselves using the Internet tool *Kahoot.*	Self-assessment sheets or the Internet tool *Kahoot* for self-assessment;
7.	1 min.	The end of the lesson	Teacher generalizes observations of the lesson.		

TASKS:

- The length of two sides of a triangle is 30 cm and 40 cm and the length of the altitude lowered to the third side is 24 cm. Find out the area of the triangle.
- The length of one leg of a right-angle triangle is 9 dm and the size of the angle next to this leg is 30°. Measure the area of the triangle.
- Measure the area of the equilateral triangle if the length of its altitude is 2 m.
- Measure the area of the triangle when its sides are equal 10 cm, 17 cm, 21 cm.
- Legs of a right-angle triangle ABK are 5 dm and 12 dm. Measure the area of a triangle ABK.
- Measure the area of the triangle ABC when the side AC is 6 cm and the altitude BD is 4 cm.

Lesson Plan 5

Subject: Mathematics

Grade/Students` age: 12-13 years old

Topic/Theme: introduction to the equations of 1st degree for students with educational difficulties

Lesson type: Individual work / work in pairs

Time: 1 week

Aims/Goals: Working language algebraic and the concept of first grade equation.

Objectives:

- Concepts:Algebraic language. Rules to solve simple equations.
- Skills: Mathematical competence, competence to work in pairs and learning to learn.

Materials/Resources/Equipment:

- Online resources/websites: See Introductory activities
- On line activities: Unicoos and the games page contained in Annex1
- Worksheets: yes
- Equipment: smart board– e-books
- Other: moodle – drive

Lesson Procedure

1. Lesson stage	Activities/tasks description	Time	Notes
Introduction/ Motivation/Warm-up/ Review	What do you know about equations? Why do we use the letter X?	10 min.	
2. Presentation	Watchthevideo https://www.youtube.com/watch?v=yGs7PhY_wOs	10 min.	
3. Practice/Activities	Activities carried out individually or in pairs. See Annex 1	4 sessions	

	Watch also that video: https://www.youtube.com/watch?v=eTT7IYC5FA8		
4. Avaluation	70% individual written test 20% evaluation of notebook 10% attitude and participation	1 session	
5. Homework	Some of the activities of Annex 1		
6. Extension	Http://matematico.es/competicion/mapa/inicio/?nivel=2 videos atUnicoos		
7. Reflection This material is intended for students who have some difficulty working equations. It is very visual exercises designed for you to reach the students deduce the rules operating the resolution of the equations of the first degree. The teacher can let students work at their own rhythm and intervene when you see that there is a problem.			

Lesson plan 6

Subject: Pithagoras Theorem

Grade/Students` 8th grade **age:**14

Topic/Theme: demonstration

Lesson type: oral exposition using the interactive board

Time: Around 45 minutes

Aims/Goals: calculate the size of sides in right angle triangles and inaccessible places distances measures

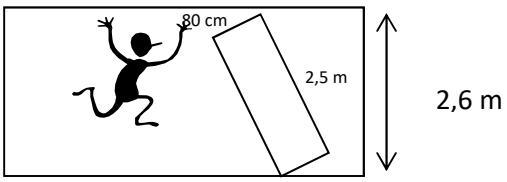
Objectives:

- Concepts: right angle triangles, area of squares and other polygons, solve equations
- Skills: application of root square, solve distance problems

Materials/Resources/Equipment:

- Online resources/websites:
- Online activities:
- Worksheets:
- Equipment: computer, interactive board,
- Other: Geogebra (free software), Quiz Faber and Turning Point.

