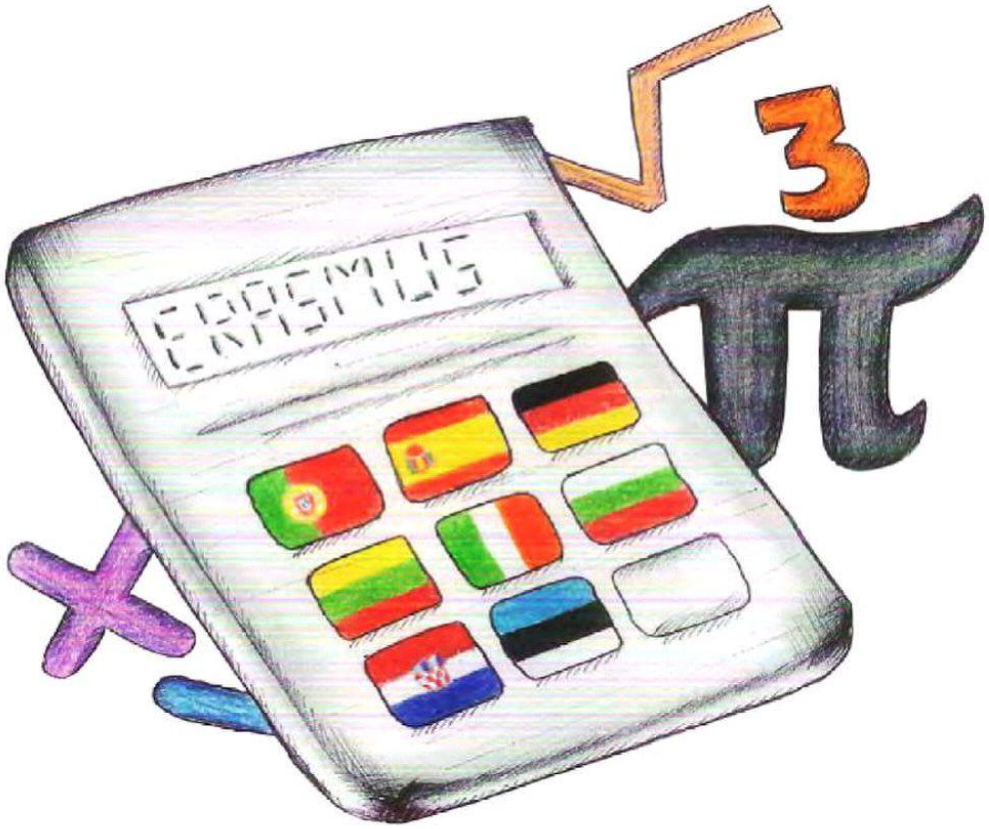
“Active, Attractive And Interactive eU Mathematics”



Sommario

Introduction

About the Guide

"Active, Attractive And Interactive eU Mathematics: Methodological Guide for innovative education" is an intellectual output created as a result of the cooperative efforts of eight European schools within a partnership under the Erasmus+ Programme, KA2 - Cooperation for Innovation and the Exchange of Good Practices, Strategic Partnerships for school education.

The leading institution is SU "Sveti Sedmochislenitsi"-Targovishte, Bulgaria.

The other partners are:

Industrijsko-obrtnička škola-Slavonski brod, Croatia

Tallinna Kristiine Gümnaasium-Tallinn, Estonia

Istituto Professionale Statale per l'Industria e l'Artigianato-Cernusco sul Naviglio, Italy

Marienschule-Brilon, Germany

Sakiu rajono Lekeciu mokykla-Lekeciai, Lithuania

Agrupamento de Escolas de Abação-Abação, Portugal

IES Pau Casesnoves-Inca, Spain

Pic. 1: First Transnational Project Meeting in Bulgaria-the project team in front of the school

The Guide, in a form of a book (electronic and printed), is based on some project results and is closely connected with the other intellectual outputs-lesson plans and an e-learning platform, both created by Mathematics teachers involved in the project. The Guide would be in an interest of Mathematics teachers, prospective teachers, teacher trainers, other interested in the field experts and stakeholders. The book will be disseminated and distributed for free: the English version-to international partners; the translated versions-to other interested stakeholders (teachers from other schools in the region and beyond, experts, teacher training centers and organizations, libraries, pedagogy universities, educational authorities, other organizations and institutions, interested in the field.

The presented in the Guide activities mainly describe teaching methods in which students play an active role, interact with different resources, collaborate and generate knowledge. It includes web-based learning, quiz competition, project-based learning, educational games, discussions, funny Mathematics activities, assignment, practical work, etc. We focus on using multimedia technology as an innovative teaching and learning strategy in a problem-based learning environment.

About the Partnership

After a detailed analysis, we have identified a need to improve the quality of education in Mathematics. Some common students` views of mathematics say it is hard and boring, it has nothing to do with real life, and it is not needed beyond schooling. Many children do not like Mathematics because of the way it was taught to them. Only a few of them realize that this school subject has a great impact on society.

On the other hand, today's students are expected to learn about and use digital technology in Mathematics to prepare themselves for their future, the work force and the challenges of everyday life.

This project has been developed in order to give teachers additional tools to help more students recognize this common language so that all students develop some basic mathematical skills.

Pic.2: First Transnational Project Meeting in Bulgaria-a work session

Our Mathematics teachers face the challenge to teach Mathematics in an innovation-driven society. Various new models of education are evolving in response to the new opportunities that are becoming available by integrating new technologies into the teaching and learning environment. From the very beginning of our collaboration and the first contacts among us we have agreed on how important it is to understand the opportunities that ICT offers and to be aware of the ways that it can enhance the teaching and learning of Mathematics. The analyses of the situation in the partner schools have shown that Mathematics teachers are still not effectively integrating computer technology in their classrooms. While planning the partnership, we have also taken into consideration the value of Mathematics education which can be found not only in its ability to help contribute to students' college/university and career readiness, but also help develop individuals who can understand the world better because of their mathematics capabilities.

Pic. 3: The project team at the Estonian school

By designing the project, the partners have focused their efforts in meeting some European Commition priorities and requirements, as if:

- Supporting ICT-based teaching and assessment practices;
- Supporting teachers in acquiring/improving the use of ICT for learning and related digital competences;
- Supporting digital integration in learning in order to increase its quality;
- Supporting inter-disciplinary approaches and cross-curricular project work;
- Promoting problem-based learning;
- Foster innovative approaches to teaching in technology-rich environment with particular focus on mathematics;
- Supporting development and availability of open educational resources;
- Promoting best practices and supporting exchanges and mobilities across Europe;
- Personalization and collaboration through digital-supported learning.

Our main project aim was:

By using effective modern teaching methods, adapted to our students' abilities and needs, and ICT-based educational tools and content, we aim to develop and implement in the partner schools stimulating teaching and learning mathematics through collaboration, cooperation and exchange of good practices.

We do believe that use of innovative strategies, including ICT, has helped motivate students to put efforts into learning Mathematics.

The project has also been designed to help teachers realize the potential of innovative methodology by using technologies in the everyday classroom practice and by investigating the added value of the collaborative approach in creating the e-learning platform and the "Active, Attractive And Interactive eU Mathematics: Methodological Guide for innovative education".

Pic. 4: Short-term joint staff training in the Estonian school

Within the partnership, our concrete objectives were:

In regards to teachers:

- To motivate them to share best practice in web-based teaching, to enable them to become confident and competent enough to create a collaborative, interactive classroom experience for their students through the use of the ICT;
- To enable staff to engage in trans-European professional development activities, experience other education systems at work and exchange ideas of good practice;
- To encourage continued teachers` professional development.

In regards to students:

- To maintain their efforts to find relationships between Mathematics and their interests and abilities and also the practical use of mathematics through using active, interactive and web-based methods;
- To promote virtual mobility and other innovative learning and communication methods and to give young people an opportunity to interact with people across different social, cultural and national boundaries;
- To develop their key competences.

In regards to the collaborative work:

- To build a partnership that supports learning and designs high-quality products to respond to partner schools` needs;
- To disseminate the project results and outcomes to other interested parties (parents, teachers, schools, the wider community and across Europe).

Pic. 5: Short-term joint staff training in the Estonian school

The problems with teaching and learning Mathematics are essentially identical for the different European counties and schools, which makes it possible to strive for a common solution carrying out the project transnationally. Developing effective e-learning courses in a collaborative way is time consuming, so the need to share the work is obvious and necessary. Through sharing experience, exchanging good practices and working together in developing the e-learning platform and others teaching and learning materials, we have had the chance to achieve our goals and results in the best possible way. We hope the project`s results and outputs, including the web-based products, will be interesting not only for the partners but also for wider professional and non-professional auditory.
As the partner organizations represent different curriculas and education systems, they all have had what to offer in terms of sharing experience and organizing activities in fields they are specialized.

Pic. 6: Second Transnational Project Meeting in Lithuania - the project team in front of the school

Within the partnership, we have focused on Mathematics both as a school subject and a universal science language. We have adopted and implemented some innovative for our schools approaches and methodologies of teaching Mathematics, in particular computer-based teaching and learning, project- and problem-based learning, practical approach, learning by doing, peer-learning, research-infirmed strategies. As modern 21st century educational institutions, we have done our best to face the need and the big challenge to rapidly find new ways to teach Mathematics and new approaches for the classroom of the future. So, we have initiated this cooperation for innovation and the exchange

of good practices as the most appropriate and fastest way to meet the needs of our schools and, on the other hand, the necessity for professional growth of Mathematics teachers in a collaborative way. The partnership has contributed to creating a modern school Mathematics closely connected with today's public life. It has also helped young people understand Mathematics with attractive, interactive and web-based methods.

Pic. 7: Third Transnational project meeting in Germany-a work session

In order to reach our goals and objectives, the partners have followed this methodology:

1) Adaptation of existing materials;

2) Creation of new ones;

3) Setting up of the e-learning platform;

4) Continuous cooperation-import of new content into the platform in a collaborative way;

5) Quality check of the content and evaluation-feedback from the target groups;

6) Import of new content into the platform;

7) Extension and adaptation to full-scale potential of the platform;

8) Continuous evaluation of project activities and results;

9) Experimental use in all partner schools;

10) Evaluation, implementation, feedback from stakeholders;

11) Continuous dissemination of the results and outputs;

12) Continuous exchange of good practices among partners and stakeholders;

13) Final reporting and evaluation.

