Study of invasive species

Presentation by:

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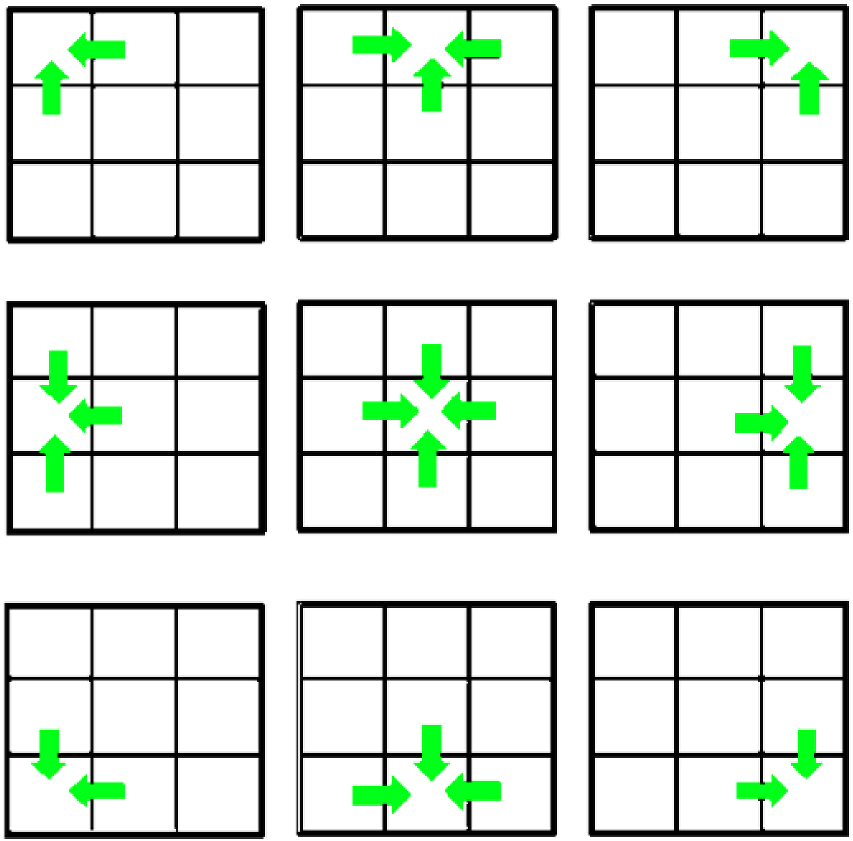
Statement

To elaborate the dispersal of the pampas grass in a simple manner, let's imagine a square-shaped garden, divided in 9 identical parcels of land. The reproduction of the plant is made, obviously, through its seeds. Each plant starts reproducing after 3 years. In the year 3, the first plant produces 5x seeds. x seeds stay in their place, while the other 4x are equally distributed to the north, south, east and west of the first x seeds, in the parcels of land next to them. The newly distributed seeds will grow into plants and will start reproducing after they reach age 3 as well, redistributing their own seeds in the same manner. Extend and generalize the process of reproduction by placing the first 5x seeds in one of the 9 parcels of land, knowing that each plant dies after 12 years.

Distribution model



Visual representation for the first 20 years (x=0.2)