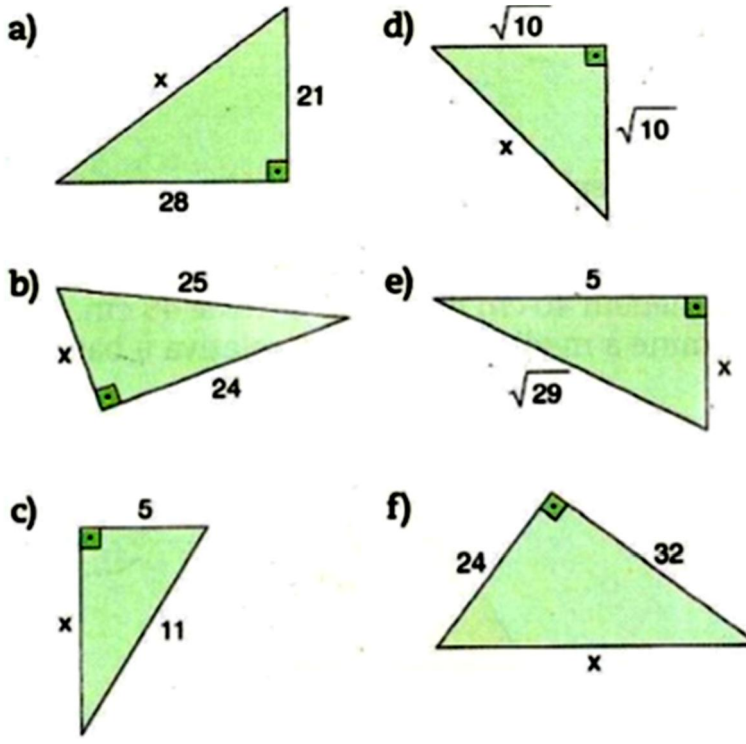
Lesson Procedure

№	Lesson stage	Activities/tasks description	Time	Notes
1.	Introduction/ Motivation/ Warm-up/ Review	Can Joe put the closet (80 cm x 2,5 m) against the wall with 2,60 m high? 	5′	
2.	Presentation	Where can we built the right angle triangle in the picture above to help Joe?	15′	

		A possible way is applying the Pythagoras´ Theorem.		
3.	Practice/ Activities ➢ Guided ➢ Independent	Guide activities (A) independent activities (B)	20´	
4.	Checking for understanding/ Assessment/ Evaluation	*Kahoot* about Pithagorean Theorem		
5.	Closure/ Homework	Woorksheet Worksheet (C)		
6.	Follow-up/ Extension	Pithagorean theorem applied to the space Pythagoras and space (D)		
7.	Reflection			

Guide activities (A)

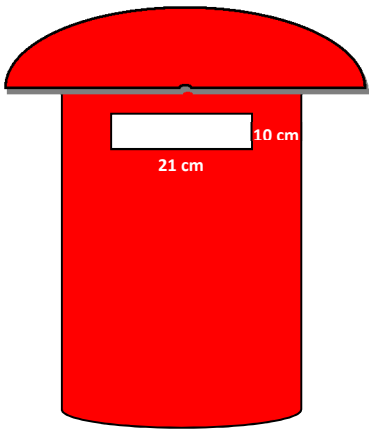
1. **Applying** the Pithagorean Theorem, **calculate** the **x** value in each right angle triangle

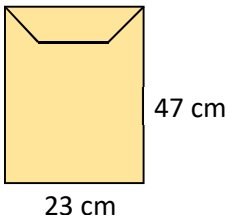


2. **Complete** the following table, knowing that ***m***, ***n*** and ***p*** are the measures of hypotenuse, leg **a** and leg **b** in a right angle triangle, respectivelly.

m	***n***	***p***
	24	32
3,7	1,2	
	1,6	3
	$\sqrt{5}$	$\sqrt{5}$

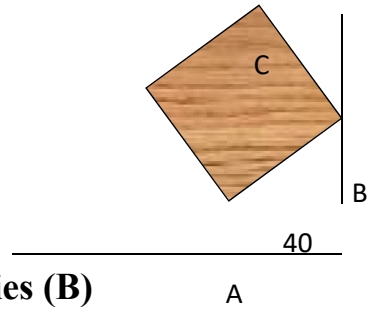
3. Does the envelope fit the mail post without being bended?