Pic. 8: Third Transnational project meeting in Germany-
the project team in front of the Town Hall

Our Mathematics e-learning platform-one of the intellectual outputs:

The Mathematics e-learning platform (open source, based on MOODLE software) has been set up by the Bulgarian partner school. After that it has been developed collaboratively by all project partners. All the Mathematics materials (exercises, Mathematical tasks and problems, assignments, short videos about specific objects, presentations, tests and other assessment materials, animated graph etc.) have been uploaded and organized by our Mathematics teachers to be tested in Mathematics lessons from some teachers and students (directly involved in the project activities) from all partner schools. All partner schools have been in charge with creating teaching and learning materials; collecting opinions and feedbacks from experts, representatives of the target groups and stakeholders; giving suggestions for platform`s improvement, outputs` evaluation.

Here is the web address of the platform: http://www.e-mathematics.eu/

A short-term joint staff training event, held in Estonia, has also been planned. Within the work programme of the training activity, the participating teachers have been involved in diverse sub-activities, closely connected with the partnership`s objectives, local project activities and intellectual

outputs, as if: physical exchange of experience and good practices; presentation of the e-platform, workshops and training activities on using it; free face-to-face discussions about the web-based teaching and learning and other innovative methods in teaching Mathematics in partner schools. Each partner institution has prepared a presentation and shared experience and good practice in using different educational software or in applying different innovations. The platform`s content and the lesson plans created by the partners so far, have been evaluated from a pedagogical point of view.

Pic. 9: Fourth (and last) Transnational Project Meeting in Italy-
the project team in front of the Town Hall

Conclusion:

All partners do believe that the e-learning platform (as a main intellectual outcome) and the innovative teaching and learning methods (some of them explained in the Guide) have set a benchmark of Mathematical knowledge and competences of the students and made learning and teaching Mathematics much more interesting, motivating and efficient. It has also given an opportunity to the pupils to be able to learn and prepare by themselves.

CHAPTER 1

IDEAS FOR WEB-BASED INNOVATIONS

The World is changing so quickly and teaching needs to adapt to the new inputs and technologies . Our students are hooked by all the new electronic devices that they use then in their daily life. Moreover the new technologies offer many potenctialities and allow us to construct dinamical pictures and make calculations more quickly. We can also share other people's works, lessons,… Nowaday, traditional lessons do not motivate student anymore, so we have to catch their attention using new tools.

In this chapter, we use Geogebra, Youtube ,Youmath, Kahoot, prezi and other apps to introduce the lessons and motivate the students. Then we can also share these experiences with other teachers all over Europe and, why not,the World. The students are invited by the teachers to prepare presentations based on lessons and share them on the Internet.

1.1. Coordinate plane - Geogebra applet

Type of innovation: blended learning; online learning; open educational resource; observation and description; relating of lessons to real life; classroom management and organization; design thinking tools and methods-problem solving; remote/virtual interaction between students; project-based learning; peer tutoring/instructions; laptops and wireless technology for anytime and anywhere learning; non-academic life skills curricula; after school programs; educational games

Theme/Topic: Coordinate plane **-** Geogebra applet

Students` age: 13-19

Time (number of sessions, min): 90 min

Goals/aims/objectives: Create Geogebra applets for easier learning.

Materials needed: Computer, projector, access to internet, Geogebra

Url of the online products used: Lesson materials made by Igor Vidović and uploaded to www.e-mathematics.eu, www.geogebra.org/phoenix25, https://prezi.com/wamkrpwoiqzj/coordinate-plane/#

Kinds of technologies used: animation; websites; gaming; Internet

Procedure:

- find mathematic problem
- create method for solving it
- imagine how your applet should look like
- create Geogebra applet using teachers guide
- upload Geogebra applet online

Products made: online event/session; Geogebra applet: Lesson materials made by Igor Vidović and uploaded to www.e-mathematics.eu, www.geogebra.org/phoenix25, https://prezi.com/wamkrpwoiqzj/coordinate-plane/#

Advantages and disadvantages of the innovation

Advantages:

- good motivation
- individual work
- encouraging thinking
- creating own applets
- making interesting mathematic

Disadvantages:

- computer literacy
- slow learning Geogebra commands

Comments/suggestions/ pieces of advice:

Teacher should have excellent knowledge in Geogebra and give students examples and guides how to create applets. This innovation can be use in regular class or extracurricular. If it is you first applet or you are basic user, then you should need more time to create it.

1.2 Quadratic inequalities

Type of innovation: learning to use the geogebra graphic part to solve quadratic inequalities.

Theme/Topic: Algebra quadratic inequalities

Students` age 15-16years old

Time (number of sessions, min) :1 week

Goals/aims/objectives : solving second degree inequalities

Concepts: Quadratic inequalities, graphic solution

Skills: mathematical competence, self-assessment and self-correction.

Materials needed:

- Online resources/websites: using geogebra
- On line activities: examples and exercises
- Worksheets: yes
- Equipment: digital board– e-books.
- Other:

Url of the online products used:

Lesson materials uploaded to www.e-mathematics.eu

Theory lesson and examples: www.purplemath.com/modules/ineqquad2.htm

Examples: http://www.shmoop.com/quadratic-formula-function/quadratic-inequality-exercises.html

Network software: www.geogebra.org

Kinds of technologies used: websites/blogs; Internet

Procedure:

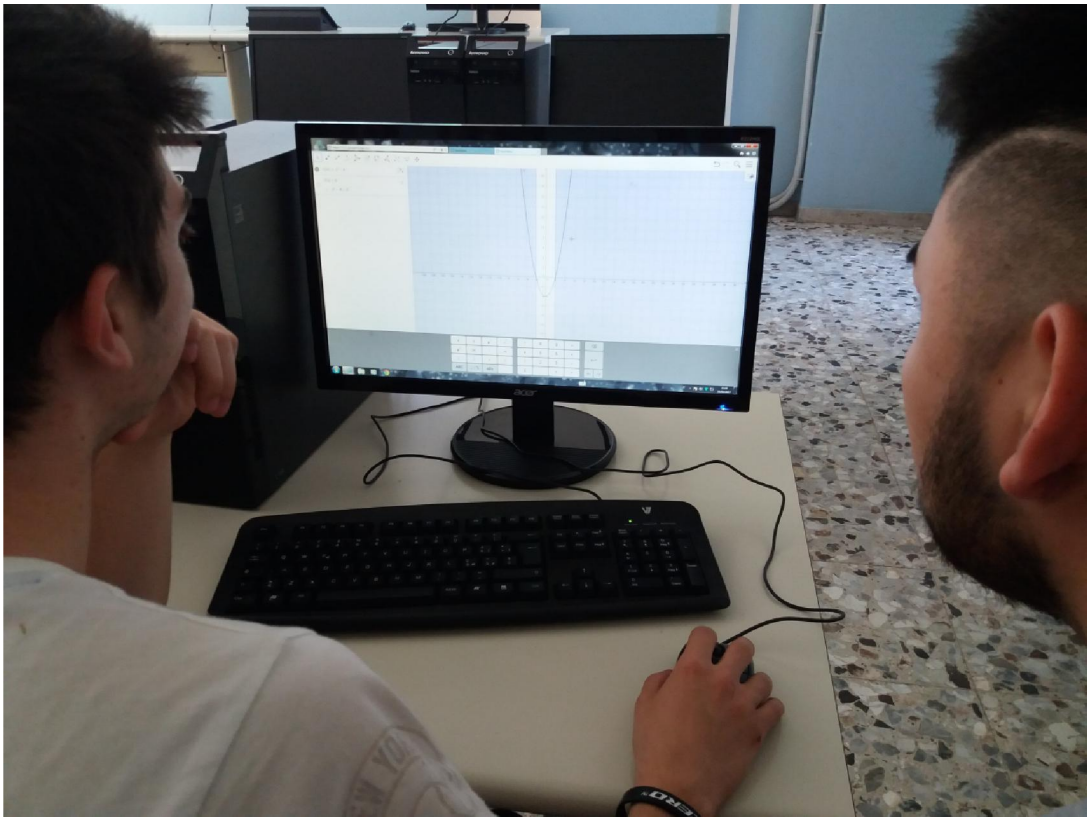
- The lesson starts with a power-point presentation uploaded to www.e-mathematics.eu
- It's now time to learn how to solve inequalities graphically using Geogebra: at first choose the working sheet "graphic calculator", and then insert the equation of the function, the program will draw the parabola. Now write the whole inequality: the program selects the section of the plane which verifies the inequality. The last step is writing the solution set.
- Finally the teacher propose to make exercises that can be corrected using the site: http://inequalities.intemodino.com/it/risoluzione-di-disequazioni-di-secondo-grado.html

Products made: exercises

Advantages and disadvantages of the innovation

- The use of the graphical method makes the exercises more understandable for those students who have difficulty understanding the math concepts.
- Online gambling exercises can encourage more students to know how to solve the quadratic inequalities.
- Self-correction and self-assessment in real time are the main advantages of on-line exercises

Webbasedquadraticinequalities1.jpg



Webbasedquadraticinequalities2.jpg

1.3 Square of binomial

Type of innovation: group work; practicing mathematical concepts, working on the web; learning a mathematical rule and being able to apply it correctly.

Theme/Topic: Algebra Square

Students` age 14-15 years old

Time (number of sessions, min) : 1 week

Goals/aims/objectives: learning a mathematical rule

Concepts: Binomial product: the square

Skills: mathematical competence, self-assessment and self-correction.

Materials needed:

- Online resources/websites: introductory activities
- On line activities: exercises http://www.kwiznet.com/p/takeQuiz.php?ChapterID=2790&CurriculumID=25&Num=7.13
- Worksheets: yes
- Equipment: digital board– e-books, Excel
- Other: cardboard, scissors, pencils….

Url of the online products used:

youtube video: **www.youtube.com/watch?v=RoLxbZw2iTQ** (the first part)

Kinds of technologies used: websites/blogs; Internet

Procedure:

- The lesson starts with a web-video showing the development of the square of a binomial using a cardboard. The immediate, straightforward geometric references help to focus on the

concept in a tangible way .The next step is the realization of the square on a cardboard . The students work in groups of three and reproduce what they have been shown in the video. This activity helps them to become familiar with the involved geometrical concepts.

