The shape of the Dunes

Colegiul Național "Emil Racoviță"
Adam Raluca, Berinde Codrin, Coman Andrada, Cocaină Mara, Năsăudean Maria, Petri Dan

The task: On an elevated support, poligonal in shape (triangle, square, rectangle, L-shaped) we distribute sand. How will the ridge lines be positioned on the sand piles?

Approach: Finding the mathematical equation of the crest - can be then applied for different shapes of the support.

Initial assumption: the slope (alpha) of the "dune" has the same value in any point on its surface, regardless of being closer or farther away from its top. We took a simple shape - an acute angle infinitely long. Let P be an arbitrary point on a crest. **Seen from above, the sand crest for an angle is its bisector** – the line equally distanced from the edges of the angle. For 3D sand dunes, **the equations of the crests in the base plane are those of their projections onto the base plane.**

Rectangle

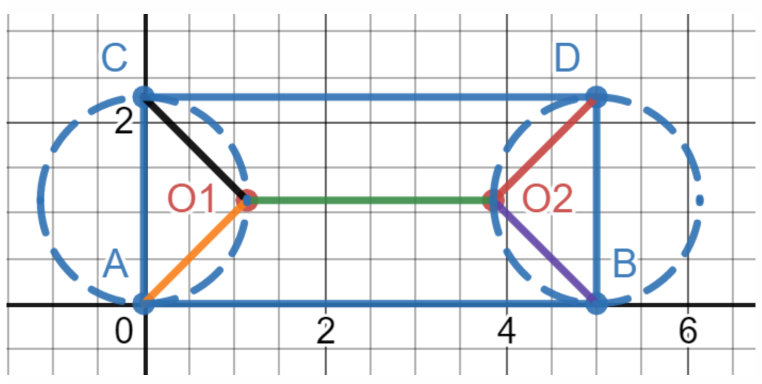
$y = \frac{l}{2} \{\frac{l}{2} \leq x \leq L - \frac{l}{2}\}$

$y = -x + l\{0 \leq x \leq \frac{l}{2}\}$

$y = -x + L\{L - \frac{l}{2} \leq x \leq L\}$

$y = x\{0 \leq x \leq \frac{l}{2}\}$

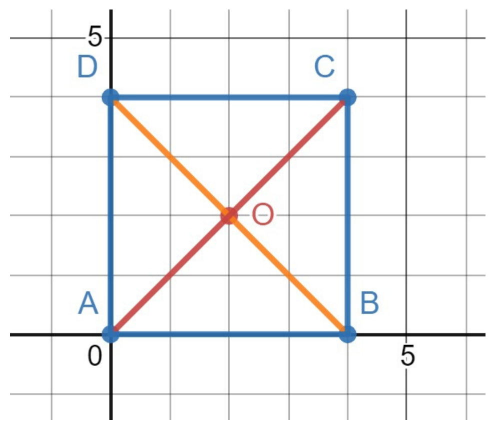
$y = x - L + l\{L - \frac{l}{2} \leq x \leq L\}$



Square

$x + y = L\{0 \leq x \leq L\}$

$x - y = 0\{0 \leq x \leq L\}$

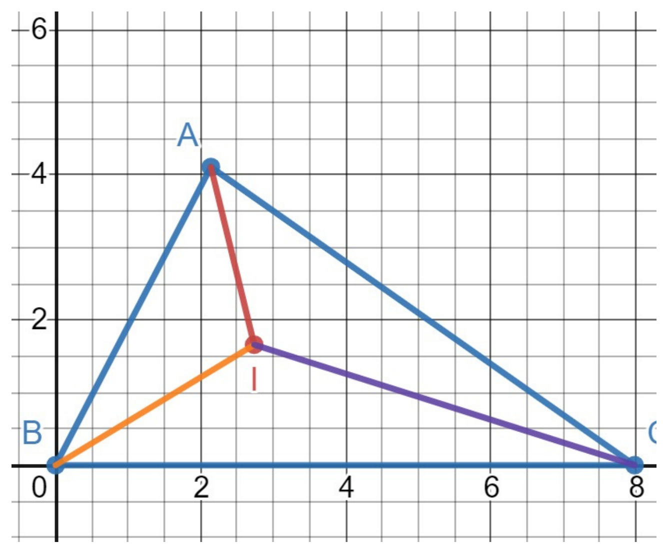


Triangle

$y = \frac{x(-ba_2 - ca_2) + cc_{1a_2}}{cc_1 - ba_1 - ca_1} \{\frac{aa_2}{a+b+c} \leq y \leq a_2\}$