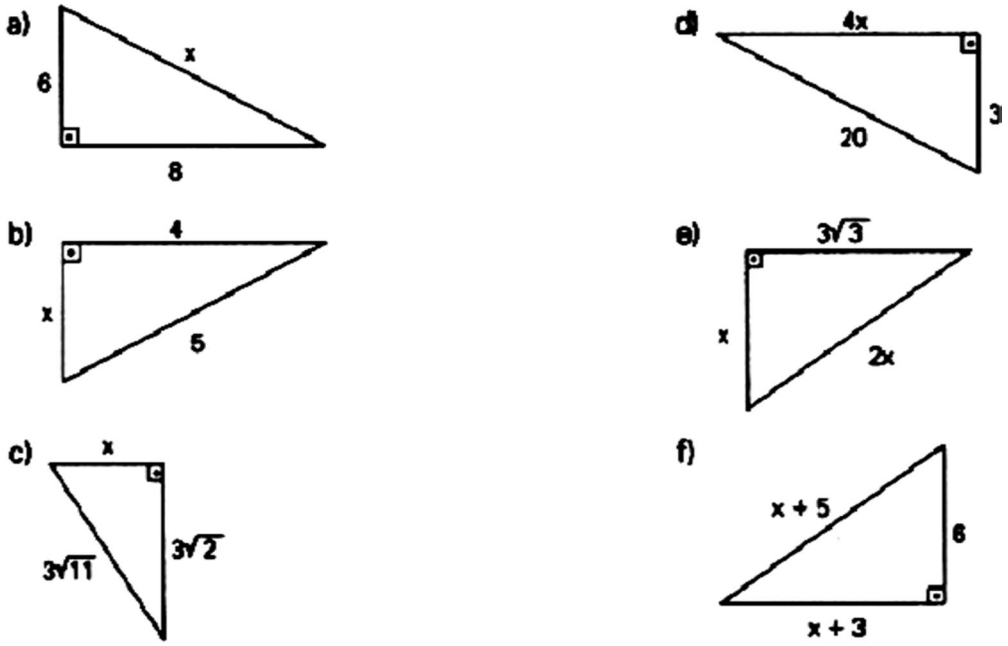
4. A cubic box with 0,5 m length is against a Wall and fixed on B like the figure shows. The length from A to the Wall is 40 cm.

   At what high is B fixed?



Independent activities (B)

5. **Applying** the Pithagorean Theorem, **calculate** the **x** value in each right angle triangle

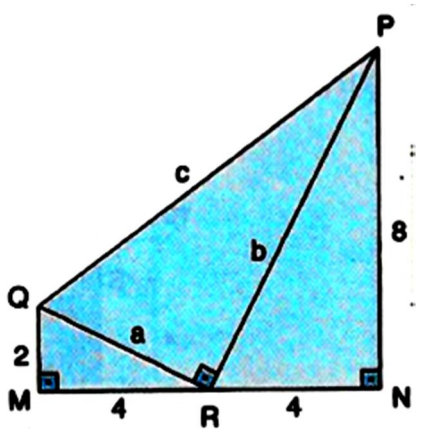


6. The following table is about lengths of triangles' sides. Which of them are right angle triangles?

Triangle	Sides' length (centimeters)		
	a	***b***	***c***
I	6	4	3
II	12,5	12	3,5
III	15	12	8
IV	37	35	12

7. Pay attention to the picture.

Calculate:



a) a $(\overline{QR})$;

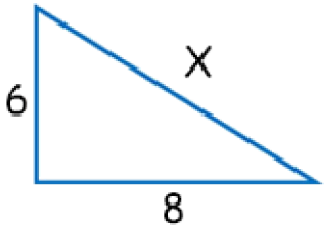
b) b $(\overline{RP})$;

c) c $(\overline{QP})$;

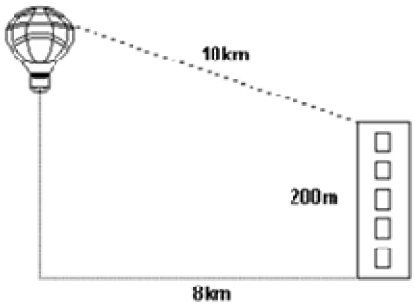
d) trapezium [QMNP] perimeter.

