





Co-funded by the Erasmus+ Programme of the European Union

#### **Totally Maths**

We proudly present the Maths work in our Erasmus+ Project Students taught Maths to peer students and then worked on the process of the creation of the lessons









### Math Class

Dries, Tibo, Gust and Kyara

Logo contest!



#### First exercise + dimensions and elements



### Diagonal Matrix



#### Symmetrical Matrix



#### Row matrix



#### Column Matrix



#### Zero matrix



### Exercise:

- 1) What's the dimension?
- 2) Give the element b23,b44,b61
- 3) Which element has the longest route?
- 4) What is special about the form of the matrice ?

A [	) G	L	Μ	S		
0 5	3 65	39	25	26	Α	
i3 0	30	14	28	27	D	
5 30	0 (	26	58	39	G	
9 14	26	0	42	13	L	
5 28	58	42	0	34	M	
6 27	39	13	34	0	S	
	A [ 0 5 53 0 5 30 9 14 5 28 6 27	A D G   0 53 65   53 0 30   5 30 0   9 14 26   5 28 58   6 27 39	A   D   G   L     0   53   65   39     53   0   30   14     5   30   0   26     9   14   26   0     5   28   58   42     6   27   39   13	A   D   G   L   M     0   53   65   39   25     53   0   30   14   28     5   30   0   26   58     9   14   26   0   42     5   28   58   42   0     6   27   39   13   34	A   D   G   L   M   S     0   53   65   39   25   26     53   0   30   14   28   27     5   30   0   26   58   39     9   14   26   0   42   13     5   28   58   42   0   34     6   27   39   13   34   0	A   D   G   L   M   S     0   53   65   39   25   26   A     53   0   30   14   28   27   D     5   30   0   26   58   39   G     9   14   26   0   42   13   L     5   28   58   42   0   34   M     6   27   39   13   34   0   S



### Adding up matrices







Solution

#### Exercises

A - B +C if

#### Determine X, Y, Z and T

 $A = \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix}, B = \begin{bmatrix} -a & 2b \\ 0 & a \end{bmatrix} \text{ en } C = \begin{bmatrix} 2a & -2b \\ a & 0 \end{bmatrix}.$ 

 $\begin{vmatrix} x & 4 \\ 2t & y \end{vmatrix} - \begin{vmatrix} y & z \\ 4t & -x \end{vmatrix} = \begin{bmatrix} 3 & 0 \\ 12 & -3 \end{vmatrix}.$ 

### Multiply with Matrices

The school is paying for the teachers who get to school by bike. They calculate the bicycle compensation a month. They get €0,15 for each kilometres.

To calculate the bicycle compensation, we need to multiply every number in the matrix A with 0,15. For example:  $240 \ge 0,15 = \pounds 36$ 



	maart	april	
	36,00	21,60	lkr 1
B = 0,15A =	15,75	9,75	lkr 2
	43,20	24,00	lkr 3

### Theory: multiplying matrices

 To solve the matrix equation with X the unknown matrix with dimension 2 X 2 we do this:



### Exercice 4

# Thank you for your your attention!

### What is an Equation?

In mathematics, the equality between two algebraic expressions is called equation, which will be called members of the equation. In the equations, they will appear related through mathematical operations, numbers and letters (unknowns).





Don't confuse it with a polynomial that they have no equality



#### - First degree equation.

3x + 2 = x + 4 3x - x = 4 - 2 2x = 2 x = 1

#### - A second degree equation.

$$x^{2}$$
+ 3x + 2 = 0



### PUT SOME ORDER





Formula  $-b\pm\sqrt{b^2}$ -4ac

This is the formula you have to know, otherwise you won't be able to do any of them

We'll call the first known number 'a', the second one 'b' and the third one 'c'

**For example:** x<sup>2</sup> + **5x + 6 = 0** a b c

Here is a video that could help you:





Complete Second Degree Equations



It is complete because we have a, b and c. To solve it we have to use the formula, and as we know where each number goes, we place it and solve it.



