Equilibrium

By Daria Şerbănescu and Flavius Budiu
Colegiul National "Emil Racovita" (Cluj, Romania)

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RESEARCH TOPIC

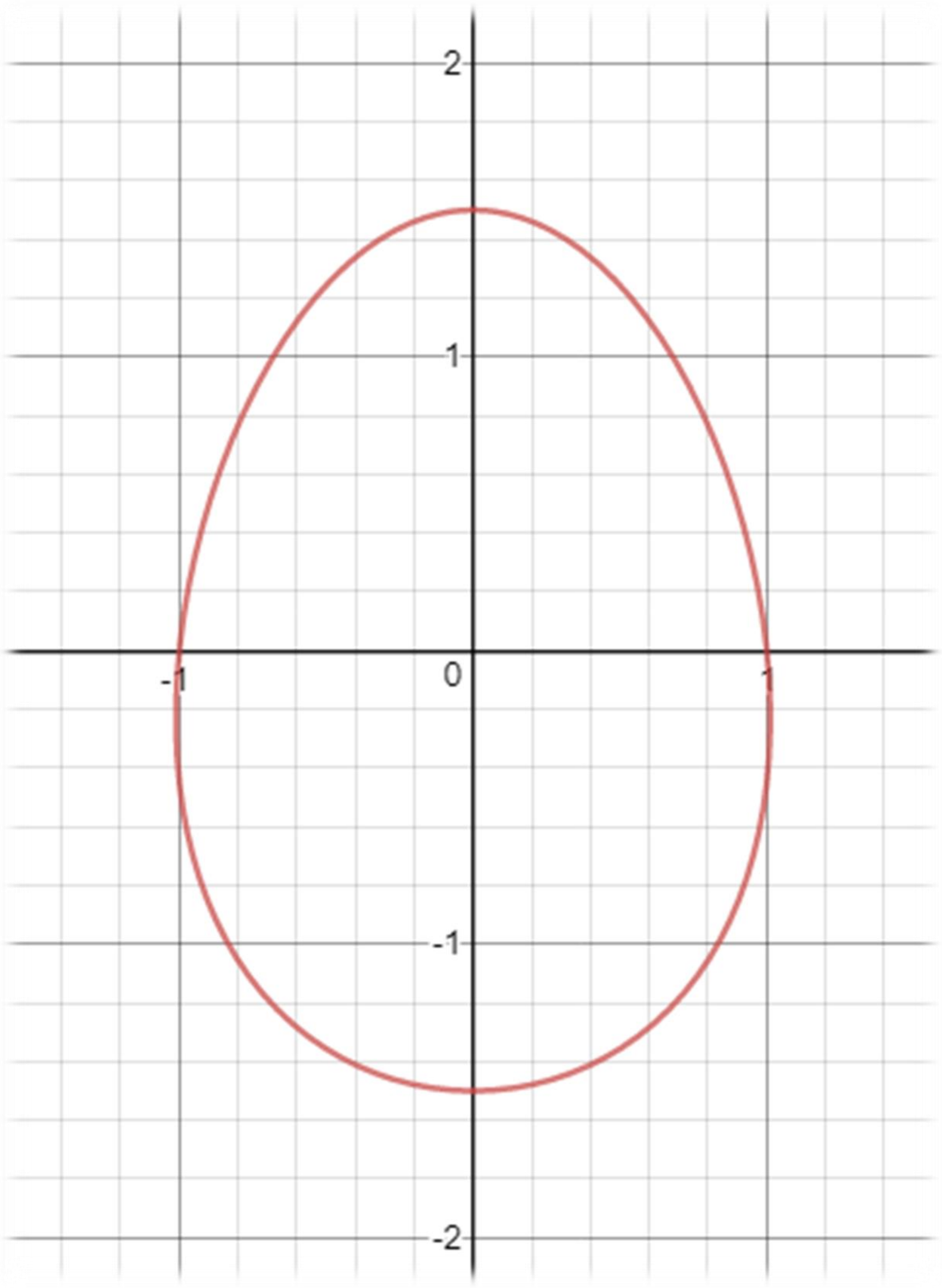
- Determine the equilibrium positions of an egg (bidimensional) or of a parabola section.