Worksheet: Pithagoras' Theorem (C)

1. Find the value of x.

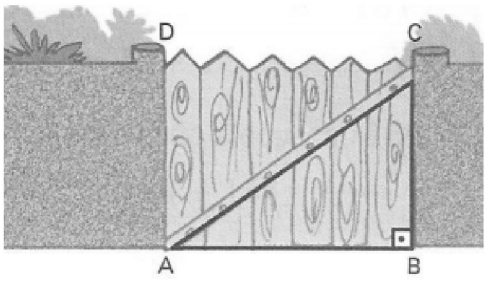


2. What should be the balloon altitude so that its distance from the top of the building is 10 km?

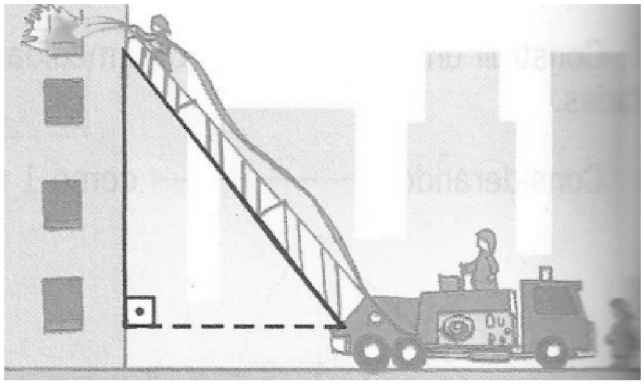


3. Calculate the barbed wire footage used to surround a triangular plot of perpendicular measurements of 60 and 80 meters, knowing the wire fence will have 2 wires.

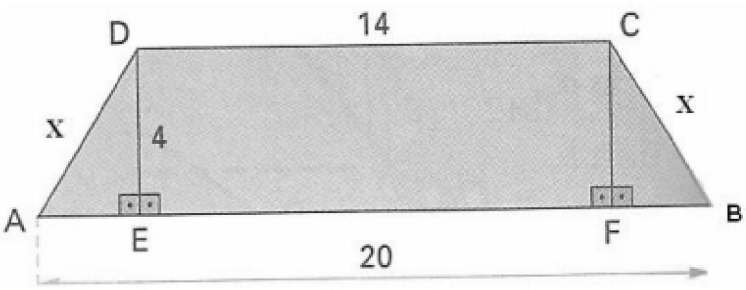
4. The entrance gate of a house is 4m long and 3m high. What length would have a wooden beam that extended from point A to C?



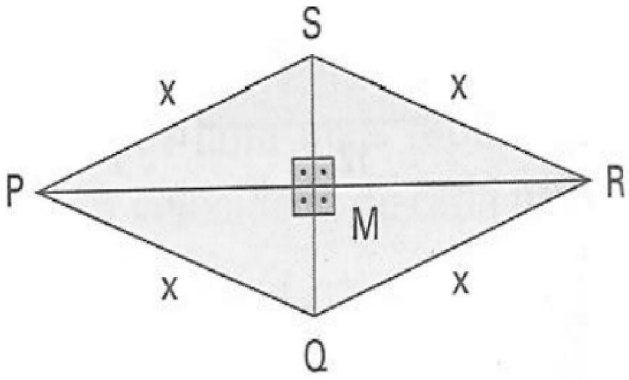
5. During a fire in an apartment building, firefighters used a 10 m ladder to reach the window of the apartment on fire. The ladder was placed 1m from the floor and 6m away from the building. What is the height of the building on fire in relation to the ground?