There are two solutions: we use the positive value in one of them, and the negative in the other one.



### Equations without solution.

 $x^{2}-2x+6=0$   $x = +1 \qquad b = -2 \qquad c = +6$   $x = \frac{2\pm\sqrt{2^{2}-4\cdot1\cdot6}}{2\cdot1} = \frac{2\pm\sqrt{4-24}}{2} =$   $x = \frac{2\pm\sqrt{-2t}}{2} \neq$ 

In this part, using the formula you'll see that the equation it doesn't have solution.

There is no solution if we have the sign *"-"* in the squared root.

Ne maze

double solution

In some cases the solution is double.

this happens when the last step is + - 0

Let's see it in the example

$$x = \frac{-4 \pm \sqrt{4^2 - 4.1.4}}{2.1}$$

$$x=rac{-4\pm\sqrt{16-16}}{2}=rac{-4\pm\sqrt{0}}{2}=rac{-4\pm0}{2}$$

🖇 emaze

# THE SPANISH TEAM **PRESENTS:** THE MAKING OF OUR MATH CLASS

## Choosing the subject:



## Dividing the tasks:



## Working on the presentation:



## Practising the class:



## Practising with public:



## Correcting mistakes:



## Doing the exposition in Berkenboom to the Erasmus students:



## Doing the exposition in Berkenboom to the regular students:







To sum up, it was a hard work that involved all of us and we're happy with the results and we enjoyed the experience.

## Graphing Functions

### First Degree Functions


#### First Degree Functions/ Linear Functions



A first degree function, or linear function is a function of a line.



It can be written in the form f(x)=ax+b



When graphed it is a straight

line

## When graphed...



The value of b is the point where the function crosses the y-axis (also known as the y-intercept)



The value of *a* is the rate of change of the function (the slope of the line)



A linear function, in the form f(x)=ax+b is the same as the equation of a line y=mx+c

#### y=mx+c







c = initial value (y-intercept)

## Linear Functions on Geogebra



## Game Time



Second Degree Functions

Quadratic functions



# What is a quadratic function?



A quadratic function, or second degree function is a function that can be described by an equation of the form  $f(x)=ax^2+bx+c$  where  $a \neq 0$ .



In a quadratic function, the greatest power of the variable is 2. The graph of a quadratic function is a parabola.

*Parabola*  $y = ax^2 + bx + c$ 



#### What are the roots of a quadratic function?

• The roots of the function is where graph cuts the x-axis.



Quadratic Functions on GeoGebra



## Game Time





### Kahoot!!



We're going to play a game of Kahoot

**Kahoot** 



## Introduction

In mathematics the Hungarian algorithm is a combinatorial optimization method that solves the assignment problem in polynomial time.

## Matrix

To use this algorithm we must use matrices. What is a matrix? It's a rectangular array of numbers, symbols, or expressions, arranged in rows and columns.

m-by-n matrix ain j changes n columns m rows a1,1  $a_{1,2}$ a1.3  $a_{2,2}$   $a_{2,3}$  $a_{2,1}$ changes a3,2 a3,3  $a_{3,1}$ . . . ··..

You are the director of a company the sellers are located respectively in the following cities:

-Austim

-Boston

-Chicago

You want your sellers to fly to three other cities:

-Denver

-Edmonton

-Fargo

From / to	Denver	Edmonton	Forgo
Austin	250	400	350
Boston	400	600	350
Chicago	200	400	250

-Where you need to send your agents so that the total cost of tickets is minimal? We introduce a matrix called the cost matrix

-A possible choice could be 250+600+250= 1100 Or maybe 250+400+350= 1000, which is even better -After trying them all, you will find that the combination with the least cost is 400+350+200=950



Now let's try it with the Hungarian algorithm!

-After trying them all, you will find that the combination with the least cost is 400+350+200=950



Now let's try it with the Hungarian algorithm!