$y = \frac{xaa_2}{aa_1 + cc_1} \{0 \leq x \leq \frac{aa_1 + cc_1}{a+b+c}\}$

$y = \frac{xaa_2 - aa_2c_1}{aa_1 - ac_1 - bc_1} \{\frac{aa_1 + cc_1}{a+b+c} \leq x \leq c_1\}$



L-shape

$(y - l_1)^2 = 2l_2x - l_2^2\{y \geq x\}\{y < -l_1\}$

$(x - l_2)^2 + (y - l_1)^2 = y^2\{x \geq y\}\{x \leq l_2\}$

$y = \frac{l_1}{2}\{l_2 \leq x \leq L_1 - \frac{l_1}{2}\}$

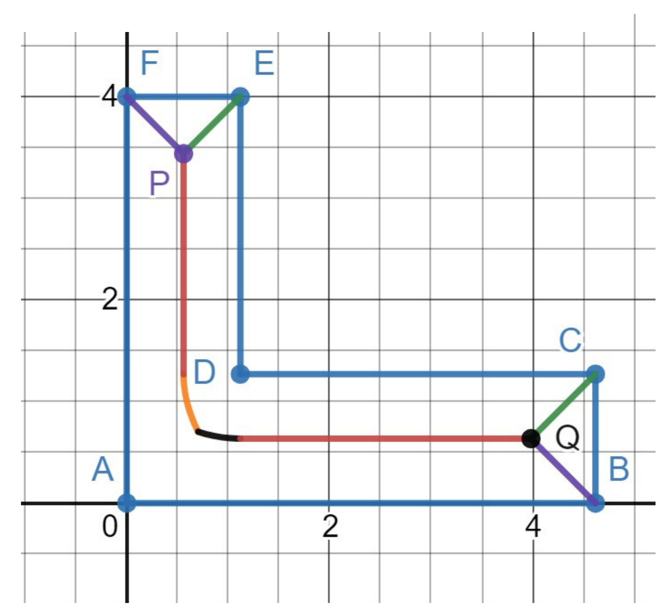
$x = \frac{l_2}{2}\{l_1 \leq y \leq L_2 - \frac{l_2}{2}\}$

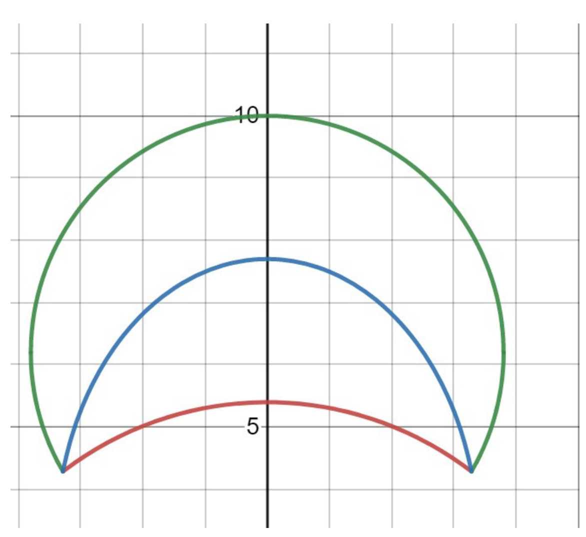
$y = L_1 - x\{L_1 - \frac{l_1}{2} \leq x \leq L_1\}$

