Untwisting braids

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Introduction

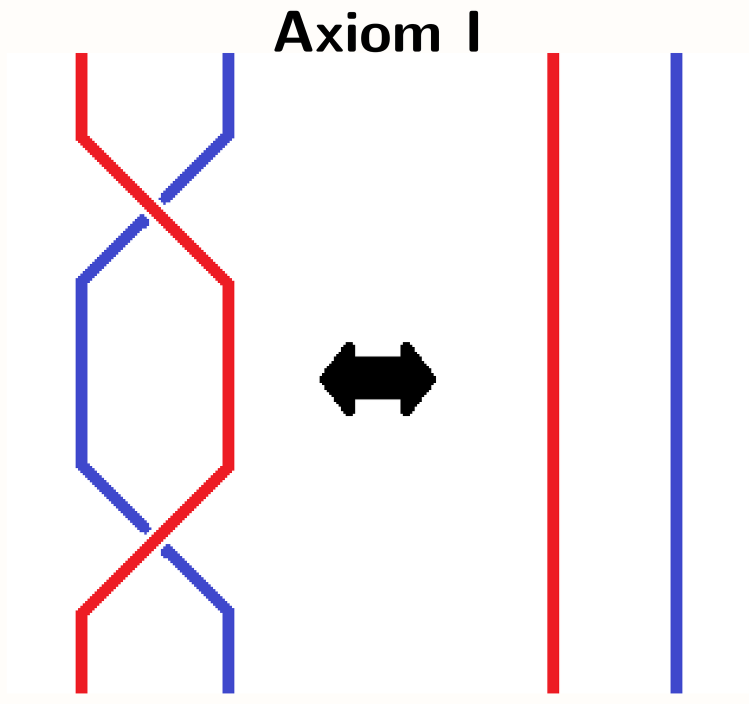
A braid consists of n vertically arranged yarns that intermingle in various crossings and are tied up and down. If we can remove all the crossings of a braid without cutting wire or moving their ends, we say that the braid is trivial. Opposite, we see two braids with three yarns, on the left a trivial one and on the right a non-trivial one:

a) Are there non-trivial braids such that if we remove any of its strands (for example if we cut it) then the braid becomes trivial ?

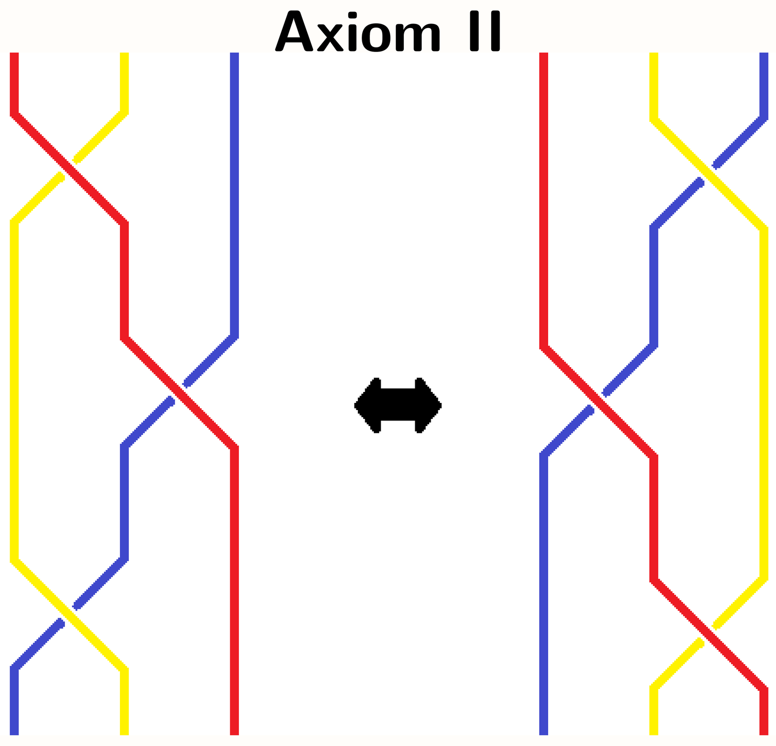
b) Is it possible to stick a braid under another braid such that the braid obtained is trivial?