#### Theorem

- If a number is added to or subtracted from all the entries of each row on column of a cost matrix, then on optimal assignment for the result in cost matrix as also an optimal assignment for the oriental cost matrix

-From each row, we find the row minimum and subtract it from all entries in that row

-Each row has at least one zero

-All entries are positive or zero

1	250	400	350	
	400	600	350	
	200	400	250	

-From each column, we find the minimum and subtract if from all entries on that column

-Each row and each column has at least one zero

0 150 100 50 250 0 0 200 50

-We draw lines across rows and columns in such a way that all zeros are covered and that the minimum number of lines have been used (in this case lines across the 1st and the 3rd row and across the 1st column)



-A test for optimality: -If the number of lines just drawn is n (number of rows of the cost matrix); we are done. If the number of lines" < n, we go to step 5.

The first case is for shorter exercises

100 50

50

100

50

#### Step 5 (only if necessary)

-We find the smallest entry which is not covered by any of the lines. Then subtract it from each entry which is not covered by the lines and add it to each entry which is covered by a vertical and horizontal line. Now we can go back to step 3.

#### **Maximization problem**

-If we have, instead of a minimization problem, a maximization problem, multiply the matrix C by -1 and proceed as above. If C is not a square matrix (there are more tasks than workers or conversely), we have to augment C into a square matrix by adding zero rows or columns.

# Why the algorithm?

The hand by hand checking method works well only when we have small problems, in fact how n grows, it increases the attempts, because there are n! ways of assigning n resources to n tasks.

-Blue = n -Green = n^2 -Red = e^n -Yellow = n!



#### What's the n! ?

n!= n\*(n-1)\*....\*1

-For example: 3!= 3\*2\*1=6



## **The Masterminds**

Buica Antonio, Levav Maya, Lizzadro Luca, Longo Tommaso, Ometto Vittorio, Pasqualin Davide, Pinarolli Leonardo, Rubinelli Johannes

#### Did you understand?

#### Who was Harold W.Khun?



#### Where and when was Harold born?

#### What did Harold win?









#### Choose the correct algorithm definition





8 guys have to make a Carbonara. So they went to the supermarket to buy 3 ingredients. They must find the less expensive



▲ 5+10+13=28	♦ 9+10+15=34
5+12+13=30	■ salmon+pike=big lake trout

## How did we work on the project?


# How did we start?

We started our work by looking for an original topic that could also involve those who were not skill in mathematics

# The idea

The idea who was recommended by our math teacher. At the beginning of the year we were without the teacher and we didn't know how to start the work.

# How did we split the work?

To work on the project we organize ourselves in two meetings a week where for an hour the whole class worked on various steps of the presentation

## Process

 We started by focusing on Harold Khun's life
 We studied the algorithm and prepared some exercises about it
 We prepared the presentation: -powerpoint

-prezi

-kahoot

## Presentation





# Trigonometry

#### Who are we?

- students from Belgium
- 16/17 years old



# BELGIV

# BELGIUM ON A MAP





- capital city
- amazing and beautiful by night
- every year flower market in Brussels' big market



# ANTWERP

- the port of Antwerp
- Sportpaleis/Lotto Arena
- 'De Meir'





- Gravensteen
- Graslei/Korenlei
- Gentse Feesten



#### **FAMOUS BELGIAN FOOD**

- Fries \_
- -
- (Brussels and Liège) waffles Chocolate and especially pralines Carbonade flamande \_
- \_
- Cuberdon \_
- Vol au vent
- Beer \_









#### THE BELGIAN COAST

- It's an attractive place
- A lot of people have a second house or apartment here
- It's the holiday place in Belgium





#### **BELGIAN INVENTIONS**

- Saxophone
- internet, Robert Cailliau was from Belgium
- Candy
- Praline
- Deodorant







#### FAMOUS BELGIAN PEOPLE

- Angèle
- Stromae
- Romelu Lukaku
- Kevin De Bruyne
- Eden Hazard
- Thibaut Courtois
- Wout Van Aert
- Max Verstappen (his mother)
- Kim Clijsters
- Nafi Thiam
- Nina Derwael