x 0,2

0

0	0	0
0	1	0
0	0	0

1

0	0	0
0	1	0
0	0	0

2

0	0	0
0	1	0
0	0	0

3

0	0,2	0
0,2	1,2	0,2
0	0,2	0

4

0	0,4	0
0,4	1,4	0,4
0	0,4	0

5

0	0,6	0
0,6	1,6	0,6
0	0,6	0

6

0,1	0,9	0,1
0,9	2	0,9
0,1	0,9	0,1

7

0,2	1,2	0,2
1,2	2,6	1,2
0,2	1,2	0,2

8

0,5	1,7	0,5
1,7	3,4	1,7
0,5	1,7	0,5

9

0,8	2,3	0,8
2,3	4,5	2,3
0,8	2,3	0,8

10

1,4	3,2	1,4
3,2	6	3,2
1,4	3,2	1,4

11

2,2	4,4	2,2
4,4	8	4,4
2,2	4,4	2,2

12

3,2	6,1	3,2
6,1	9,8	6,1
3,2	6,1	3,2

13

4,8	8,4	4,8
8,4	12	8,4
4,8	8,4	4,8

14

7	12	7
12	17	12
7	12	7

15

10	16	10
16	22	16
10	16	10

16

14	22	14
22	30	22
14	22	14

17

20	30	20
30	41	30
20	30	20

18

29	40	29
40	56	40
29	40	29

19

40	55	40
55	77	55
40	55	40

20

56	76	56
76	106	76
56	76	56

year in which the first plant of year 0 dies

The number of seeds in each plot in the first six years

							x	
	1			1		x	$x+1$	x
							x	
	$2x$			$3x$		$2x^2$	$2x^2+4x$	$2x^2$
$2x$	$2x+1$	$2x$	$3x$	$3x+1$	$3x$	$2x^2+4x$	$5x^2+4x+1$	$2x^2+4x$
	$2x$			$3x$		$2x^2$	$2x^2+4x$	$2x^2$

C++ Algorithm

Input and Output

Input:

Two integers:

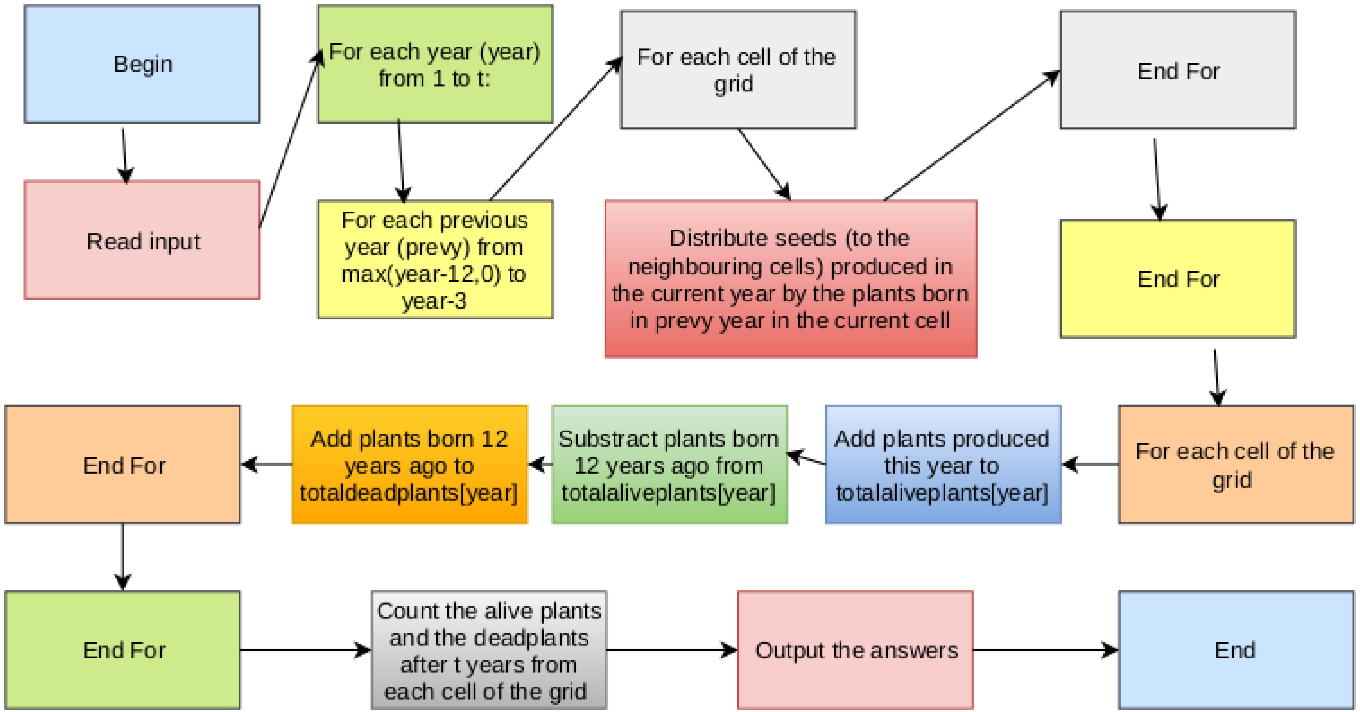
t- the number of years that have passed

Output:

- a matrix of polynomials representing the number of living plants in each cell after t years have passed
- a polynomial representing the number of living plants after t years
- a polynomial representing the total number of dead plants after t years

The ‘x’ from the polynomials represents the number of seeds that a plant produces in a year, divided by 5.

Logical schema of the algorithm



C++ program

Algorithm Analysis

- Time complexity: O(t*g)
- Memory complexity: O(t*g)
- g is the maximum degree of a polynomial
- t must be smaller than 50

Special approach I.