INTRODUCTION

- First of all, we took the equation below* to describe an egg. We thought that the egg would have 2 points of equilibrium, symmetrically positioned to the axis of symmetry (in our case, OY).

1 $\frac{y^2}{e_c} + x^2 \cdot t(y) = r^2$

2 $t(y) = 1 + py$



METHOD

- From the centre of the XOY system, we draw rays at each 1°, starting with OX. Intersecting these rays with the "outline of the egg" we obtain 360 points notated with $P_i(x_i, y_i)$ i∈{1, 2, ...,360}.
- Using these points we divide our surface in triangles with one vertex at the origin O, ΔOP_iP_{i+1}.

Step 1. Finding the centroid of each triangle

- Knowing the formula for the coordinates of the centroid[1], we apply it for each of our triangle, obtaining:

$$x_{C_i} = \frac{x_i + x_{i+1}}{3};\ y_{C_i} = \frac{y_i + y_{i+1}}{3}$$

- Finding the centroid we also find the centre of gravity[2] (G_i=C_i).

METHOD

Step 2. Calculating the area of each triangle

- Using Gauss's shoelace formula we can calculate the area of each $\Delta\ OP_iP_{i+1}$ triangle.

$$A_i = \frac{1}{2}(x_i y_{i+1} - y_i x_{i+1})$$

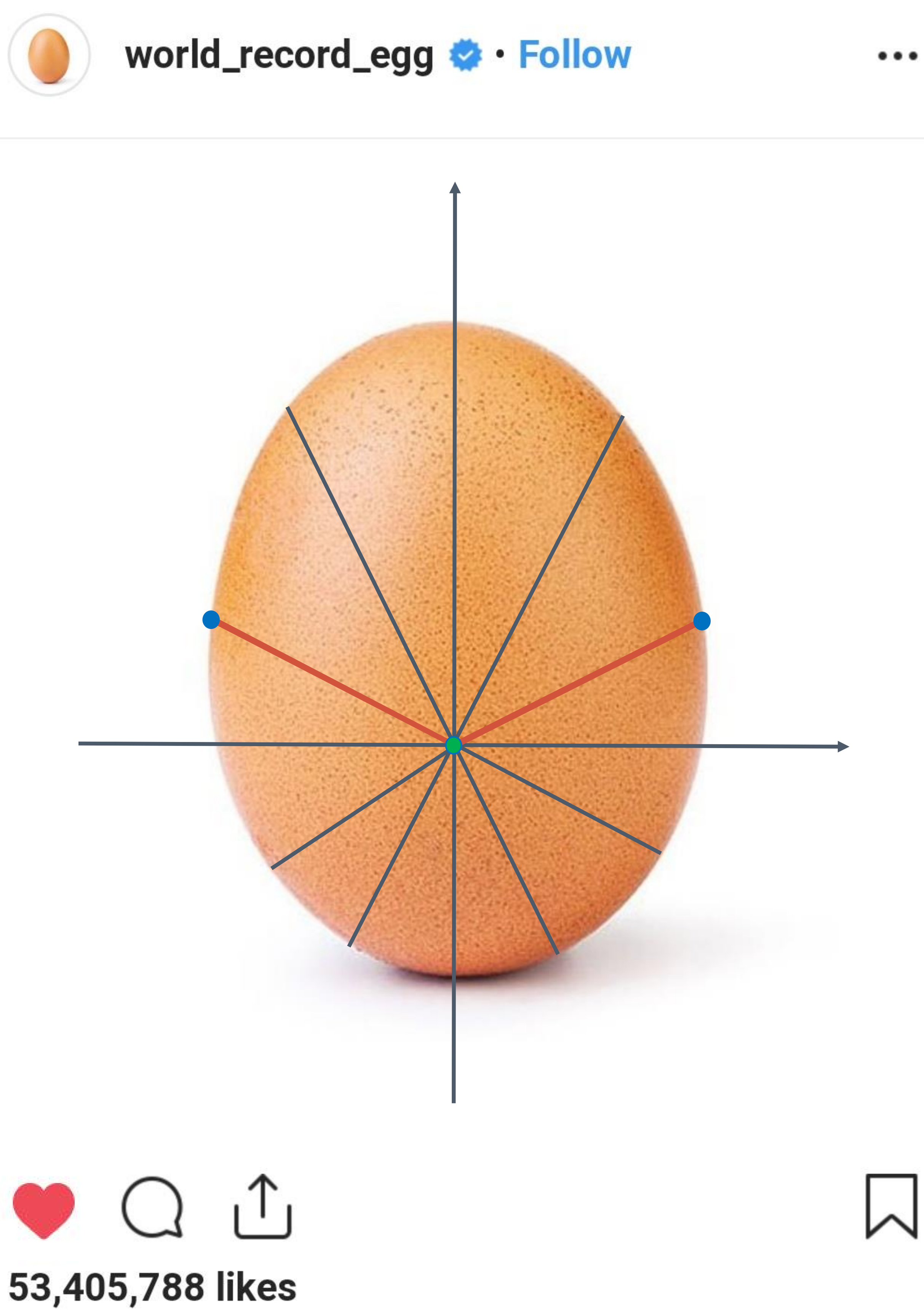