#### Some mathematical terms in English

Teorema de pitágoras	Pythagorean theorem
distancia	distance
ángulo	angle
triángulo rectángulo	right-angled triangle
suma de ángulos	angle sum
proporción	ratio

#### Why do we need trigonometry?

#### Trigonometry

• to obtain unknown angles/ distances

#### Pythagorean theorem

• to obtain unknown distances

#### What do we need?

- right-angled triangle
  - opposite side (= lado opuesto)
  - hypotenuse (= hypotenusa)
  - adjacent side (= lado adyacente)

#### • 2 known values

- o 2 distances
- o 1 distance and 1 angle



#### The basics

Pythagorean theorem: a<sup>2</sup>+b<sup>2</sup>=c<sup>2</sup>

Sine  $\alpha$ = opposite side / hypotenuse (SOH) Cosine  $\alpha$ = adjacent side / hypotenuse (CAH) Tangent  $\alpha$ = opposite side / adjacent side (TOA)



#### Now it's your turn...

• a right-angled triangle

#### What do we know?

opposite side= 10 cm

adjacent side= 15 cm

tan<sup>-1</sup>(O / A)= tan<sup>-1</sup>(10 / 15)

*α*= 33,7°



#### Now it's your turn...

• a right-angled triangle

#### What do we know?

opposite side= 8cm  $\alpha$ = 50°

 $\tan \alpha = O/A$  $A = O/\tan \alpha$  $A = O/\tan (50^{\circ})$ 

=>



A= 6,7 cm

#### Values to remember

α	<b>0</b> °	30°	45°	60°	90°
sine	0	1/2	√2/2	√3/2	1
cosine	1	√3/2	√2/2	1/2	0
tangent	?	?	?	?	?

#### What is tan (0)?

=>  $\tan \alpha = \sin \alpha / \cos \alpha$ =>  $\tan 0^\circ = \sin 0^\circ / \cos 0^\circ$ =>  $\tan 0^\circ = 0 / 1$ =>  $\tan 0^\circ = 0$ 

α	<b>0°</b>
sine	0
cosine	1

#### What is tan (30)?

=>  $\tan \alpha = \sin \alpha / \cos \alpha$ =>  $\tan 30^\circ = \sin 30^\circ / \cos 30^\circ$ =>  $\tan 30^\circ = (1/2) / (\sqrt{3}/2)$ =  $(1/2) \cdot (2/\sqrt{3})$ =>  $\tan 30^\circ = 1/\sqrt{3}$ 

α	0°
sine	1/2
cosine	√3/2

#### What is tan (45) ?

=>  $\tan \alpha = \sin \alpha / \cos \alpha$ =>  $\tan 45^\circ = \sin 45^\circ / \cos 45^\circ$ =>  $\tan 45^\circ = (\sqrt{2}/2) / (\sqrt{2}/2)$ =>  $\tan 45^\circ = 1$ 

α	45°
sine	√2/2
cosine	√2/2

#### What is tan (60)?

=>  $\tan \alpha = \sin \alpha / \cos \alpha$ =>  $\tan 60^\circ = \sin 60^\circ / \cos 60^\circ$ =>  $\tan 60^\circ = (\sqrt{3}/2) / (1/2)$ =  $(\sqrt{3}/2) . (2)$ =>  $\tan 60^\circ = \sqrt{3}$ 

α	60°
sine	√3/2
cosine	1/2

#### What is tan (90)?

=> tanα = sinα / cosα
=> tan 90° = sin 90° / cos 90°
=> tan 90° = 1 / 0
=> tan 90° = /

α	90°
sine	1
cosine	0

# Thank you for your attention!

# the progress



- Preparing a game
- Too complicated for students
- We scratched the idea

# All are work for nothing

- Prepared our PowerPoint and text
- Wrong subject
- Teacher's fault

# **New lesson**





# We were ready to go



## **Lesson in Marbella**


















It depends of how many vertex we have, the domo will seem more or less as as a sphere

#### And can I build a DOME with this figure (an icosaedrom)?



#### YES, but it doesn't look like a sphere at all

#### A DOME IS TO GET MORE POINTS OF THE SPHERE FROM AN ICOSAHEDRON

#### HOW?

#### VERTEXES OF THE SPHERE

#### CENTERS OF THE SIDES

☆NEW VERTEXES!