Axioms

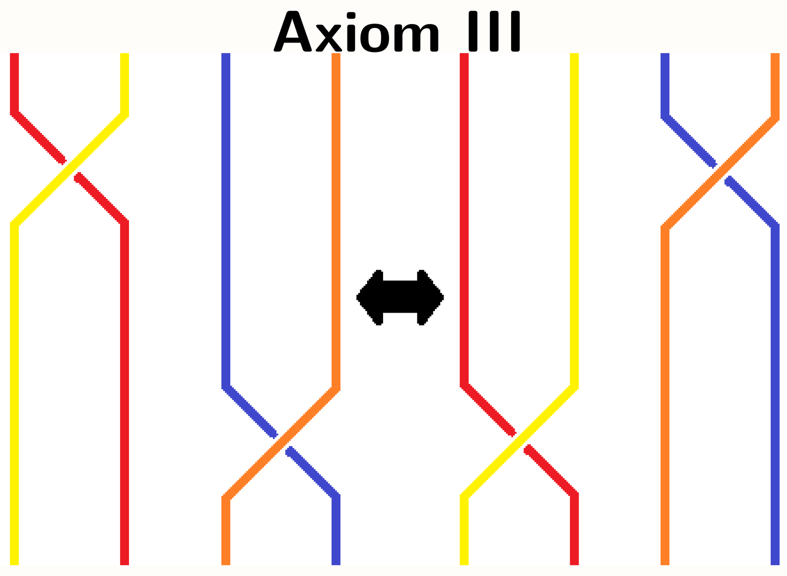
Axiom I



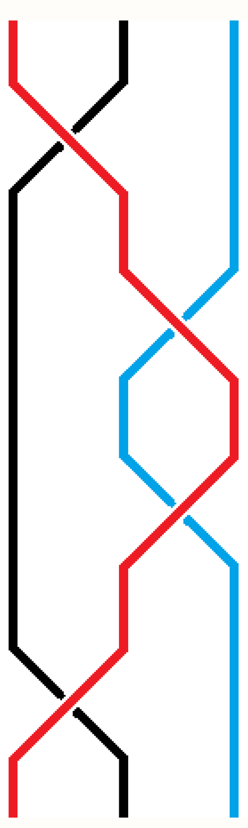
Axiom II



Axiom III



Notations



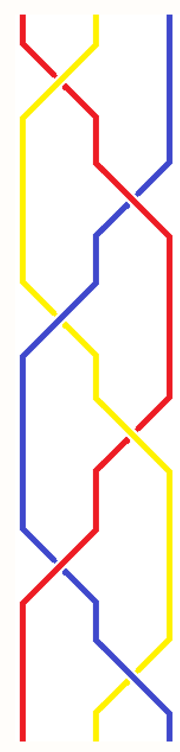
(2, 1)

(3, 1)

(3, 1)