- The next level is the teacher's theoretical explanation stressing the importance of the formula as it, allows a quick computing solution. The innovation of this methodology consists in making the students memorize a formula consisting of some symbols which sound like a" nursery rhyme" .With this TEACHING STRATEGY the students cannot miss the basic necessary steps TO UNDERSTAND THE RULE.
- The students are then asked to solve some easy exercises on the board with the teacher's help while as homework they will be assigned exercises on the web.
- Finally other exercises will be solved on an exceI sheet which gives the students the possibility to self-correct and self-assess their work.

Products made: exercises

Advantages and disadvantages of the innovation

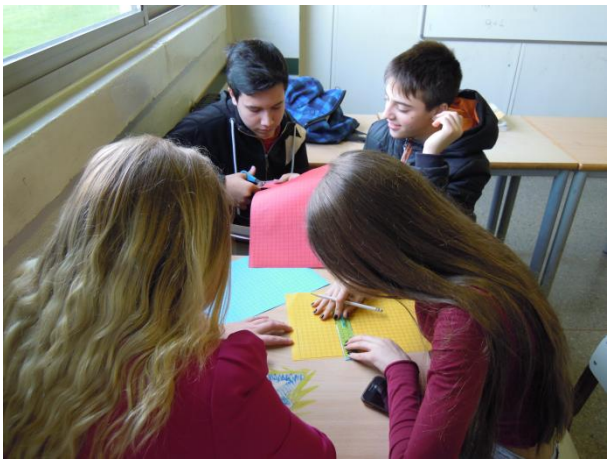
- learning by doing is a great advantage: redoing what has been shown only twice stimulates the students' attention for details. While working in small groups helps those students who normally feel embarrassed in front of the rest of class.

Comments/suggestions/ pieces of advice

The topic is objectively easy,in addition it has been treated in different ways to allow the students a deeper understanding of the rule which becomes easier to memorize and apply.

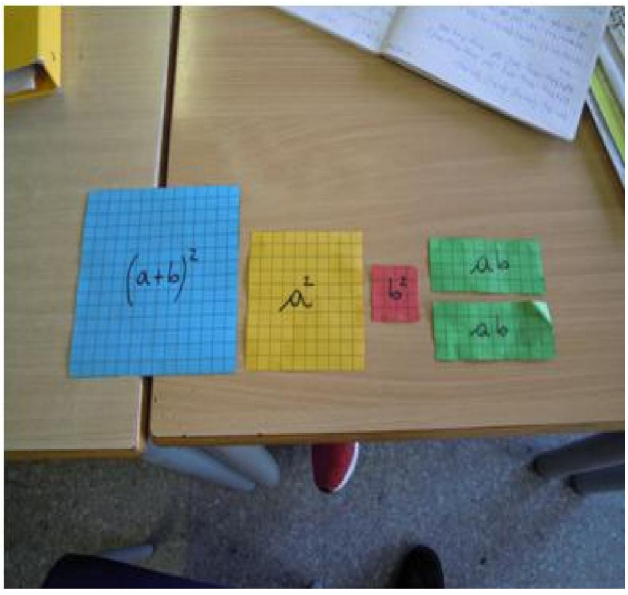
We are attaching a few photos that show the task done in teamwork as well as the results.

Webbasedsquare1.jpg



Webbasedsquare2.jpg

Webbasedsquare3.jpg



Webbasedsquare4.jpg

CIRCLE SEAGULL CIRCLE

$$(a+b)^2 = a^2 + 2ab + b^2$$

1.4 Axial symmetries

Type of innovation: Group work; put math concepts into practice, work on the web, interactive concept learning using didactic software

Theme/Topic: Axial symmetries

Students` age 16-17years old

Time (number of sessions, min) 8 hours

Goals/aims/objectives: Learning calculation methodologies of symmetric images with respect to an axis

Concepts: Axial symmetry.

Skills: Mathematical competence, self-assessment and self-correction.

Materials needed:

Online resources/websites: Introductory activities, activity of deepening

- On line activities: Exercises
- Worksheets: yes
- Equipment: Digital board, e-books, software Geogebra
- Others: notebook, squadrette, riga, matite e almeno due penne di colori diversi.
- Exercise book, ruler, set square, pencils at least two different colored pens

Url of the online products used:

youtube video: http://www.raiplay.it/video/2016/10/Bolle-esegue-i-tre-balletti-Prototype-95cfaf70-a732-4948-86d7-82bd8888a652.html

with no comment youtube video: https://www.youtube.com/watch?v=2zXNr2P5O6s

youtube video with comment:https://www.youtube.com/watch?v=P-ys-3Drp4s

Theory lesson on Youmath: http://www.youmath.it/formulari/formulari-di-geometria-piana/2252-simmetria.html

Video on the concept of axial symmetry: https://www.youtube.com/watch?v=OtwqT2wUUfc

Exercises in motions with Geogebra: http://geogebra.altervista.org/isometrie.htm

Recovery exercises on Zanichelli site: worksheet

http://online.scuola.zanichelli.it/bergamini-files/Biennio/Recupero/bergamini_trasfcartesiane_R2_14V.pdf

Site rich in symmetry evocative images divided according to their content

http://www.matematita.it/materiale/index.php?p=cat&sc=640

Kinds of technologies used: websites/blogs; Internet

Procedure:

- The lesson starts with a video showing the ballet dancer Roberto Bollein“ La mia danza libera” in order to attract the students’ attention on the strong bond between Math , in particular Geometry, and dance. After that a power point presentation will deepen the concept of Axial Symmetry and show the connections with reality ,after that the students are left to their considerations until the next lesson.-
- The second lesson starts with a silent tutorial (made with Geogebra) on the construction of the symmetric as regard to the axis of a polygon which will help the students grasp the definition of axial symmetry and the related properties. Next the students with the teacher’s help will express formulaically the definition and the related characteristics on the board.
- At the end of the lesson the teacher suggests some useful sites both for theory and activities.
- After that 6 hours will be devoted to the solution of exercises on the algebraic calculus of symmetric figures and their graphic representation on the Cartesian plane using ruler and compass and on the graphic construction of axial symmetries using Geogebra software.

Products made: Power point presentation exercises and formulary

Advantages and disadvantages of the innovation

- The use of the new digital technologies fosters new didactic methodologies and new learning possibilities which facilitates the students’ creativity , research , discovery, experimentation , involvement and motivation. It changes the teaching context from a

notion classroom taught lesson to a an interactive and social one, Learning by doing(as John Delivery used to say).

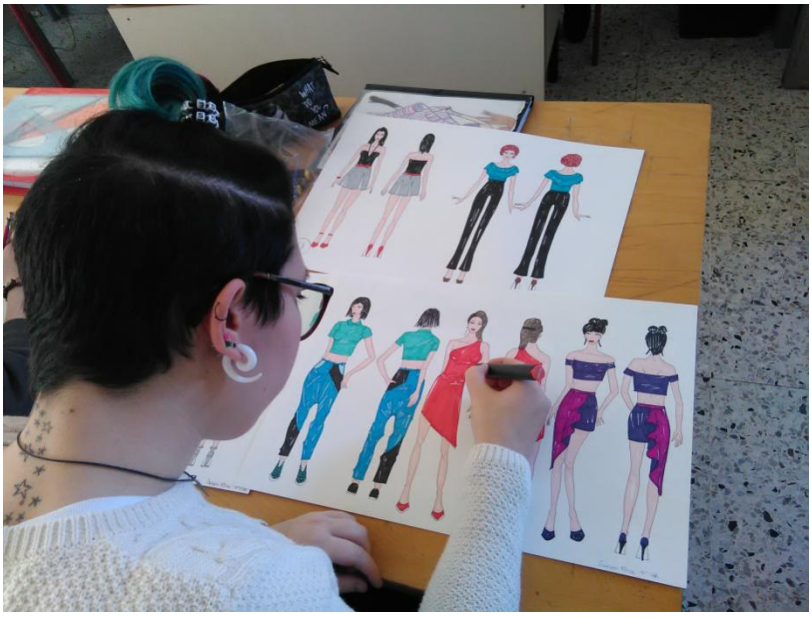
- The main advantages of the on-line exercises are the self correction and self assessment in real time.

Comments/suggestions/ pieces of advice

For the first time students are given powerful means for the acquisition of knowledge in an interactive, social and collaborative way. These new learning forms takes advantage from the personal creativity and the shared contribution of knowledge, and from each member skill and experience

Webbasedaxialsymmetries1.jpg

Webasedaxialsymmetries2.jpg



Webbasedaxialsymmetries3.jpg

CHAPTER 2

IDEAS FOR TRANSCURRICULAR INNOVATIONS

“Why do we always learn only mathematics in our mathematics lessons? What is it for?”

Do your students ask you the same questions sometimes? So we try to give you in this chapter some new ideas how to combine mathematical topics with other subjects.

During our two-years-project every country has developed lessons, involving several subjects like for example physics, arts, handicraft or methods (methods-learning, discussing, brainstorming, mind-mapping, …) which can also be used in different subjects.

The aims of transcurricular learning are to

- combine different subjects
- realize that mathematics is part of real-life
- discover mathematical problems in other subjects
- change the presentation level (symbolic, iconic and acting)

We present some ideas which were innovative for the math teachers in the partner schools. So in one example from the Estonian school, younger students train first the calculations skills and control their solution in programming a bee, running the right way. So by the combination of mathematical and software capacities, students are more motivated to do this exercises.

In the German school teachers invented creative methods in order to give concrete examples for abstract mathematical topics. Solving simple linear equations by using a model made of matches and match boxes is not only more motivating but shows also in a sustainable manner how to solve equations. The teachers made the experience that students remember the different steps better than before.

Furthermore there are some more examples which show how to motivate students by using methods which are originally from other subjects or which can be also used in other subjects.

2.1. Beebot Math & Programming

Type of innovation (blended learning; open educational resource; observation and description; group work; classroom management and organization; methods-problem solving, cooperative learning; peer tutoring/instructions; laptops and wireless technology for anytime and anywhere learning; flipped classroom; outdoor learning, etc.)

Theme/Topic- Calculation, programming

Students` age :10-14 years

Time (number of sessions, min): 1 lesson - 45 min,

Goals/aims/objectives: To organize and repeat the calculating by the head and the order of the calculations. To develop logical thinking , problemsolving and teamwork.