# how is it calculated?











#### **ASSEMBLY DIAGRAM**

A=1,052R

B=0,519A

C=0,556A



Example: If we build a Dome of 10m of radium: The icosahedron will have the side A=10x1,052=10,52m

The type B bar will have: **B**=0,519x10,52=**5**,46m

The type C bar will have :C=0,556x10,52=5,85m

#### OUR MATHS LESSON

Spanish team











#### **BUILDING THE FIRST PIECES**





#### FINISHING THE DOME









#### THE DAY OF THE **PRESENTATION**







#### FINALLY WE SHOW OUR DOME







# GEOMETRY

BY EUREKA SECONDARY, IRELAND.

# WHO ARE WE?





## INTRODUCTION

#### HOW DO WE RECOGNISE SIMILAR TRIANGLES?



**Right angle triangle** 

**Isosceles triangle** 



**Equilateral triangle** 

### **SO.... WHAT ARE SIMILAR TRIANGLES?**

IN SIMILAR OR EQUIANGUALR TRIANGLES, ALL THREE ANGLES IN ONE TRIANGLE HAVE THE SAME MEASURE AS THE CORRESPONDING THREE ANGLES IN THE OTHER TRIANGLE.



#### EXAMPLE OF A SIMILAR TRIANGLE

LOOK AT THE CIRCLED DIGITS AND TRY AND SEE IF YOU NOTICE ANY CONNECTION BETWEEN THEM?



12









# THE RATIO OF THE SIDES



	AC	BC		DE	DF	EF
DE	DF	EF	or	AB	AC	BC







### **ANGLES IN SIMILAR TRIANGLES**



vertically opposite



# **REAL LIFE EXAMPLES**

# KAHOOT

Kahoot!

# SUM UP

#### WHAT ARE SIMILAR TRIANGLES?

#### HOW DO WE RECOGNISE SIMILAR TRIANGLES?



Class plan

#### Similar triangles

- 1. Introduction to us name, age etc.
- 2. Triangle game students are given triangles to match with, the only instruction is to match the triangles, our aim is to see if students can discover similar triangles
- 3. Introduction into similar triangles
  - What are similar triangles? We are going to use the triangles from the game to explain what similar triangles are
  - And how do we recognize these similar triangles? All three angles are equal in triangles, but the sides are in proportion. They are not the same length but the match up.
- 4. Explaining triangles
  - In similar or equiangular triangles, all three angles in one triangle have the same measure as the corresponding three angles in the other triangle.
- 5. The scale ratio of triangles
  - An example of finding the ratio
  - o Three examples for the students to work through themselves
- 6. The ratio of the sides
  - Showing examples on the power point

- o Worksheet for students to complete
- 7. Showing real life examples of similar triangles Looking at a lighthouse and ship example on the power point
- 8. Real life problems (handouts)- 3 questions regarding similar triangles and one in relation to ratio of the sides
- 9. Kahoot
- 10. Answering questions
#### How did we prepare our class?

#### **Difficulties**

The first problem we ran into was that a lot of people quit the project at the last minute. Because of this we had to find new students who were able to participate.

#### **Preparation**

When we found those new students we only had one week left to organize the whole project. Luckily our teacher Mr Steven helped us a lot in the preparation. To anticipate the possible problems we could encounter we already gave the lesson to a Belgian class at our own school. Thanks to this we were able to optimize our class and make sure all the students understood what we were telling them.



