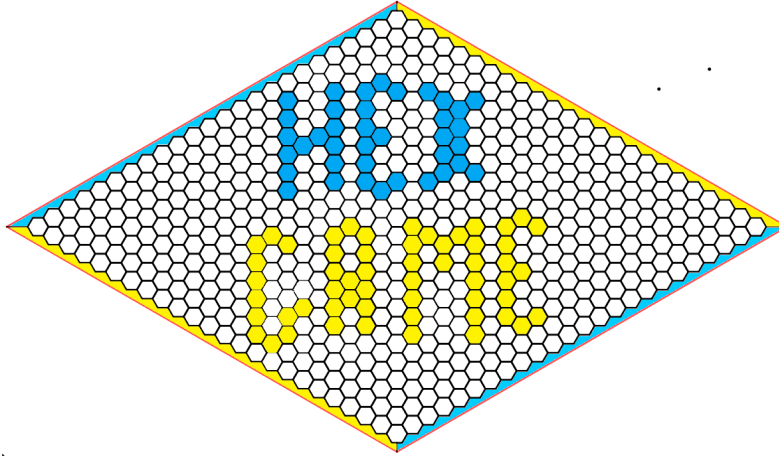# Subject n° 9

2021 / 2022

Name, family name and level of education: Mady MARTIN, Charline CLADEL   10th

School: Bellevue Marie – Rivier   Ales

Teachers: Sebastien CASTAGNEDOLI, Laure VIALA

Researchers: Mr. Serge Dumont, professor at Nîmes University, from the research laboratory at the Montpellier Institute, Alexander Grothendieck, CNRS UMR 5149, University of Montpellier.

Name, family name and level of education: Nathan THELU, Sacha ROBERT VIGANI and theis MARTINEZ   10th

School: Val de Durance of Pertuis

Teachers: Hubert PROAL

Researchers: Yves PAPEGAY (INRIA Sophia Antipolis)

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1/ Presentation of the subject

The Hex game is a universal board game invented by two mathematicians , the Danish Piet Hein (1905-1996) and the American John Nash (1928-2015) in the 1940s .It is certainly the first abstract combinatorial game that used these two forms. It is a playful board game that engages reflection through various mathematical facets. The goal of the game is to face an opponent, the blacks and the whites, on a "diamond-shaped board" composed of hexagonal boxes. All dimensions on the diamond side are possible, the most traditional is the one composed by 11 hexagons, but there are also values 13 or 19. The winner being the first who succeeds in forming a continuous path connecting the two edges. In spite of rules with a childish appearance, thinking and strategies take a much more important turn to win. Our mission this year was to find out if finding a winning strategy is possible.

2/ Announcements of the conjectures and results obtained

After several games, when we play against the computer, we notice that when we play our first pawn in the middle of the deck, the opponent sticks it to ours.

We also saw that the adversary in our relationship creates himself a passage to win, he follows our path glue his pawns to ours. Thus it creates an advantageous path to win while preventing us from passing. It is therefore necessary to be attentive to his game but also to that of his opponent which can be more important to avoid losing.

We must also be attentive because the opponent can create a false course to distract us while thinking about his way to win.

3/ Hex game's strategy

## 1/ The strategy and the reflection at the core of the game

Starting at the beginning of our research, we started to play on a smaller board to maximize our chances of finding a winning strategy, these strategies become harder and less numerous the bigger the board.

Big board   **Little board**

Figure 1
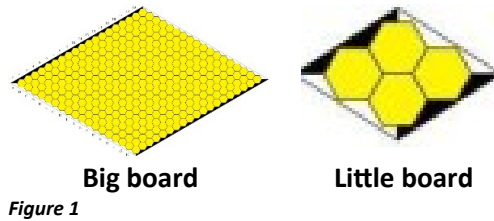
For example, if we take the smallest tray that exists (2x2) we will notice that there is no winning strategy for whites.

Figure 2
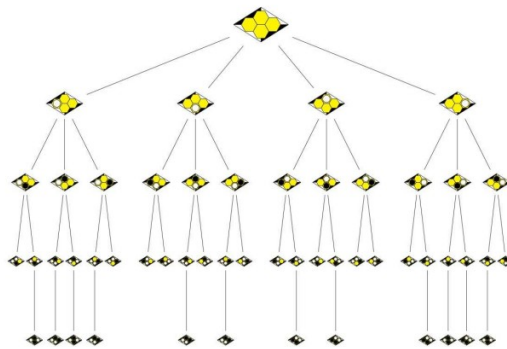
No matter the strategy used by the whites, they would always end up losing because the board is not big enough to adapt its game to the following blacks because they already win.

Figure 3
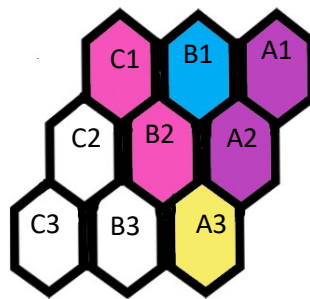
For a board (3x3), there is only one way to win. If players do not play B1, they cannot avoid a default in three moves.

During our games, we noticed that several game modes were much more advantageous than others. First of all, if we place all our pawns right next to each

other, it will be much easier to block the chain and thus block the opponent. We then set out to place our pawns so that they have at least two empty squares in common this is called the link strategy.

To be more modern, it is possible to use a computer and more specifically its heuristics (algorithm that allows to quickly solve complex optimization difficulties, without designing a formal modelling) with the help of programming techniques derived from artificial intelligence.

As we saw in the previous point (Announcements of the conjectures and results obtained), the opponent can make a wrong path to beat us and thus win.

Here, the whites made a false course to the left and blocked the blacks to win.

Figure 4

2/ Probability at the service of chance

Probability theory in mathematics is the study of phenomena characterized by chance and uncertainty. It forms with statistics the two sciences of chance which are an integral part of mathematics.

Many other probabilistic results lead to the following observation: if chance is, by definition, the unpredictable, a repeated chance nevertheless contains regularity. There are laws of chance, certainty in the uncertain.
In this game on a board of 8 out of 8, from the first pawn played, the opponent has 1 chance out of 64 to place himself well and so on until the end of the game . The probability that the player knows from the beginning whether the placement of his pieces will be winners.

We learn to play the game of Hex with at the beginning the chance mixed with the strategy. We use the best known winning strategies by sometimes randomly

winning after simply trying to block our opponent . Only correlations can be accurately predicted, but not separate events or exact quantities.

PROBABILITY: the probability that A = win or B = lose is realized is obtained by summing the probability (A or B) = P (A) + P (B) − P (A and B)

We can also link the conjecture «P ≠ NP» to it, is a reasoning that leads to the existence of this winning strategy which is a reasoning by the absurd.
The reasoning by the absurd consists in demonizing the truth of a speech by proving the contrary of the latter.

4/ Conclusion

Finding a winning strategy is possible only up to a certain tray size. It is impossible to predict the outcome of our game every time. Strategy is the basis of the game, without winning strategy turns out to be complicated. The strategies were discovered and invented in order to allow the players to win by associating their reflection with the strategies. Thus, the game of Hex is the reflection of the reflection and strategies as well as the random that sometimes generates success.

The best strategy consists to placing the first hexagon in the center of the board, then using the "safely connected" method. We tried to make a code that could play the game using our strategy, but we encountered problems implementing it. Backtracking, trees, and artificial intelligence were required in order to make the code work properly.