Materials needed: Beebots robots (or similar easily programmable robots)

Url of the online products used: Youtube tutorials by own choice, https://www.bee-bot.us/

Kinds of technologies used (robotics; social media)

Procedure: Teacher must prepare on workboards/playingmats with mathematical written tasks on playingfield. Students calculate by head the results and write the results on the controlpaper. After that they must find the corresponding answer on the playingfield and program the robot to follow the path to correct answer. The final point will be known by the teacher. Depending on the age of students the tasks may be easier or more complex.

Advantages and disadvantages of the innovation

Comments/suggestions/ pieces of advice : At first students can solve tasks made by teacher, after some practice they can start produce similar tasks to each other or younger students. Its important to have the controlsheet for the teacher to check the calculations and final result.

Transcurriculabeebot1.jpg

Transcurriculabeebot2. Jpg

2.2. Learning by doing – understanding by practical experiences

Type of innovation observation and description; group work; relating of lessons to real life; cooperative learning; project-based learning; outdoor learning

Theme/Topic: Learning by doing – understanding by practical experiences

Students` age: 10 - 16

Time (number of sessions, min): minimum 1 session

Goals/aims/objectives: improvement of understanding mathematical topics, doing practical experiences, getting concrete examples of abstract problems, improvement of the capacity to generalize from concrete examples

Materials needed: depending on the topic, for example matches and match-boxes, tape, ropes

Url of the online products used: www.unterricht-als-abenteuer.de (only in German), https://www.youtube.com/watch?v=yZEg7cnDejM (how to use a pantograph - in English)

Kinds of technologies used: depending on the method: Internet resources

Procedure:

First, teachers have to search for examples to give to students concrete examples of the mathematical topic. For example we use matches and match-boxes to show to our students how to solve linear equations. In the match boxes there is an unknown number of matches; "x". In a first step, one student plays the role of scales holding a certain number of matches and match-boxes in both hands. What do the students have to do to solve the problem? The human "scales" have to stay in balance. In a second step, a pencil represents the equal sign. In a last step, these concrete operations are translated in algebraic operations.

Another example consists in building a "human" graph of a function by students. A Cartesian coordinate system is built by two ropes in the schoolyard for example. Each student will represent one solution of a function. The students are looking for here places in the coordinate

system. Later they draw the same graph into their exercise books. Another innovation is to build a pantograph in order to introduce to the topic: enlargement and reduction of shapes.

Products made oral/written presentation; scenario

Advantages and disadvantages of the innovation

This kind of innovation has a lot of advantages. First of all practical exercises are very motivating. Students like working with their hands. Secondly it is more easy to students to understand mathematical facts by practical exercises. Doing a task, they are thinking about the reasons: Why am I standing at the same place in the Cartesian coordinate system as my partner? How can I scale up and down drawings? Thirdly students will remember these exercises for a long time. When they will have to solve an equation a few months later, they will remember the matches and the match-boxes and find the correct way.

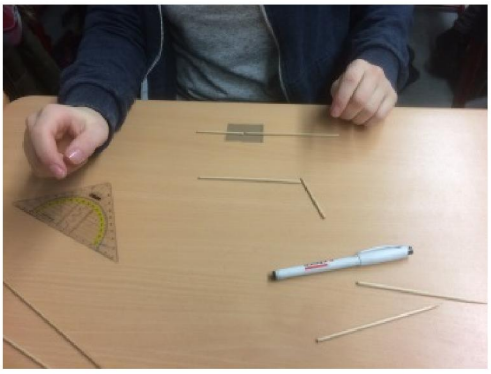
But there are also disadvantages. It will take time, to build a pantograph for example. Sometimes teachers think that it takes less time to explain the solution of a problem than to let students find the solution by themselves. But to tell the truth it might be better that students don't only learn but understand things.

Of course, teachers have to organize a lot of material, but once bought you can use it several times.

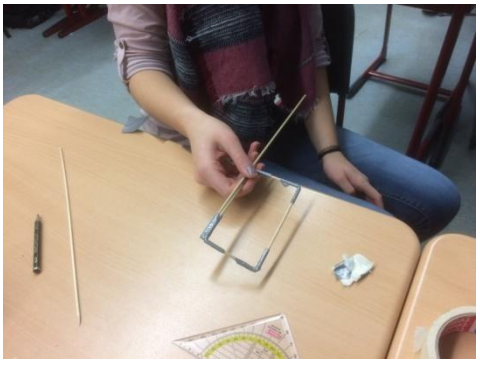
Comments/suggestions/ pieces of advice

Of course teachers have to be well prepared to this kind of lessons. They have to try the practical exercises by themselves in order to optimize them and to be ready to answer to questions or to be able to react to problems.

It goes without saying that teamwork between teachers in preparing this kind of lessons and between students in solving problems is very useful.



Pantograph1.jpg



Pantograph2

2.3. Let's make our own business

Type of innovation: Entrepreneurship and business; relating of lessons to real life; cooperative learning/instructions; integration of different subjects; team work and pair work;

Theme/ Topic: A practical business planning

Students` age: 14-19

Time (number of sessions, min): depending on structure of business/complexity of planning (1-4 lessons, 45min-180min)

Goals/aims/objectives: knowledge systematization and consolidation; collection of various information from different subjects on the Internet; find solutions to problems, teamwork and co-operation.

Materials needed: internet, spreadsheet and presentation program.

Kinds of technologies used: it depends on the complexity of the tasks which are given to students. Teamwork, working in pairs

Procedure:

The students will be given a task to choose a future business for themselves and to execute imaginary 1 year business plan, taking into account all the needed resources/costs and the connections between them. Students will investigate on internet what are the main business advantages in their region (handcraft, wood, metal, services etc) and will decide what company to build. They should start mapping all the mathematical resources they need to start a company- for example: equipment cost and room rental, staff salaries, materials for production, production time and advanced calculations about the governmental taxes, risks, profit etc.

Advantages and disadvantages of the innovation:

Approach the problem in a team/ pair work, cooperative learning inside/ outside the school and relating of lessons in real life is an advantage. Students naturally must explore the world around them, look for answers to all questions and practically examine the knowledge they have learnt at school.

Comments/suggestions/ pieces of advice

The teacher/s should prepare the frames on the topics/aspects covered by the students in their plans. Depending on the students age and knowledge teacher should choose how deep and complex should the plan be. Depending on the region/country the factors of government taxes may be different that is why it is difficult to prepare flexible samples for international use.

TranscurricularBusiness1

TranscurricularBusiness2

CASHBOOK **Name:** My Business **Month:** January

Details			Income				Expenses						BANK BALANCE	R
Date	Description	Ref	Capital	Caps	T-Shirts	Total	Asset	Drawings	Vehicle	Phone	Bank Fee	Total		
Jan-01	**Opening Balance**												**400.00**	
Jan-01	Telephone	1								55.00		**55.00**	**345.00**	R
Jan-05	Payment from Mr Jay	2		120.00		**120.00**							**465.00**	R
Jan-05	Bank fees	3									5.00	**5.00**	**460.00**	R
Jan-11	Petrol	4							40.00			**40.00**	**420.00**	R
Jan-15	Purchase of stock	5					450.00					**450.00**	**-30.00**	R
Jan-15	Cash paid in by owner	6	2,000.00			**2,000.00**							**1,970.00**	R
Jan-21	Cellphone Topup	7								25.00		**25.00**	**1,945.00**	R
Jan-21	Truck Repairs	8							100.00			**100.00**	**1,845.00**	R
Jan-21	Payment from Mrs Jones	9			150.00	**150.00**							**1,995.00**	R
Jan-31	Bank Interest received	10											**1,995.00**	R
Jan-31	Cash taken by owner	12						400.00				**400.00**	**1,595.00**	
	TOTALS		**2,000.00**	**120.00**	**150.00**	**2,270.00**	**450.00**	**400.00**	**140.00**	**80.00**	**5.00**	**1,075.00**		
												Closing Balance	**1,595.00**	

2.4. Coordinate plane artwork

Type of innovation: Group work; multi-disciplinary learning; project-based learning; cooperative learning; learning by doing

Theme/Topic: Coordinate plane artwork

Students` age: 13 years

Time (number of sessions, min): 40 min

Goals/aims/objectives:

- Familiarize students to the Cartesian coordinate system and its many uses
- Use terms related to rectangular coordinate system
- Value the importance of René Descartes works in plotting points
- Learn to locate and plot points on the rectangular coordinate system
- Knowledge transfer
- Creativity development
- Active students` involvement

Materials needed: graph paper, drawing tools, colored pencils, computers, multimedia projector

Kinds of technologies used: Microsoft PowerPoint

Procedure

The teacher creates a PPT to present and demonstrate the steps students must follow in order to locate and plot points on the rectangular coordinate system. After that, he/she asks the students to draw pictures using ordered pairs of numbers that identify points on the coordinate plane (Handout).

Teacher`s instructions:

1. Take a sheet of graph paper.
2. Draw the “x” axis.
3. Draw the “y” axis.
4. Mark the ordered pairs on the graph (plot the points).

5. Connect the points to make a picture.
6. Color the picture.

Students work in groups.

Products made: Pictures on a graph paper

Advantages of the innovation

- Increased motivation
- Collaboration
- Positive attitude
- Practical use of Mathematics
- Multidisciplinary approach

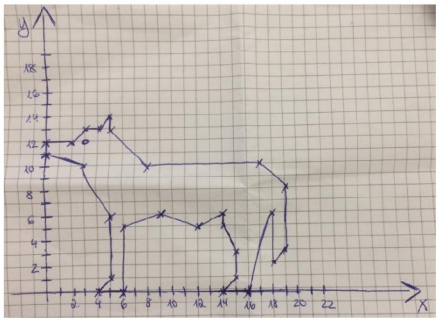
Comments/suggestions/ pieces of advice

Possible follow-up activity: Students continue their practice on using the coordinate plane by drawing a picture on a coordinate plane and then writing out directions (using coordinates) for that picture to be replicated exactly by another student, who will not see the picture but will follow the directions.