Step 3. Generalization of the formula

To find the centre of gravity (G) we need to calculate the weighted average of all G_i points, also considering the area of the triangle it represents.

$$x_G = \frac{\sum_{i=1}^{360}(A_i x_{G_i})}{\sum_{i=1}^{360} A_i} \Leftrightarrow x_G = \frac{\sum_{i=1}^{360}\left(\frac{1}{2}(x_i y_{i+1} - y_i x_{i+1})(\frac{x_i + x_{i+1}}{3})\right)}{\sum_{i=1}^{360} A_i} \Leftrightarrow x_G = \frac{1}{6}\frac{\sum_{i=1}^{360}((x_i y_{i+1} - y_i x_{i+1})(x_i + x_{i+1}))}{\sum_{i=1}^{360} A_i}$$

$$y_G = \frac{\sum_{i=1}^{360}(A_i y_{G_i})}{\sum_{i=1}^{360} A_i} \Leftrightarrow x_G = \frac{\sum_{i=1}^{360}\left(\frac{1}{2}(x_i y_{i+1} - y_i x_{i+1})(\frac{y_i + y_{i+1}}{3})\right)}{\sum_{i=1}^{360} A_i} \Leftrightarrow y_G = \frac{1}{6}\frac{\sum_{i=1}^{360}((x_i y_{i+1} - y_i x_{i+1})(y_i + y_{i+1}))}{\sum_{i=1}^{360} A_i}$$

- Calculating the distances d(G, A_i), where i∈{1,...360} and knowing that an object is in stable equilibrium when its centre of gravity is the closest to the surface it is standing on[3], finding the smallest distance from the centre of mass to the outline of the egg will show us the points of equilibrium.