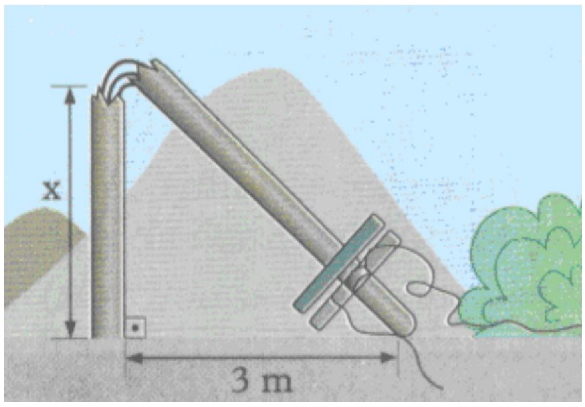
6. Analyzing the isosceles trapezoid, determine the measure "x", the perimeter and the area.



7. In a diamond, the diagonals cut each other in half, meaning the meeting point of the diagonals is the diagonal midpoint of each one. In PQRS diamond, the longest diagonal measures 80cm and the smaller one measures 18cm. Determine the value of "x", the perimeter and the area.

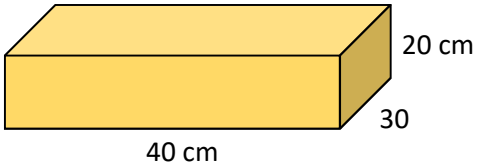


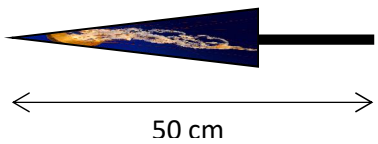
8. In a recent windstorm, a lamp post 9 meters high was broken at a point distance x from the ground. Part of the pole above the fracture leaned over and touched its upper end the ground at a distance of 3 m from the bottom of the same. So the hypotenuse worth (9-x) meters. How high land the broken pole?



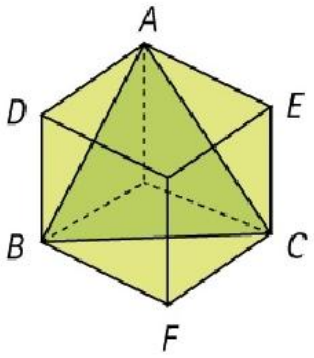
Pythagoras' Theorem in Space

8. Does the umbrella fit in the box?

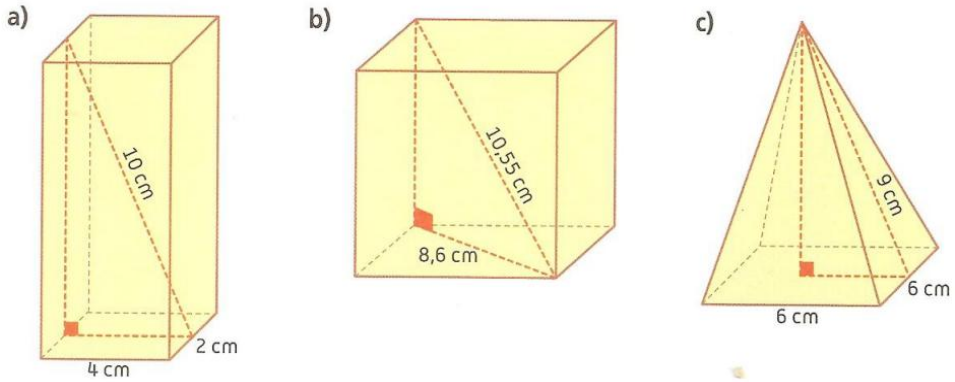




9. The following represented cube edges measure 10 cm. Calculate the perimeter of the triangle [ABC] defined by three of its vertices.



10. Calculate the volume of each solid (to one decimal place).



11. The bases of a prism are rectangular 10 cm by 7 cm and diagonals of the major faces measuring 26 cm.

Calculate:

a) The prism height;

b) The prism volume;

c) The perimeter of the triangle [ABC].