Our plant will produce only 4 seeds, which will be equally distributed to the north, south, east and west of the initial parcel, without leaving any in its current position. The reproduction starts immediately, in the first year of the plant’s life. The development of our new garden, considering that each plant dies immediately after reproducing, will follow this course:

Special approach I.

$$A_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$A_2 = A_1{}^4 = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{pmatrix} \qquad A_3 = A_2 \cdot A_1{}^3 = A_1{}^7 = \begin{pmatrix} 0 & 8 & 0 \\ 8 & 0 & 8 \\ 0 & 8 & 0 \end{pmatrix}$$

$$\dots$$

(using mathematical induction method)

$$\Rightarrow A_n = A_1{}^{1+3(n-1)}$$

Special approach II. (garden without fence)

As it can be deduced from the rules, the plants will be arranged in a rotated-square shape. Each of the 4 sides of the square will correspond with the $(n+1)^{th}$ line of Pascal's Triangle, where n represents the year's index.

1

4 4

6 16 6

4 24 24 4

1 16 36 16 1

4 24 24 4

6 16 6

4 4

1

Special approach II.

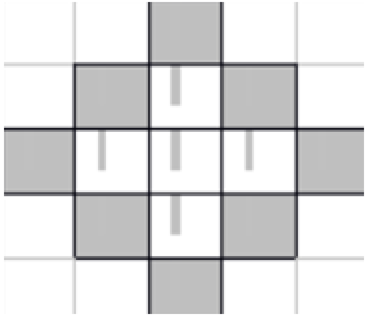
But what if the garden had no fence? Suppose we have an infinite number of parcels in our garden. Analysing the structure, we can see that for each year there are rhombus shapes with the number of squares equal to the sum of the numbers of an arithmetic progression with the ratio 4.

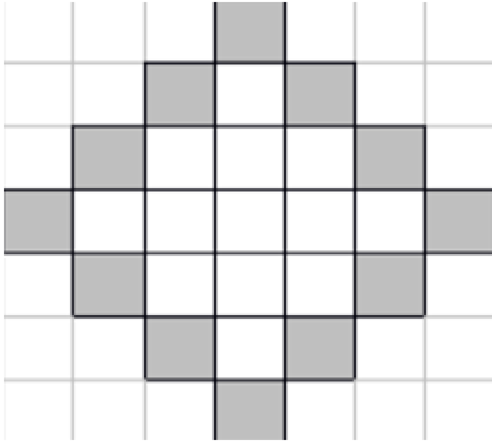
Formula: $1 + 4 + 8 + 12 + 16 + ... + 4(k - 1) =$

$$= 1 + 4(1 + 2 + 3 + 4 + ... + k - 1) =$$

$$= 1 + 4k(k - 1) / 2 =$$

$$= 1 + 2k(k - 1)$$

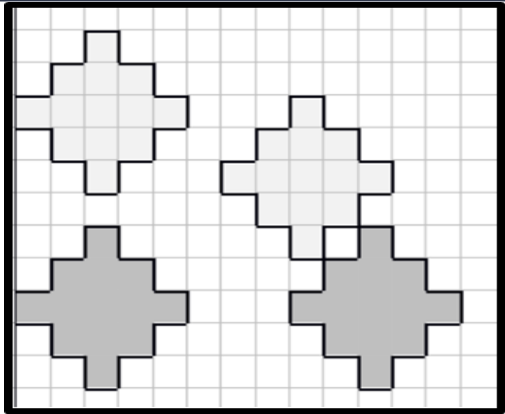


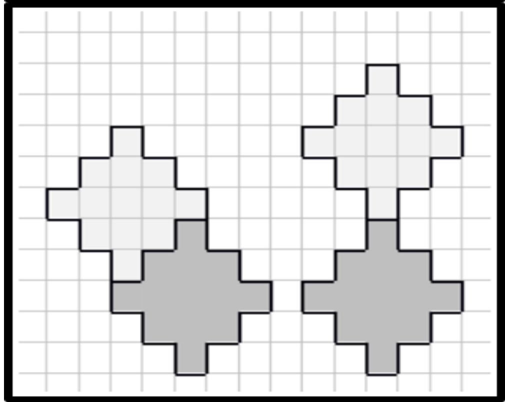


Special approach III.

Statement: If we had more plants in a garden and they'd stop expanding when they touch another plant, in how many years will all the plants stop expanding? (one expansion stops when it meets another expansion).

Special approach III.