Handouts: a table of values:

1. x= 6; y =0	2. x=6; y=5	3. x=9; y= 6	4. x=12; y=5	5. x=14; y=6	6. x=14;y=5
7. x=15; y=3	8. x=15; y=1	9. x=14; y=0	10. x=16; y=0	11. x=18; y=6	12. x=18; y=2
13. x=19; y=3	14. x=19; y=8	15. x=17; y=10	16. x=8; y=10	17. x=5; y=13	18. x=5; y=14
19. x=4; y=13	20. x=3; y=13	21. x=2; y=12	22. x=0; y=12	23. x=0; y=11	24. x=3; y=10
25. x=5; y=6	26. x=5; y=1	27. x=4; y=0	28. x=6; y=0	29. oko x=3; y=12	

dog.jpg



2.5. Brainstorming

Type of innovation: Work in group. Technique to make decisions by consensus.

Theme/Topic: Brainstorming

Students` age: From 8-10 years

Time (number of sessions, min): 1 session

Goals/aims/objectives: Help the class group to take decisions by consensus**.**

Concepts: The purpose of the brainstorming session is to work in a group to define a problem and find, through a participatory intervention, the best plan of action to fix it.

Skills: Working group to make decisions and reach consensus.

Materials need:

- A problem to be solved.
- A class group with potential to work as a team.
- A board, large sheets of paper and thick markers to write.
- A moderator (teacher) that encourages students to make suggestions without imposing their own opinion.

Basic rules:

- The facilitator leads each session.
- The facilitator asks the participants suggestions (students).
- It is not allowed to criticize any suggestion.
- All suggestions are written on the board (even more ruined).

Procedure:

1. Definition of the problem:

- Ask for suggestions on what the problem is more important.
- It is not allowed to criticize any suggestion.
- Write on the board all the proposed problems.
- Divide the problems that are similar or related.
- Sort by priority (most important at top).

2. Creating a target:

- Spend the definition of the problem (and the solution is given).
- The solution to the problem is before you set the goal.
- Define the goal as the solution to the problem.
- Write the goal on the board.
- Remind the group that the goal is that they have chosen.

3. Defining the target

- Explain the difference between goal and objective. (The facilitator should know that an objective is measurable, finite and has a completion deadline).
- Ask the group to propose goals.
- Write on the board all the objectives.
- Divide the objectives that are similar or related content, and sort them by priority (most important at top).
- Remind the group that they generated the main objective.

4. Identify resources and constraints:

- Ask the group to propose resources and impairments and write them on the board.
- Group the resources that are similar or related content, and sort them by priority (most important at top).
- Remind the group that they generated the list, not you.

•Group the impediments that are similar or related content, and sort them by priority (most important at top).

•Remind the group that they generated the list.

5. Identify a strategy:

•Ask the group to propose strategies and write them on the board.

•It is not allowed to criticize any suggestion.

•Divide the strategies that are similar or that are related, and sort them by priority (most important at top).

•Remind the group that they generated the list.

•Choose the strategy that head the list.

6. Make a summary of the decisions of the group:

•The problem

•The goal

•The objectives

•Resources

•the impediments

•Strategy

2.6. Mathematic for energy transformation

Type of innovation: group work; practicing concepts in the fields of mathematics and electrotechnics; learning a mathematical concept and being able to apply it correctly.

Theme/Topic: Mathematic for energy transformation

Students` age: 18 years old

Time (number of sessions, min) : 1 week

Goals/aims/objectives : Use mathematics to calculate the current passing through the wire, absorbed power, cosϕ, power factor correction and engine torque.

Concepts: Complex numbers, trigonometry, sinusoidal waves, mechanical transmissions.

Skills: electrotechnical competence, mechanical competence, mathematical competence, self-assessment and self-correction.

Materials needed:

- Worksheets: yes
- Other: paper, pencils, calculator

Url of the online products used:

youtube video: https://www.youtube.com/watch?v=LtJoJBUSe28

Kinds of technologies used: PC

Procedure:

- We started introducing the mathematical concepts of complex numbers, sinusoidal waves, trigonometry.
- We than explained the basic formulas and principles of electrotechnics.

- After that got in detail and explained what kind of transmissions, belts and reducers exist in the field of mechanics.
- Than that we did an actual example of calculation of: the current passing through the wire, absorbed motor power, cosϕ, power factor correction and engine torque.
- Themselves: the current passing through the wire, absorbed power, cosϕ, power factor correction and engine torque.
- The students, in small group, prepare a power point presentation on the topic.

Products made: power point uploaded to www.e-mathematics.eu,

Advantages and disadvantages of the innovation

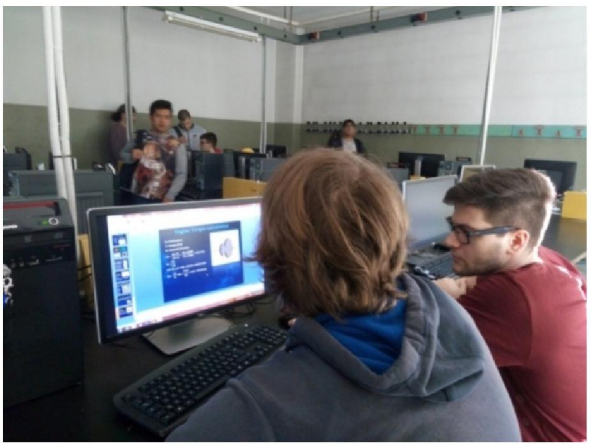
- Working in small group represent the main advantage in fact this way they can cooperate and share their experiences and knoledge in order to realize the final product.
- This lesson shows that matematics is the common basis of these two subject, electrotechnic and mechanics.
- The complexity of the arguments treated may be too high for the students that have no knowledge in the field of electrotechnics and mechanics

Comments/suggestions/ pieces of advice

The topic is objectively complex and need to be treated with an easy approach.

Energytransformation1.jpg

Energytransformation2.jpg



2.7. Vectors

Type of innovation: blended learning; observation and description; group work; relating of lessons to real life; multi-disciplinary learning; personalization; cooperative learning peer tutoring/instructions; after school programs and educational games.

Theme/Topic: Vectors

Students` age: 16-19

Time (number of sessions, min): 180 minutes

Goals/aims/objectives: Use concept of vectors learnt in mathematics to create easier method for solving real life problems in other fields of studies.

Materials needed: Computer, projector, access to internet, notebooks, Smartphones

Url of the online products used: Lesson materials made by Igor Vidović and uploaded to www.e-mathematics.eu, https://www.geogebra.org/m/A8HKjzTd , prezi.com/pox4i1i0q-sh/present/?auth_key=6bscuny&follow=b8adn0rhmzwi&kw=present-pox4i1i0q-sh&rc=ref-184480623, play.kahoot.it/#/k/f77b613c-85c6-4827-bd5d-c649046efbef

Kinds of technologies used: animation; websites/blogs; gaming; Internet; smart phones

Procedure:

- See animation about vectors magnitude and direction.
- Learn the concepts of vector.
- Solve easy mathematic problems.
- Create and solve harder vector problems from other fields of studies.

Products made: report; PREZI; teacher resource; geogebra applet; quiz
Lesson materials made by Igor Vidović and uploaded to www.e-mathematics.eu, https://www.geogebra.org/m/A8HKjzTd , prezi.com/pox4i1i0q-

sh/present/?auth_key=6bscuny&follow=b8adn0rhmzwi&kw=present-pox4i1i0q-sh&rc=ref-184480623, play.kahoot.it/#/k/f77b613c-85c6-4827-bd5d-c649046efbef

Advantages and disadvantages of the innovation:

Advantages:

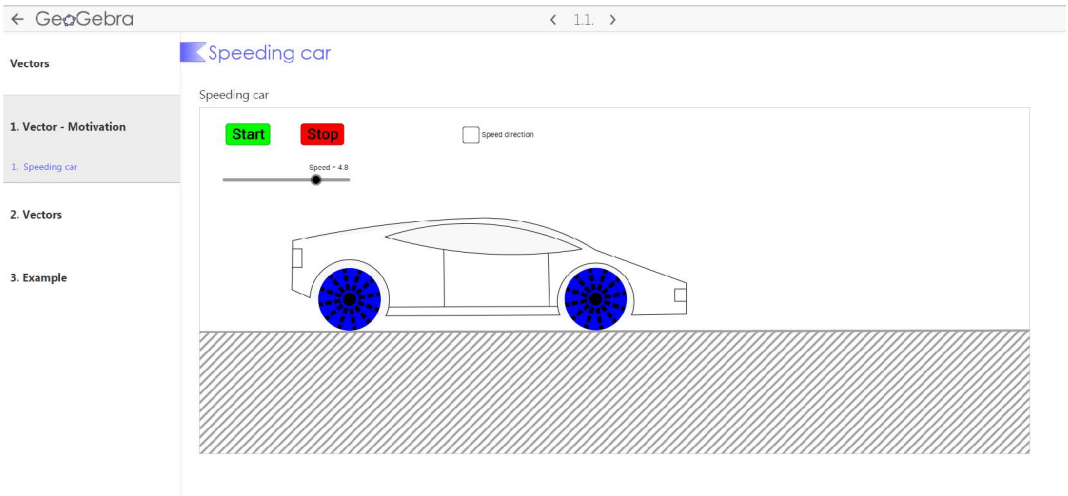
- good motivation
- individual work
- easy and fast learning
- encouraging thinking
- creating own problems and solving them
- apply mathematic in real life

Disadvantages:

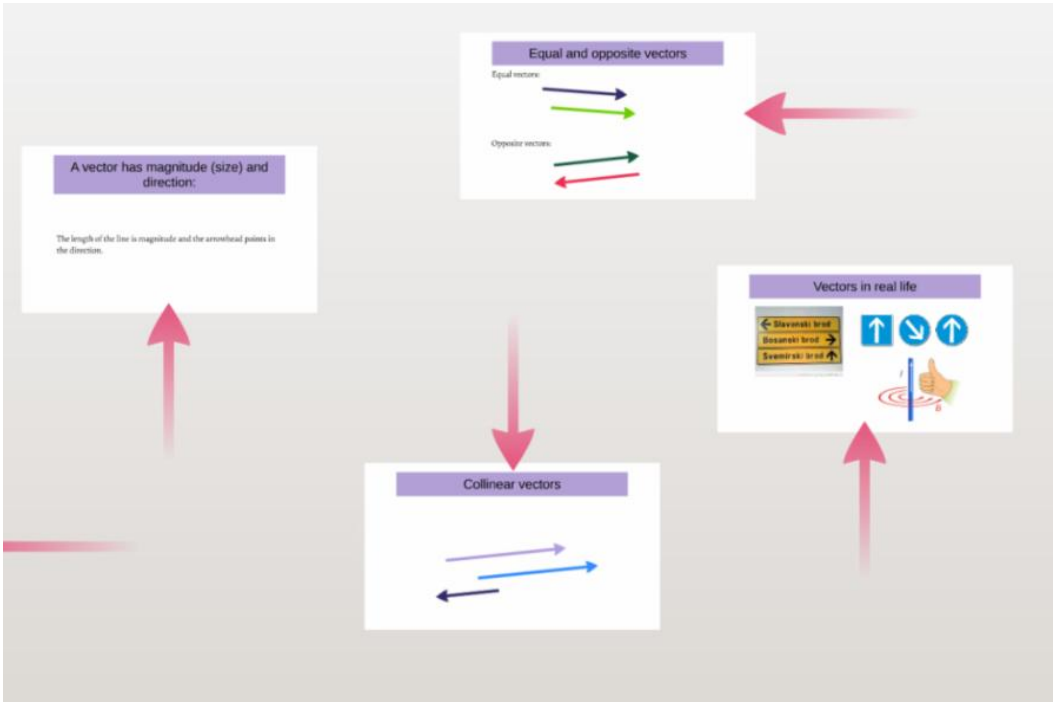
- lack of time
- different knowledge
- not all can create their own problem