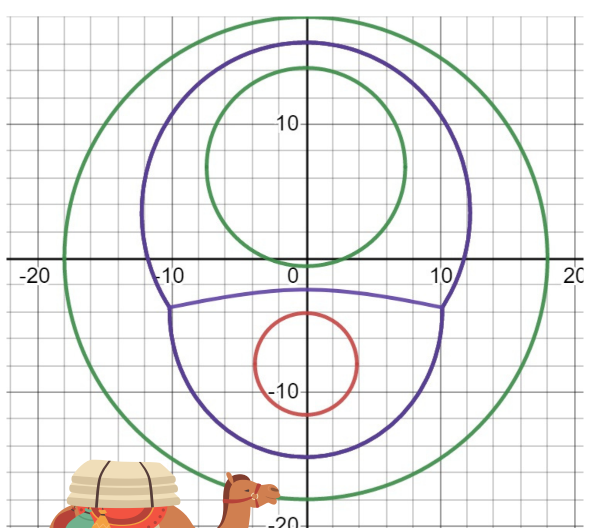
$y = L_2 - x\{0 \leq x \leq \frac{l_2}{2}\}$

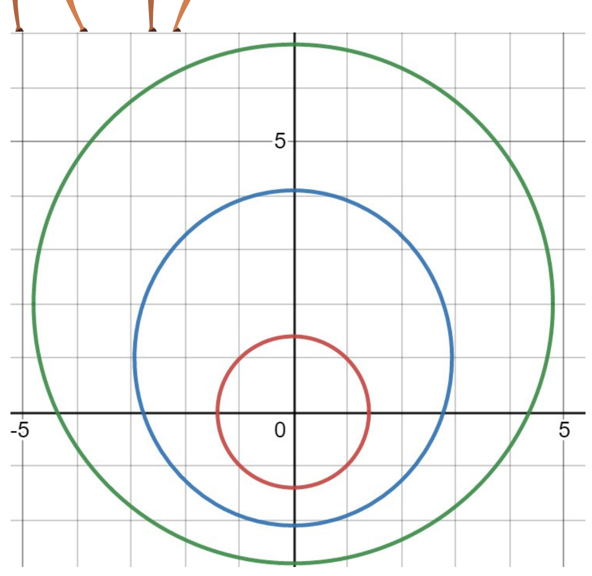
$y = x - L_1 + l_1\{L_1 - \frac{l_1}{2} \leq x \leq L_1\}$

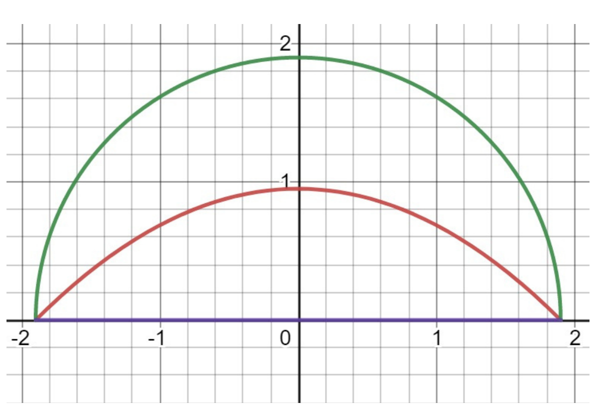
$y = x - l_2 + L_2\{\frac{l_2}{2} \leq x \leq l_2\}$











Semicircle

$$y = -\frac{1}{2r}x^2 + \frac{r}{2}$$

Crescent

$$\sqrt{x^2 + (y-d)^2} - r_1 = r_2 - \sqrt{x^2 + y^2}$$

Circle from which we cut a circle

$$\sqrt{x^2 + (y-d)^2} - r = R - \sqrt{x^2 + y^2}$$

Circle from which we cut two circles

Upper crest

$$\sqrt{x^2 + (y-d_1)^2} - r_1 = R - \sqrt{x^2 + y^2}$$

Middle crest

$$\sqrt{x^2 + (y-d_1)^2} - r_1 = \sqrt{x^2 + (y-d_2)^2} - r_2$$

Bottom crest

$$\sqrt{x^2 + (y-d_2)^2} - r_2 = R - \sqrt{x^2 + y^2}$$