I. We build a matrix with the distance between every 2 plants equal to ($|x_A - x_B| + |y_A - y_B|$) / 2 because each plant expands. We also build an array which represents the values of the distances between each plant and the closest one to it.

II. We find the minimum value in this array (this will correspond to the plant that first stops expanding). We change the values in the matrix with 2 * distance – minimum on the lines and columns which contain the initial minimum value.

Complexity is O(n^2) which means we can manage a parcel with 1000 plants in 0.1 seconds.

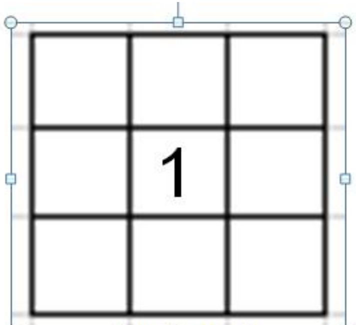
Special approach IV.

We can consider another approach for this subject. Our group worked on a different model, in this model we place a bag of seeds with a value of 1 at the center of the garden and each year the bag divides itself by five. 4/5 of the bag will (if possible) be equally distributed to the north, south, east and west of the initial parcel while 1/5 will stay in it. The seeds don’t grow plants so none of them die or disappear. We then studied the movement of the seeds in the garden.

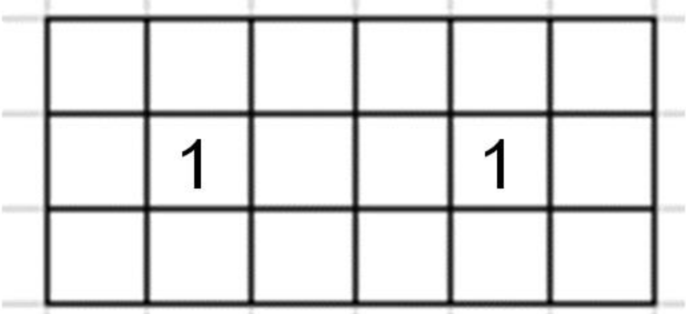
Special approach IV.

We created 2 programs, in Python, representing this model, having as input « a », representing the number of years that have passed and as output the number of seeds in each parcel that can be transformed into a percentage.

The first program is for a garden of 9 parcels with 1 bag and the second is for a garden of 18 parcels with 2 bags.



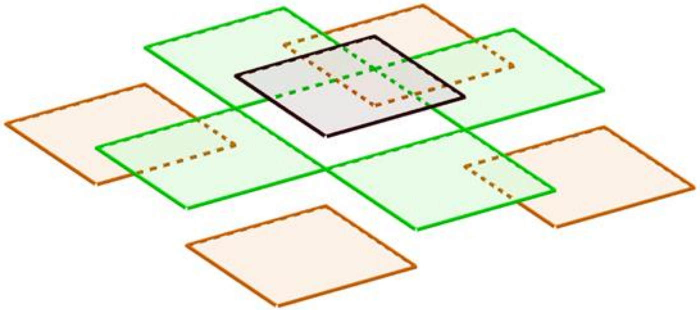
Program 1



Program 2

Special approach IV.

We took the outputs of the first program for the years 1 to 10 and used them to create a visual representation of the number of seeds in each parcel (in geogebra). This is what year 10 would look like.



The values are :

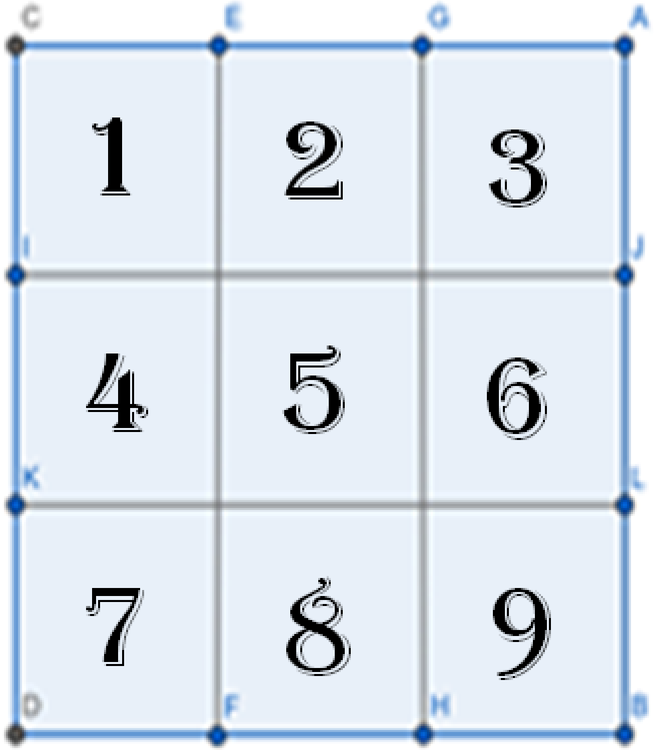
The corners (orange) ~8,3% of the seeds

The borders (green) ~12,5% of the seeds

The center (black) ~16,6% of the seeds

Special approach V.