Comments/suggestions/ pieces of advice:

This innovation is easier to use in different classes (Physics, Mechanical engineering…). Due to the different student knowledge instead of individual work you can make them to work in pairs our teamwork



Vector1.jpg


Equal and opposite vectors
Equal vectors:
Opposite vectors:
A vector has magnitude (size) and direction:
The length of the line is magnitude and the arrowhead points in the direction.
Vectors in real life
Slavonski brod
Bosanski brod
Collinear vectors

Vector2.jpg

CHAPTER 3

IDEAS FOR PROJECT-BASED INNOVATIONS

This chapter contains 6 descriptions of innovations on project-based lessons. Using these kind of lessons teachers intend to involve students aged 13-19 in the production of maths materials and their use in real life. These innovations promote the use of cooperative learning of maths, the integration of different subjects and the funny and practical use of mathematics. The main idea of a project-based lesson is to introduce different maths concepts, motivate students to do research inside the classroom and use the acquired knowledge outside the classroom. These lessons lead students to the practical use of maths. Lessons are motivating and appealing.

3.1. 3D school model

Motivation: Industrial and trade school SlavonskiBrod doesn't own any documentation in shape of ground plan or a 3D model what encouraged students to this project. Namely, at our school we educate students in the program drafting technician. In one of the mobilities in this project I detected an excellent solution in one of the institutions: in the place of the old school, they have made a model of the school. That gave me the idea to transform student's acquired skills and knowledge in mathematics into product useful to the school and local community.

Type of innovation: blended learning; online learning; group work; relating of lessons to real life; classroom management and organization; design thinking tools and methods-problem solving; project-based learning; outdoor learning; school programs; educational games; curriculum innovation, etc.

Theme/Topic: 3D school model

Students' age:13-19

Time (number of sessions, min): Minimum of 70 school lessons in duration of 45 minutesfor a school that doesn't own any documentation needed to make a model.

Goals/aims/objectives: Apply the acquired skills and knowledge in the making of the model:taking measurements, sketching, drawing, construction, assessment, trigonometry and geometry, IT, programming…

Programs: AutoCAD, Solid Works, scientific calculator, distance measuring aid, cardboard, spray paint in the colours of the school, printer, plastic foil, cotton wool, wooden sticks.

Url of the online products used:

https://www.youtube.com/watch?v=zYMtqaSILhE

https://www.youtube.com/watch?v=V8Y-0hz544g

https://www.youtube.com/watch?v=vSrO-uPhMBE

Kinds of technologies used:websites; digital cameras; gaming; GPS devices; Internet; iPad; iPods; Learning Response Systems; social media; video cameras; virtual classroom;web cameras; smart phones.

Procedure:

1. Dividing into groups of 2-3 students
2. Clearly defining the task: each group takes measurements of the assigned part of the school
3. Sketches
4. Choosing the scale: scale (1:)
5. Assessment: sizes that cannot be measured due to inaccessibility
6. Certain angles on the roofs are calculated using the trigonometry of the rectangular triangle, and certain sides using Pythagoras's theorem
7. Ground plan and sketches - parts of the school in 2D.
8. Transfer to the computed where the 3D model is made – Auto Cad
9. Using the 3D model, real model is made using cardboard.
10. The model is painted using spray paint
11. Realistic decoration of the environment.
12. Comparison with the 3D model
13. Demonstration to the parents and the local community on the mathematics workshop "Mathematics evening", School day or Math month.
14. Posting on twin space and YouTube channel.

Products made:

1. Technical drawings
2. 3D model in AutoCad, usable by the other students during their education
3. 3D model made out of cardboard

Advantages and disadvantages of the innovation:

- student motivation needs to be very high, which is as much an advantage as it is a shortcoming

- practicing acquired skills and knowledge
- team work and responsibility
-growth of self-confidence
- working on programs used in real industry - Solid Works, AutoCAD
- shortcoming is that it takes a lot of time
- an excellent product for school that have a 3D

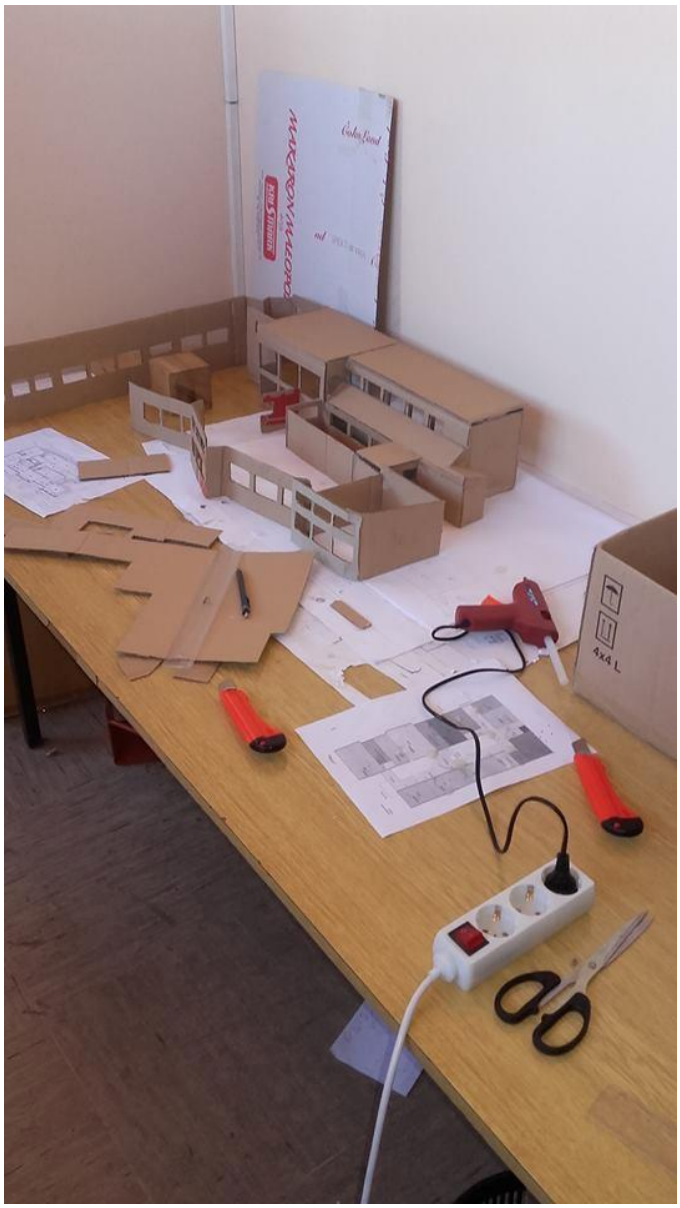
Comments/suggestions/ pieces of advice:

You should definitely make an effort so your students can experience this. Some students gave up after 10 lessons because the work included is really demanding, but you shouldn't give up. After this one, students were very curious on what will we make next year, there are already some ideas. We have even applied for a project to acquire a 3D printer. Everything can be transferred to an etwinning project. Carpenters can make a model out of wood, photographers can make a film, CNC operators and metal wokers can make a metal version but in a smaller scale due to expensive material…

We were ecologically oriented since all the cardboard was given to us by the local shops.

Distance measuring aid is very useful and it speeds up the process.

Hand-outs: Flyer with a short description of the project, the list of students and skills they have perfected, as well as the photos of the model to be shown to the interested parties (parents, friends, future employers etc.), certificate issued by the school for the skills acquired in the making of the model and the participation in the project.


4x4 L

3.2. A practical trip-lesson

Type of innovation: trips and excursions asactive teaching; relating of lessons to real life; cooperative learning/instructions; integration of different subjects; team work and pair work;

Theme/ Topic: a practical trip-lesson

Students' age: 13-19

Time (number of sessions, min): depending on the route

Goals/aims/objectives: knowledge systematization and consolidation; ability to use maps/ a tape-measure; collection of various information from different subjects on the Internet; find solutions to problems.

Materials needed: chalk, tape-measures, calculators,maps.

Url of the online products used: online maps onwww.maps.lt

Kinds of technologies used: it depends on the tasks which are given to students.

Procedure:

- The first step must be done by the teachers' team (the team is made according to the integration of different subjects). First teachers have to discuss about the route, the length of a trip, places to visit and decide how to reach these places. Moreover they have to consider proper tasks based on a topic and useful means to do tasks. It is useful for a teacher to visit those places and to assess potential threats, positive/ negative conditions and other important factors. Then with the help of a geography teacher, students find maps of places on the Internet, decide on an upcoming route, calculate the distances, look for places to visit and relax or find places to stay for a night if needed etc. With the help of a history teacher, students get to know the historic objects, search for Internet websites and find information about important historical events in these places. If a teacher thinks it is useful, students can

make a presentation about places to visit. Depending on the route and places, more teachers are possible to work.