#### <u>The lesson itself</u>

The lesson itself went well except for a few problems. The students weren't really participating in the beginning of the lesson so we started picking out random people and this helped a lot. The students learned something new but it wasn't too complicated for them.

The Kahoot that we had prepared was online, so we could do it the way we prepared him, because phones aren't allowed in the school. Because of this, we used colored paper. In this way, it was a bit difficult to see who the winner was, but now we could ask them better why they thought something was right or wrong.

#### How did we prepare our class?

#### **Examples**



To explain the constant function we used the example of a wine cellar which always has to have the same temperature. Another example we gave was with a phone company who always charges the same amount of money every month.





To explain the linear function we used the example of farmer Patrick who has a piece of land. On the piece of land there are clovers. If he buys more land he will have more clovers.

To explain the linear function we used the example of a car driving. The car is driving and then encounters a traffic light. It reaches zero speed and then when the traffic light goes green again he speeds up. He leaves again.





To explain the exponential function we used the example where you have a cake of 10 pieces, when you are on your own, you have the 10 pieces for yourself. When there are 2 people, there are 5 pieces of the cake and when you are with 10 people, everyone has 1 piece. Another example we used was to build a house. When you're with not so many people, it will take a long time to build the house. When you are with a normal number of people, it will take a normal time to build the house and when you are with many people to build the house, the house will be built in a short time.

# BECGUN

## BELGIUM ON A MAP





- capital city
- amazing and beautiful by night
- every year flower market in Brussels' big market
- royal family -> King Fillip



#### ANTWERP

- the port of Antwerp
- Sportpaleis/Lotto Arena
- 'De Meir'



## GENT

- Gravensteen
- Graslei/Korenlei
- Gentse Feesten



#### FAMOUS BELGIAN FOOD

- Fries
- (Brussels and Liège) waffles
- Chocolate and especially pralines
- Carbonade flamande / stoofvlees
- Cuberdon
- Vol au vent
- Beer







time Duoel VEDETT Statut

#### THE BELGIAN COAST

- It's an attractive place
- A lot of people have a second house or apartment here
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#### **BELGIAN INVENTIONS**

- Saxophone by Adolphe Sax
- internet, Robert Cailliau was from Belgium
- Candy
- Chocolate
- Deo









#### FAMOUS BELGIAN PEOPLE

- Angèle
- Stromae
- Romelu Lukaku
- Kevin De Bruyne
- Eden Hazard
- Thibaut Courtois
- Wout Van Aert
- Nafi Thiam
- Nina Derwael



# this was

# BELGUI

# Functions

**BY BELGIUM** 

### Constant functions (Y=A)





## Linear functions (X=Y)



Quadratic functions (Y=X<sup>2</sup>)



## Quadratic functions (Y=-X<sup>2</sup>)







Exponential functions (Y=1/X)





### REPRESENTING LINEAR FUNCTIONS







y=2x+1

#### TABLE OF VALUES

Х	Y
0	1
1	3
-1	-1



y=-3x +2

#### TABLE OF VALUES

Х	Y
-1	5
0	2
2	-4















$$y=-3x+2$$

y=2x-1





y=1/2x

y=-x-2

And now we are going to do a kahoot!

# Thank you all for listening! It was a pleasure being here today.

т т



# THE SPANISH TEAM PRESENTS:

THE MAKING OF OUR CLASS

#### Choosing the subject:



#### Dividing the tasks:



### Working on the presentation:



#### Practising the class:













#### Practising with public:



#### Doing the exposition in Kells to the regular students:




It was a hard work, but we are all pleased and happy with the results and the experience:)

# Functions x4 x=92) ("=14! X=92) (JN 50-X-1)

 $\frac{x+y'=ab+4c}{A \rightarrow a} \begin{pmatrix} a \end{pmatrix} c(x,y) \begin{cases} xy=c \\ y \end{pmatrix} c(x,y) \begin{cases} xy=c \\ y \end{pmatrix} c(x-cy) = a \end{cases}$ 

22 -----

984 + X

D(x)=04014.

