TOPIC 2
Percolation
2021-2022

Soil model & water flowing metrics

The soil is **modeled as a 50x100 grid** with a **certain density** of blocked cells. The water is on the top layer and it will fill every adjacent (i.e. if they have a common side) cell that is empty. We will assume that one such operation of filling a cell takes **one time unit**. Let **t be the time** (the amount of time units) needed for the water to complete the **shortest path between the first and the last line** of the matrix.

Our task is to establish the **relation between t and d** as a function based on simulations.

We consider the **density** to be **specific for each individual cell**, giving us a matrix such as the following:

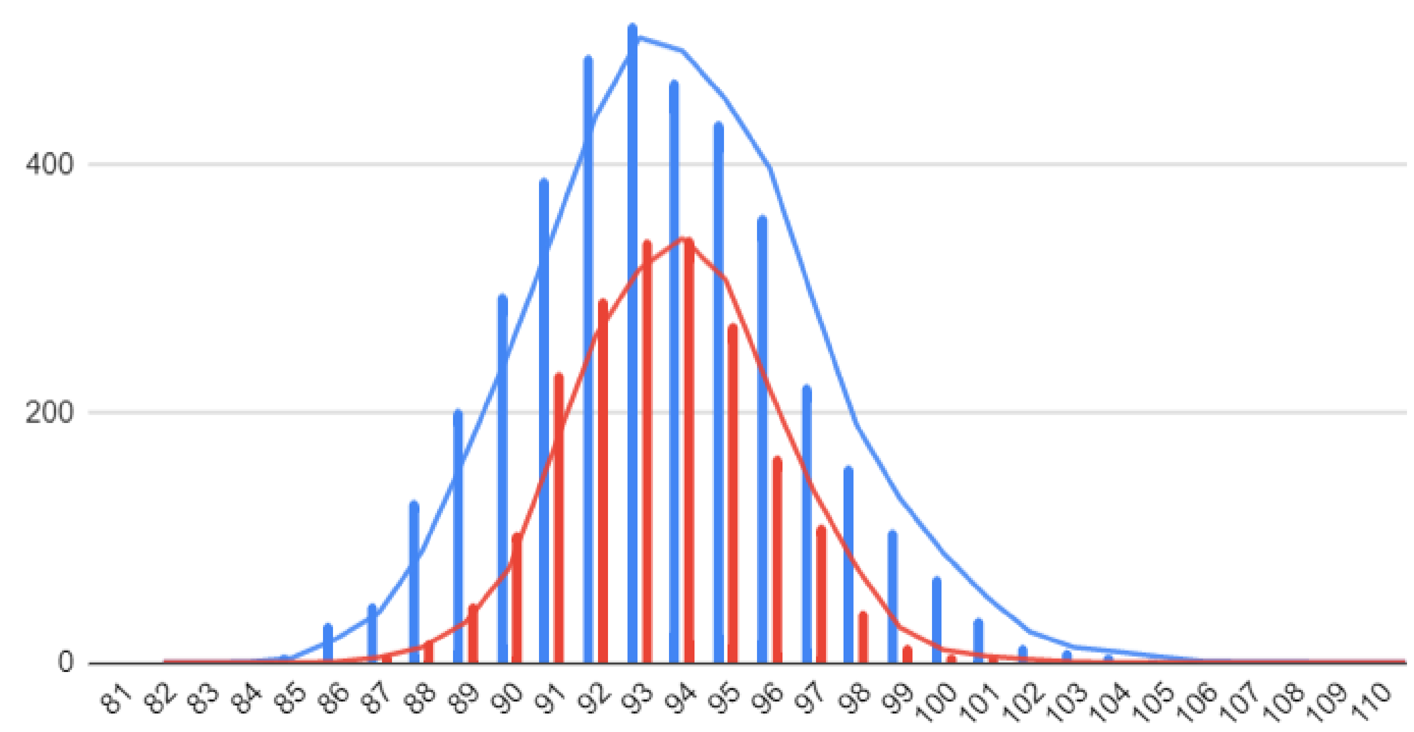
85	63	83	62	40	62	30	41	34	99	68	33	56	2	85	96	55	97	4	14
87	5	85	32	83	66	40	9	97	52	47	66	12	63	70	9	28	26	57	31
24	3	14	25	12	96	7	31	23	11	54	46	7	44	18	27	91	50	91	69
36	84	9	72	93	72	58	85	24	6	24	40	85	87	0	59	53	80	41	40
88	6	22	88	2	42	86	58	3	45	14	87	71	99	20	52	11	80	93	46
81	8	19	79	8	97	76	71	42	23	6	82	41	46	71	48	99	95	55	26
37	22	57	71	12	43	95	78	68	98	25	68	22	15	94	43	93	64	49	0
98	38	6	26	10	58	37	0	61	12	19	63	59	82	80	7	58	7	98	89
74	88	7	5	29	52	40	95	71	10	76	7	79	8	1	73	29	25	89	92
42	7	56	64	56	13	1	30	3	13	63	84	12	91	96	58	98	30	61	67