- Secondly, choose the best means of transport for a trip because it is relevant for planning the time of a trip. We went by bicycles because of longer distances to reach famous historic places. It was a 35-kilometres bicycle trip.
  The description of a practical day-trip-lesson, tasks and solutions is on the link: http://www.e-mathematics.eu

Products made: presentation was made by students and published on the our platform http://www.e-mathematics.eu/course/index.php?categoryid=8

Advantages and disadvantages of the innovation:

Approach the problem in a team/ pair work, cooperative learning inside/ outside the school and relating of lessons in real life is an advantage. Students naturally must explore the world around them, look for answers to all questions and practically examine the knowledge they have learnt at school. The development of skills in every possible way is one of the most relevant aims of education around the world. According to this aim, our teachers try to employ active teaching methods by integration of lessons of several subjects. The application of each method should be carefully justified and help the teacher know the class better, plan the lesson carefully, educate skills of critical thinking, reflection, learn independently, use the experience, solve problems and deal with ideas. Students are more engaged, involved and encouraged to think, to interest and to cooperate when learning is active. The advantage is high motivation of students to learn mathematics and other subjects that are usually boring if you teach them in class in a traditional way.

However, the time is spent on a trip inefficiently if students only play or observe something during this period. Students should practically use the knowledge learnt at school and learn several subjects on excursions. Students will have problems if the trip is not planned properly. To prepare for a good trip/excursion you need a lot of time and knowledge plus lots of teachers of several subjects.

Comments/suggestions/ pieces of advice:

Success is guaranteed by a harmonious preparation of teachers' team work, a successful management of students before the trip, collecting materials and presenting it. It is useful to

take account of the readiness of a class if you use a trip as an active method for teaching mathematics. After the accurate evaluation of students' characteristics and teaching aims, the best teaching method can be chosen. Each trip is unique and interesting for students and teachers.

3.3. A unit of length

Type of innovation: observation and description; group work; relating of lessons to real life; classroom management and organization; design thinking tools and methods-problem solving, cooperative learning; project-based learning

Theme/Topic: Creation of a unit of length in teams of four students

Students' age:13-19 years old

Time (number of sessions, min):12 sessions

Goals/aims/objectives: touse the different units of measurement of the metric system.

Concepts: Units of measurement – Changes of units in the metric system (length, mass and capacity)
Skills: mathematic competence, teamwork competence and the competence of learning to learn.

Materialsneeded:

- Online resources/websites: introductoryactitivities
- On line activities: thatquizhttps://www.thatquiz.org/tq-9/math/measurement/
- Worksheets: yes
- Equipment: digitalboard–e-books
- Other: cardboard, scissors, crayons….

Url of the online products used:
youtubevideos
-En su justamedida: https://youtu.be/srAzK4jqZPE
-Tareaplus_ Cómo se midió por primeravez la Tierra: https://youtu.be/UeIQnjOEGUY
-Eratòstenes: https://youtu.be/mPPb7EJSZ9w

Kinds of technologies used: websites/blogs; Internet

Procedure:

- **Define your dimension.** Decide what you want to use as a unit of measurement of your system.This will be your dimension and you have to name it. You will also have to make a model and a few copies.

  Define all the divisors of this dimension so that it can measure distances and lengths that are longer or shorter than your dimension. You will have designed your scale of measurement, in other words, the equivalences.
- Once you have decided the dimension and its divisors, we will measure the following using your system:
  - The height of all the members in the group.
  - The length of the palms and the feet of each member in the team
  - The length and the width of the table.
  - The length of the corridor in front of the classroom.
  - The length and the width of the windows of the classroom.

  Once you have measured everything (but not before), you will use a measurement tape to measure the length of your dimension and of its divisors.

  **BEFORE BEGINNING WORK think of which material you will need and how you will present your work.

Products made: poster and oral exposition

Advantages and disadvantages of the innovation

-cooperative work - heterogeneous groups: they can help each other, and every pupil can do what they can.

-learning by doing is a very good advantage

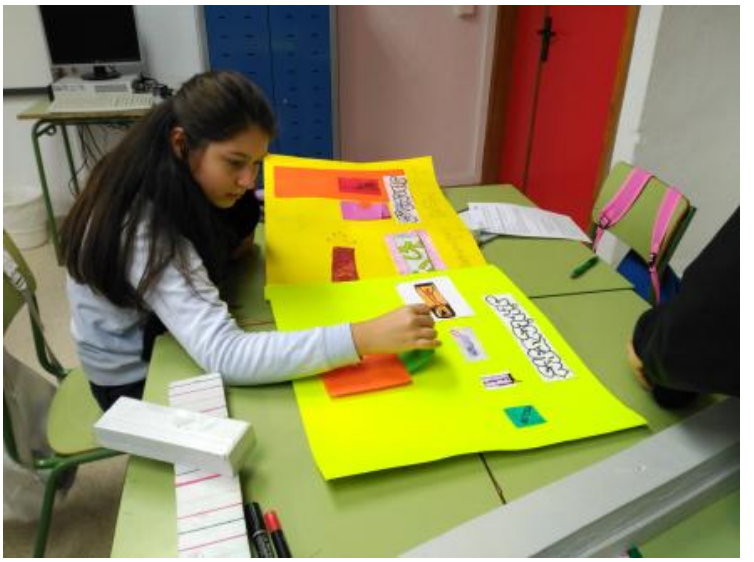
-spending more time designing and planning could be a disadvantages.

Comments/suggestions/ pieces of advice:

Team work and the invention of a dimension of length of their own has been useful to motivate measurement, to calculate and to change units without difficulty.

This topic, which is quite hard, has been engaging for students, and the marks have been higher than in previous years.

We are attaching a few photos that show the task done in teamwork as well as the results.



3.4. Statistics using Moodle

Type of innovation: Use the spreadsheet to work statistics

Theme/Topic: data Interpretation

Students' age: 13-14 years old

Time (number of sessions, min): two weeks (8 sessions)

Goals/aims/objectives: to use a spreadsheet in order to deal with the representation of statistics concepts.

Concepts: to represent a statistics series with a bar graph or with a histograph and calculate the main parameters (arithmetical average, the median, the rank, the standard deviation and the variation coefficient).

Skills: mathematics competence, computer competence and the learning to learn competence.

Materialsneeded:

- Online resources/websites: spreadsheet (office)
- Online activities: Moodleactivities
- Worksheets:
- Equipment: laptops and smartboard

Kinds of technologies used: Internet, spreadsheet.

Procedure:

The math teacher will be a presentation of the topic based on some data closer to students, for example the number of hours they are with the phone, the number of hours watching TV

... Once begun working with data, they have to be moved to the worksheet to start the processing of data with this tool.

The aim is to work histograms statistical concepts and make the spreadsheet.

Products made:

Students must print the various graphs and histograms made in order to compare the different ways of presenting some data.

Advantages and disadvantages of the innovation

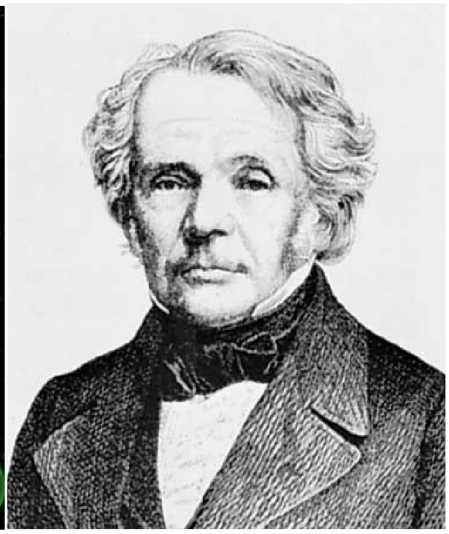
The advantages of working with a spreadsheet statistic is that students are more motivated because the activities are more attractive.

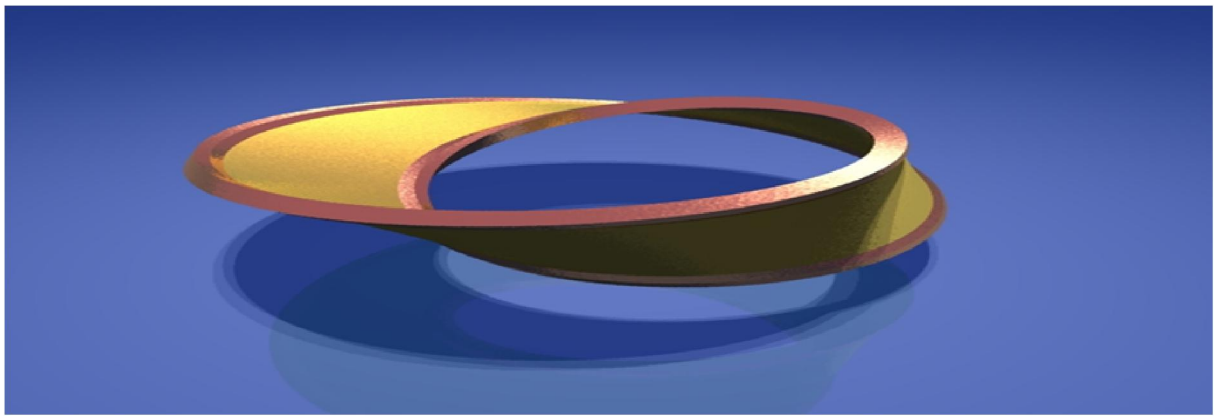
The disadvantages are that there are students who carry a faster rate than others and this can cause some disorder.

Comments/suggestions/ pieces of advice:

Students love working this statistics topic with the spreadsheet. In this way they learn the basic statistics concepts and they also learn to use a spreadsheet.