To solve the problem we drew the plot and divided it into 9 equal squares, just like in the picture and we indexed them with numbers from 1 to 9. In the first stage we plant seeds on plot 5 and those adjacent to it (2,4,6,8) equally. If we note the number of seeds with s, on each of the 5 plots we will have 1/5 s. Because the dementias will germinate, and the amount of seeds that will reach each plot will be different, we note as follows:



Special approach V.

t - number of seeds remaining on the main plot
p - number of seeds remaining on the plots adjacent to 5
q - number of seeds remaining on the plots not adjacent to it 5

$\mathbf{t_1}$- number of seeds remaining on an adjacent plot (2,4,6,8)
$\mathbf{p_1}$- the number of seeds remaining on a plot adjacent to this plot time
$\mathbf{q_1}$ - the number of seeds remaining on a plot not adjacent to this plot time

$\mathbf{t_2}$ - number of seeds remaining on a non-adjacent plot (1,3,5,7)
$\mathbf{p_2}$- the number of seeds remaining on a plot adjacent to this plot time
$\mathbf{q_2}$ - the number of seeds remaining on a plot not adjacent to this plot time

a - the number of the seeds from the plot 5

b - the number of the seeds from the plots 2,4,6,8

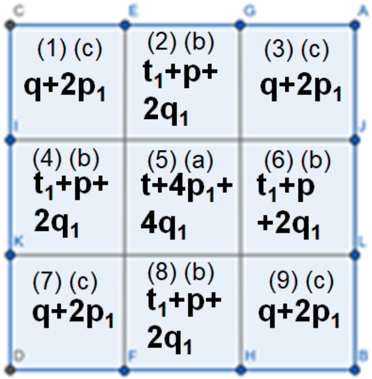
c - the number of the seeds from the plots 1,3,7,9

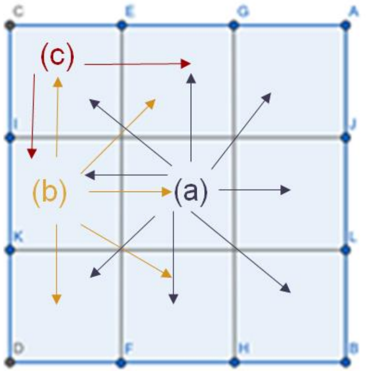
Special approach V.

$a_1 = t+4p_1+4q_1$
$b_1=t_1+p+2q_1$
$c_1=q+2p_1$

$a_2=t_1'+4p+4q$
$b_2=t_1'+p'+2p_2+2q_1'$
$c_2=t_2'+q'+2p_1'$

$a_3=t''+4p_1'+4q_2'$
$b_3=t_1''+p''+2q_1''+2p_2'$
$c_3=t_2''+q''+2p_1''$





P(0): $a_1= t^1+4p^1+4q^1$
P(1): $a_2= t^2+4p_1^{\,2}+4q_2^{\,2}$
P(2): $a_3= t^3+4p_1^{\,3}+4q_2^{\,3}$
…
P(n-1): $a_n= t^n+4p_1^{\,n}+4q_2^{\,n}$

P(0): $b_1= t^1+p^1+2q_1^{\,1}$
P(1): $b_2=t_1^{\,2}+p^2+2p_2^{\,2}+2q_1^{\,2}$
P(2): $b_3=t_1^{\,3}+p^3+2q_1^{\,3}+2p_2^{\,3}$
…
P(n-1): $b_n= t_1^{\,n}+p^n+2q_1^{\,n}+2p_2^{\,n}$

P(0): $c_1=q^1+2p^1$
P(1): $c_2=t_2^{\,2}+q^2+2p_1^{\,2}$
P(2): $c_3=t_2^{\,3}+q^3+2p_1^{\,3}$
…
P(n-1): $c_n=t_2^{\,n}+q^n+2p_1^{\,n}$

We observe that between the 2nd and the 3rd layer, the steps are similar, so using mathematical induction:

Thank you for your attention!