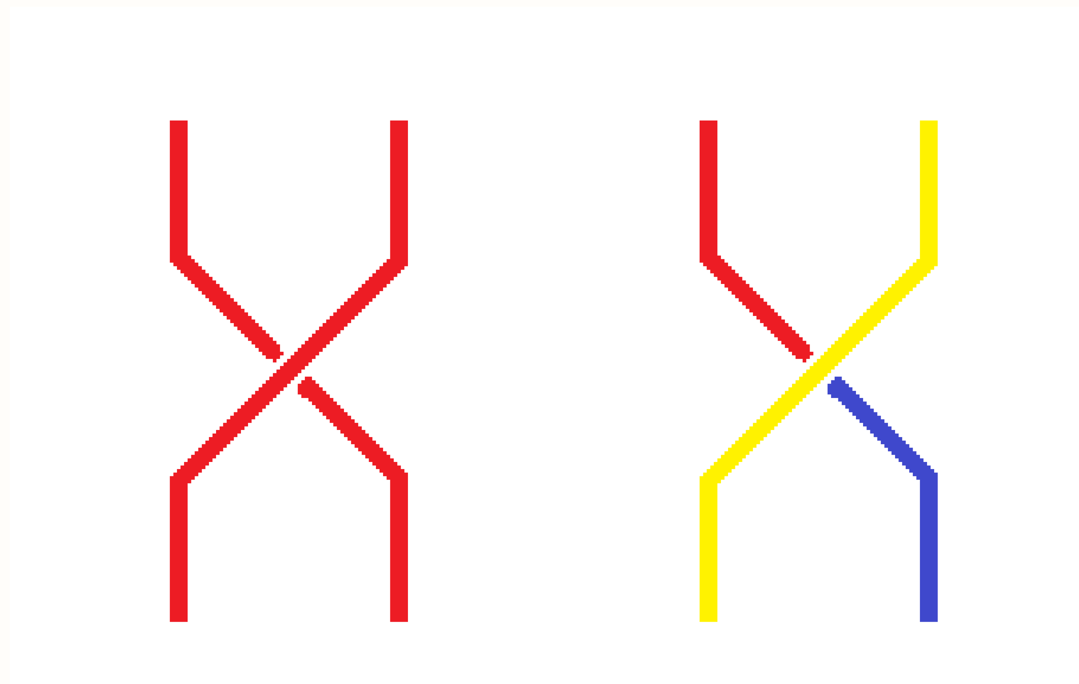
(2, 1)

Borromean braid

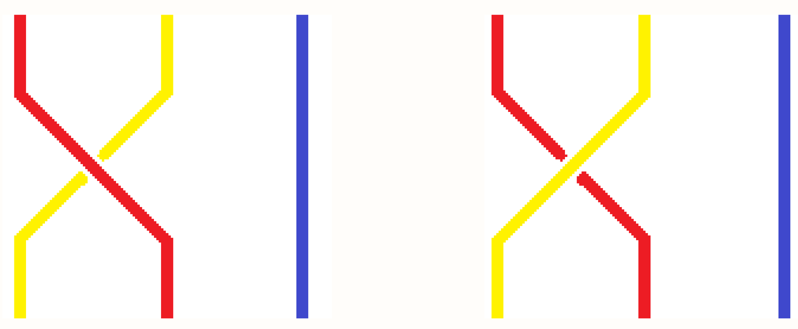


(1, 2)
(3, 1)
(2, 3)
(1, 2)
(3, 1)
(2, 3)

Tricolorability



The group



x $\quad$ x^{-1}

$G = \langle x_1, x_2, \ldots, x_n \rangle$

$|x_i| = \infty$

$\langle x_i \rangle \cap \langle x_j \rangle = \{e\}$, for $i \neq j$

Let x and y be generators. Then:

Axiom I

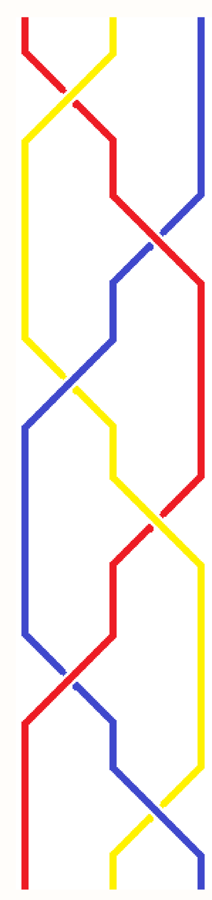
There exists x^{-1}.

Axiom II

$xyx = yxy$, if x and y are adjacent.

Axiom III

$xy = yx$, if x and y are not adjacent.



$(x^{-1}y)^3 \neq e$