3.5. Topology using Mobius strip





Type of innovation: after school programmes; multi-disciplinary learning; learning by doing; funny and practical use of Mathematics; demonstration and observation

Theme/Topic: Topology: Mobius stripactivities

Students' age: 14 years

Time (number of sessions, min): 40-45 min (1 lesson)

Goals/aims/objectives:

- Introducing the Mobius strip as a topological construction
- Investigating and studying properties of the Mobius strip
- Consolidate knowledge about solids (cylinder)

- Investigating some real-life applications of Mobius strips
- Introducing topology to students and its differences from classical geometry
- Making students believe Maths is funny

Materials needed: strips of paper, pencils, scissors, tape; computer, multimedia projector, Internet

Url of the online products used:

https://www.youtube.com/watch?v=JNtKcK27x1s

https://www.youtube.com/watch?v=wKV0GYvR2X8

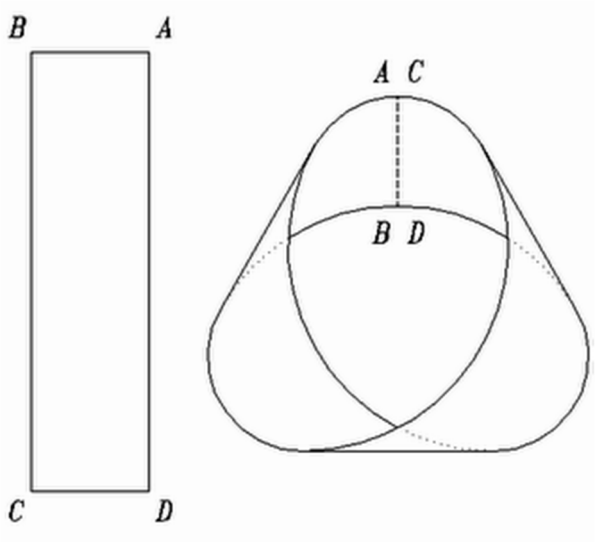
https://www.youtube.com/watch?v=mh3eMt09EAs

Procedure:

Students watch the 1st YouTube video with some teacher`s comments, if needed. They follow the instructions from the video and the teacher`s ones as well.

Teacher`s instructions:

1. To make a Mobius strip, take a strip of paper, give one side a twist and then glue or sellotape the two ends together.
2. Draw a line along the middle of the Mobius strip. Cut along this strip.
3. Take the new object that you've made and cut it in half again.
4. Make yourself a new Mobius strip. This time, draw a line along it a third of the way in from the edge. Cut along the line.



Students watch the other 2 videos and do the same as in them.

Products made: different kinds of Mobius strips` shapes

Advantages of the innovation

- Increasing motivation
- Having fun while studying Maths
- Seeing the practical application of the Mobius strip in the real life
- Learning by doing predictions and experiments
- Easy to do
- Developing thinking skills

3.6. How to determine inaccessible distances

Type of innovation: observation and description; relating lessons to real life; building instruments to measure angles; cooperative learning peer tutoring/instructions.

Theme/Topic: Trigonometry of a triangle rectangle

Students' age: Students from 14 to 15 years

Time (number of sessions, min): 90 minutes

Goals/aims/objectives: To determine inaccessible distances, such as the height of a pole, a tree, the school, the width of a river, etc.

Materials needed: Paper, thick cardboard, scissors, glue, adhesive tape, straw, string, an object to stretch the string (weight), tape-measure, writing material.

Url of the online products used: A lesson done by Jorge Novais and uploaded to the project platform: http://www.e-mathematics.eu/course/view.php?id=101

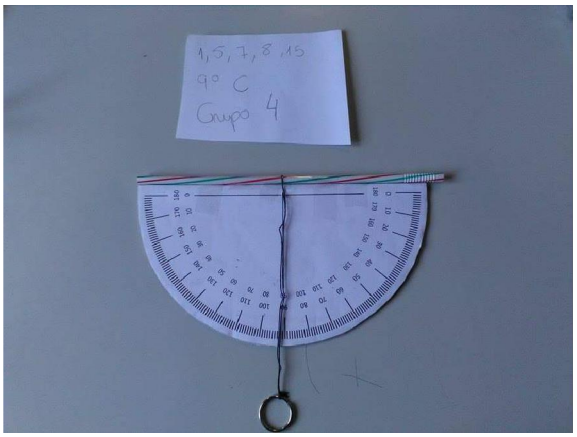
(Pictures:How_to_determine_inaccessible_distances_5)

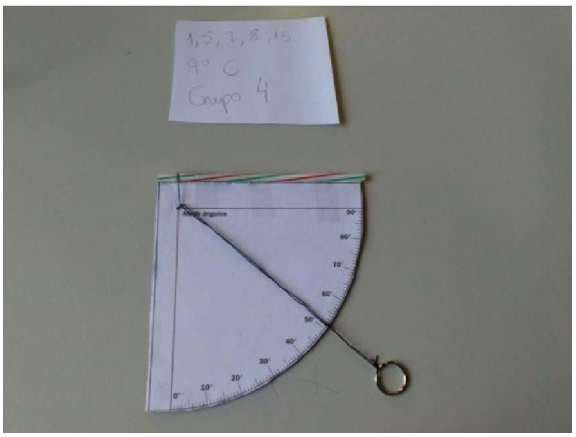
Kinds of technologies used: Construction of a measuring instrument.

Procedure:

- The teacher teaches the concepts related to trigonometric ratios of a triangle rectangle.
- The teacher sets a challenge to students: to measure the height of the school, a tree, a pole, etc.

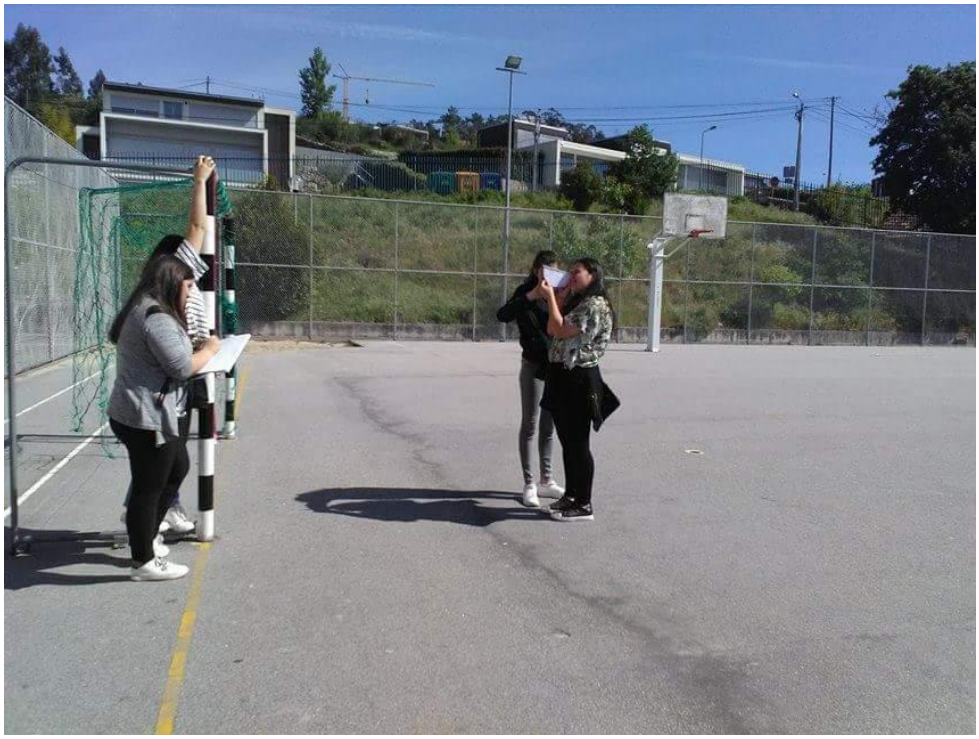
- Students construct a quadrant, an instrument to measure angles.





(Pictures: How_to_determine_inaccessible_distances_1 + How_to_determine_inaccessible_distances_2)

- The teacher and the students leave the classroom and go to the school grounds.
- The students use the quadrant to measure angles and calculate the height of the school, a tree, a pole, etc.



(Pictures: How_to_determine_inaccessible_distances_3 + How_to_determine_inaccessible_distances_4)

Products made: A video that illustrates the different steps students took to complete the task.

Advantages and disadvantages of the innovation:

Advantages: to change the teaching-learning process that is usually more conservative;the students develop their own knowledge; the students apply mathematics outside the classroom context, they apply mathematics in real life situations.

Disadvantages: the students reveal lack of autonomy in the accomplishment of the task.

Comments/suggestions/ pieces of advice:

Due to the extension of the mathematics programme, this activity would be better explored as an extracurricular activity.

3.7. Why Maths is important for us?

Type of innovation: Mathematical mind mapping (handdrawn or computer mind maps)-a visual method of representing, organizing and understanding information; a visual representation of radial thinking; a technique for solving Maths problems; a special form of note-taking.

Theme/Topic: Suitable for many different themes and topics, both made and used by teachers or made by students. In the sample the theme is "Why Maths is important for us?".

Students` age: 13-19

Time (number of sessions, min): 45 minutes

Goals/aims/objectives: brainstorm the topic; brainstorm ideas quickly and easily; help students find relationships, remember what they have learnt; solve problems and find solutions/answers creatively; increase students` thinking power, memory and creativity.

Materials needed:

Mind maps samples

Handdrawn mind map: A4/A3 paper sheets; colour pens/pencils/markers; relevant pictures/images (optional).

Computer mind map: computer, Internet/Mind Map software

Url of the online products used:

Videos to present mind mapping to the students:

https://www.youtube.com/watch?v=34LJtRaycF4

https://www.youtube.com/watch?v=4wZ5wV5dPZc

https://www.youtube.com/watch?v=wLWV0XN7K1g

Kinds of technologies used: https://www.mindmup.com/#storage (for making computer mind maps)

Procedure:

1. Preparation/Students` motivation.

Watch videos. Explain the mind map structure: central topic – branches – sub-branches. Show samples.

(Mind Map software demonstration if the students` activity is making a computer mind map.)

2. Brainstorming/Revision (depending on the theme/topic).

3. Making the mind maps (group/team work).

4. Presentation: Students (in groups) present and explain their own work.

5. Results/Final discussion.

6. Display of the mind maps made by students.

Products made: Handdrawn or computer mind maps created by the students.

Advantages and disadvantages of the innovation:

Advantages**:**

- Due to the map's layout, it's easy to collect ideas and group them. Further ideas can later be added at appropriate places in the map.
- The map helps you not to lose sight of the overall picture.
- Easy to learn and fun to use.
- Group concepts together using natural associations.
- More ideas are generated in a shorter period of time.
- Helpful for a deeper understanding of the topic.
- Make students smarter.
- Allowa review of any Maths topic.
- Great tool for visual learners.
- Develop communication skills.

Disadvantages:

- Takes time.
- Might be difficult for others to understand.

Comments/suggestions/ pieces of advice:

- Start with making handdrawn mind maps first.

- Ask students to use larger sheets of paper for more complex problems.

- Divide the class in group of 3-5 students (group/team work).

- Good for using in many ways: at the end of a topic (as a summary); as a student activity; learning game; thinking tool, etc.

- Mind maps can be created by using PREZI software as well.

Handouts: -

A sample of a computer mind map on the theme "Why Maths is important for us?":