10 APB "T", 24 + x + 2+32 + x 3

(10) Va'+B! =X

### Meet our group!



Anna, 16	Niamh, 15	Alicia, 16
Kasey, 16	Eabha, 16	Emma, 16
	Eva, 16	



### Introduction



### What are functions?



How can we recognise a quadratic function?



How do we solve a quadratic function?



What are the steps to figure out inputs and outputs on the calculator?



### **Function Notation**





Key Words

This function takes an input value and multiplies it by 3.

## How to Graph a Quadratic Function

- Turn on calculator and press Mode Setup.
- Press 3.
- Then add in x<sup>2</sup> and press equals.
- Once that is done put in the first part of the sum -3.
- Then add in the end 3.
- We will be going up in steps of 1 so press equals.
- Then once that is all done you will have your table of results.
- Then graph your Function.

## Example



x	f(x)
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



.



### Group Work

# Worked Example





#### 2016

#### **Question 14**

The function h(x) below gives the approximate height of the water at Howth Harbour on a particular day, from 12 noon to 5 p.m.

$$h(x) = 10x^2 - 50x + 130,$$

where h(x) is the height of the water in centimetres, and x is the time in hours after 12 noon.

(a) Draw the graph of the function  $h(x) = 10x^2 - 50x + 130$ on the axes below, for  $0 \le x \le 5$ ,  $x \in \mathbb{R}$ .

#### (Suggested maximum time: 20 minutes)



Source: www.theirishlandscape.com. Altered.



(b) Use your graph in part (a) to answer the following questions.

(i) Find the height of the water at 12 noon.

#### (ii) Estimate the height of the water at its lowest point.

(iii) After 12 noon, how long did it take before the water was at its lowest point?

The graph on the right shows the approximate height of the water in centimetres at Crookhaven on a different day, from 12 noon to 6 p.m. The graph is symmetrical.

On this day, the height of the water at 12 noon was 180 cm, and the height of the water at the lowest point on the graph was 0 cm.

(c) Taking x as the time in hours after 12 noon, this graph is given by the function

$$g(x) = ax^2 + bx + c,$$

where  $a, b, c \in \mathbb{Z}$ , and  $x \in \mathbb{R}$ .

(i) Find the value of c.



#### (ii) Hence, or otherwise, find the value of a and the value of b.



Q14	Model Solution – 45 Marks	Marking Notes
(a)	<b>▲</b> <i>y</i>	Scale 15D (0, 4, 9, 13, 15)
	160	Accept correct graph without work.
	140	Award a linear graph at most Low Partial
	120	Credit.
		Low Partial Credit
	100	substitution for $x$ in $h(x)$ .
	80	Mid Partial Credit
	60	<ul> <li>h(x) evaluated correctly for any three values of x ∈ {0,1,2,3,4,5} (Accept points</li> </ul>
	40 -	shown on the graph)
	20	High Partial Credit
		• 6 points on the graph of $h(x)$ plotted
	0 1 2 3 4 5	<ul> <li>5 points on the graph of h(x) plotted and joined correctly</li> </ul>
	c lib	Full Credit –1
2.11		• Curve with a flat bottom, otherwise correct
(b)	(i) 130 cm	Scale 15C (0, 5, 12, 15)
	(ii) 67.5 cm	Accept correct answers without work.
	(iii) 2·5 hours	Accept answers taken from either the graph or the function
		In (ii), tolerance of $\pm 3$ units on <i>y</i> -axis, but not in next box up or down.
		Low Partial Credit
		1 part correct
		<ul> <li>Relevant line on graph (either a vertical line from the lowest point or a horizontal line from the lowest point)</li> </ul>
		<ul><li>High Partial Credit</li><li>2 parts correct</li></ul>
		   Full Credit –1
		Unit(s) incorrect or omitted, otherwise fully correct