In this example we are 95% confidence that the mean of all the values that t can take (i.e. the expected value) is in between 14.218 and 14.225 (over 1000 observations)

The problem of finding the **shortest path in a matrix** is often encountered in mathematics, especially in **graph theory**. However, when dealing with probabilities, taking into account all the possible matrices would render any thorough approach obsolete. Therefore, our approach consists of making **inferences based on a sample distribution**.

If we take into consideration only the successful cases, by plotting different sample means on a chart, according to the **Central Limit Theorem**, we would come up with a **normal distribution** as the number of tests goes on to infinity.

Below there is a chart obtained by creating samples of 25 (blue) and 50 (red) respectively over a testbed of 100 000 cases (based on a randomly generated 50x100 matrix).



Normal Distributions

Every normal distribution is characterized by 2 values

-μ(Mu) which represents the distribution mean is **the expected value**

-σ² (Sigma squared) represents the variance of the distribution

Sample Distribution

When dealing with only a part of the total possible martices, we deal with a sample distribution, which is characterized in turn by

-x̄(x barred) represents the sample mean

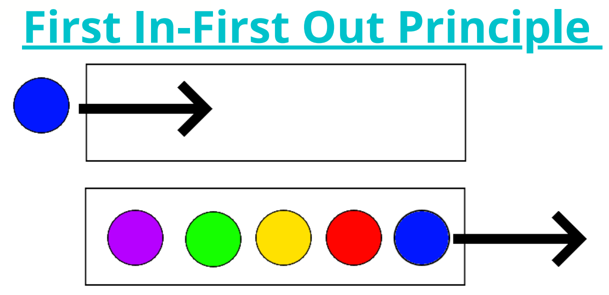
-s² (s squared) represents the sample variance

-s represents the standard deviation of the sample

Sketch of our method of simulating the flow of water through a matrix

```
procedure PBFS
  for aux←1,n do
    Q.enqueue(0,i)
  end for
  while Q!=∅ do
    (i,j)←Q.dequeue()
    for (ni,nj)∈neighbors(i,j) do
      if visited(ni,nj)=false
        if random<p_success do//p_success=1-cell_density
          Q.enqueue(ni,nj)
          reached←reached+1
          if reached=last_line do
            success!
          end if
      visited(ni,nj)←true
      end if
    end for
  end while
end procedure
```

First In-First Out Principle



Interpretation of results

In order to approximate the expected value of time required for the water to cross the matrix we must first create a sample and then make inferences on its mean accordingly. Thus, we run the code displayed above multiple times (as many as we find convenient) and calculate the arithmetic mean of the valid values (<u>note that there might be cases in which the water does not reach the last line of the matrix</u>). After we obtain the **sample mean**, we can move onto the **standard deviation** of the sample as displayed below:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad s^2 = \frac{\sum_{i=1}^{n} (\bar{x} - x_i)^2}{n-1}$$

Then, depending on the **confidence of the interval** in which we want the expected value to be placed, we select the **t value** (from a t table) and then formulate the result:

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

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