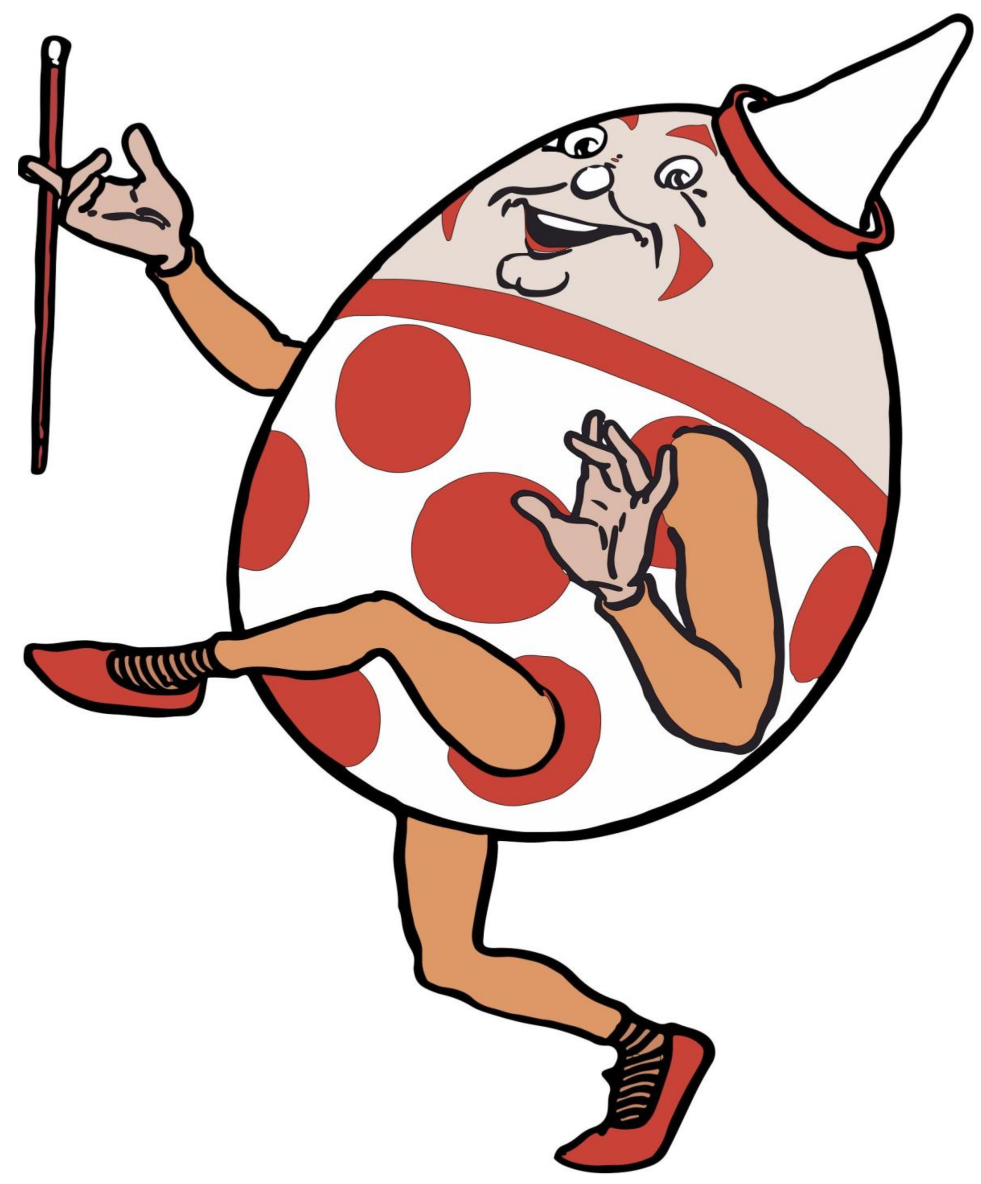


PROPRIETIES USED

- [1]The formula for the coordinates of the centroid of a triangle is:

$$x_C = \frac{x_1 + x_2 + x_3}{3}$$
$$y_C = \frac{y_1 + y_2 + y_3}{3}$$

- [2]In case of a rigid, homogeneous object, the centre of mass will be located at the centroid, together with the centre of gravity.
- [3]An object is in stable equilibrium when its centre of gravity is the closest to the surface it is standing on.



NOTES

- We consider the egg as being solid and homogeneous.
- * Knowing that the equation $x^2 + y^2 = r^2$ describes a circle of radius r, to obtain an ellipse we divide y^2 by e_c, the eccentricity of an ellipse, where e_c=9/4. To develop the shape of an egg we have to multiply x^2 with t(y), where t(y)=1+py, -0.2≤p≤0.2 and p describes the constant which controls the pointiness of the oval.

CONCLUSIONS

- To determine the equilibrium positions of an egg we divide the egg into triangles of whom we find the centre of gravity and area. Using these, we calculate G's coordinates.
- The smallest distance d(G, P_i), i∈{1,...360} will correspond to two points on the egg situated symmetrically across the OY axis, the points of equilibrium.