Q14	Model Solution – 45 Marks	Marking Notes
(c)	Method 1	15D (0, 4, 9, 13, 15)
(i)&(ii)	Part (i)	Accept correct answers without work.
	(0, 180):	Low Partial Credit
	$a(0)^{2} + b(0) + c = 180$	• Work of merit,
	$\Rightarrow$ c = 180	e.g. identifies (0,180), (3,0), or (6,180); relevant substitution in g(x); relates c to y-intercept;
	Part (ii)	attempt at relevant shifting of graph;
	(3, 0):	Mid Partial Credit
	$a(3)^2 + b(3) + 180 = 0$	• Finds $c = 180$
	$\Rightarrow 9a+3b = -180$	• Finds E1 and E2 and E3
	$\Rightarrow$ 3 a + b = -60	• Finds $a = 20$ • $(x - 3)^2$
	(6, 100)	High Partial Cradit
	(6, 180): $\pi (C)^2 + h(C) + 180 = 1$	• Finds c and E2 and E3
	$a(6)^{-} + b(6) + 180 = .$	• $20(x-3)^2$
	$\Rightarrow 500+00=0$	• Finds <i>a</i> or <i>b</i> , having found <i>c</i>
	$\rightarrow$ $0 u + b = 0$	
	E3 – E2:	CLUX
	$\Rightarrow$ 3 a = 60	
	$\Rightarrow$ a = 20	
	E2: $b = -60 - 3$ (20)	
	$\Rightarrow b = -120$	
	OR	
	Method 2	
	Quadratic has 2 roots at $x = 3$	
	$\Rightarrow$ $g(x) = a (x-3)^2$	
	$= a (x^2 - 6x + 9)$	
	$= ax^2 - 6ax + 9a$	
	(0, 180):	
	$a(0)^2 - 6a(0) + 9a = 18$	30
	$\Rightarrow a = 20$	
	$\Rightarrow g(x) = 20x^2 - 120x + 1$	80
	i.e. $a = 20, b = -120, c = 1$	80
	OR	

Q14	Model Solution – 45 Marks	Marking Notes
(c)	Method 3	See previous page.
(i)&(ii) cntd	The shifted quadratic graph through (0,0) and (3,180) is of the form $y = ax^2$	
	$\Rightarrow$ $a(3)^2 = 180$	
	$\Rightarrow$ a = 20	
	Shift quadratic 3 units back to the right:	
	⇒ $g(x) = 20 (x-3)^2$ = 20 ( $x^2 - 6x + 9$ )	
	$= 20x^2 - 120x + 180$	
	i.e. <i>a</i> = 20, <i>b</i> = -120, <i>c</i> = 180	



# Our Lesson Plan



• We decided to start the lesson by recapping key words, so that the students know which words they need to know for the lesson

Evaluation:

It was good to tell the students what the key words are so they would be able to know what to expect.

We could have included more key words.

### Matching Game

- We used GeoGebra to graph the functions for the matching game
- We decided to use this to start our lesson to see what prior knowledge that the students had of the topic

Evaluation:

It was a fun way to get them interacting and working together. However, one of the graphs was wrong.

### **Graphing Functions**

- Graphing Functions was next in our class, we decided to add this as a follow on from the matching game
- We decide to make it into a race to add excitement to the class

Click to add text

Evaluation:

It made everyone get involve and it was a different method of graphing functions for them.

We could have used graph paper and make sure they graph the function correctly, by plotting the points.

### Kahoot

- We added in the Kahoot to test the student's knowledge on what we have taught them during the lesson
- We also thought that it would be a bit of fun!

Evaluation:

It was a fun way to assess what they had learned from the lesson. If we had laptops or phones we could have had a winner.

### Worked Example

- We added in the worked example to relate what we were teaching to real life examples
- It also allowed the students to recap what we had taught during the lesson

Evaluation:

It was a real-life example and their answers showed us what they had learned.

We could have swapped the Kahoot and the worked example around to finish the lesson on a fun activity.