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# Measuring distance

Today there are many ways to measure distances, some requires more effort than others, and in this report a few methods will be used to estimate the distances between cities.

## Abstract

The knowledge about planet Earth has grown, in the last 50 years. Nowadays we have the luxury to only do a quick Google search for finding out the distance between countries, but back in the days navigators and mathematicians used spherical trigonometry and maps. This project involves a comparison between a digital method, such as Google Earth and Wolfram alpha, and a mathematical method, spherical trigonometry. The first phase of the investigation is consisted of a background description of the famous mathematician Menelaus of Alexandria. The second phase is a comparison of distance between Söderköping and Bad Harzburg, using both a digital and mathematical method. The third phase is instead a comparison of distance between Söderköping and Peniche, using once again both digital and mathematical methods. The results varies depending on the chosen method, but the most reliable way to calculate the distance between different cities are the digital methods, as Google Earth and Wolfram Alpha.

# Introduction

Which procedure is the most efficient and reliable when measuring the distance between two cities? Our period of time provides us with websites that help us measure different distances, with the help of satellites. Before digital methods were possible and enough developed for the human kind to use, using map books and spherical trigonometry was the only useful way for calculating the distance between countries.The best way to measure a distance is debatable but in this report the different methods will be discussed and used to reach a final conclusion.

## Aim of the study

The purpose of this study is to approximate the distance between Söderköping and Bad Harzburg as well as Söderköping and Peniche with different kinds of methods, and also analyze the similarities between the results.

### Research questions

- *How long is the distance between the swedish city Söderköping and the german city Bad Harzburg, when using digital methods?*
- *How long is the distance between the swedish city Söderköping and the german city Bad Harzburg, by using an atlas?*
- *How long is the distance between the swedish city Söderköping and the german city Bad Harzburg, when using mathematical calculations?*

- *How long is the distance between the swedish town Söderköping and the portuguese city Peniche when using digital methods?*
- *How long is the distance between the swedish city Söderköping and the portuguese city Peniche, by using an atlas?*
- *How long is the distance between the swedish town Söderköping and the portuguese city Peniche when using mathematical calculations?*

## Background

### Menelaus of Alexandria

Menelaus of Alexandria was a greek mathematician and astronomer who first defined the formula for a spherical triangle. The greek mathematician lived in the island Alexandria but moved to Rome later in life. Historians believe that is was here Menelaus established the basis of the theorem. His important work is *Sphaerica*; a book collection consisted of three parts explaining the geometry of the sphere. In his first book, *Book I*, he created the basis for a mathematical treatment of spherical triangles. In *Book II* he described the astronomical interest of the theorem. The last book, *Book III* contains trigonometric ratios which consist of introducing Menelaus's theorem to the basis of trigonometry and astronomy (Sampaolo 2017, Hunt 2000).

**Spherical Trigonometry**

In normal euclidean trigonometry the sum of the angles is always 180°. Whilst spherical trigonometry differs. The angle sum can reach up to 540 degrees. To calculate a distance in euclidean trigonometry one can use pythagoras theorem. However, spherical trigonometry has another mathematical formula because all sides are bent. The spherical equivalent of the Pythagoras theorem is cos c = cos a cos b but this formula is only possible to use if there's a 90 degree angle in the triangle. Normally spherical trigonometry can be determined in the formula:

$$\cos\left(\frac{c}{R}\right) = \cos\left(\frac{a}{R}\right)\cos\left(\frac{b}{R}\right)$$

*Source: (Google n.d.)*

**The spherical law of cosines**

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

*Source:Chalmers (n.d).*

**Arc-length**

$$b = \frac{\alpha}{360} 2\pi r = vr$$

*Source: (Formelsamlingen n.d.)*

**Scales**

Map scales compares the size of a representative item on the map to the size of the object in real life. There are many different scales but what they all have in common is that the scales are adapted to the area that the map is showing. A scale is smaller when its denominator is larger, for instance 1:1000000 is a smaller scale than 1:00000 (Fuechsel, n.d.).

There are two different types of scales, linear scales and areal scales (Fuechsel, n.d.). It is important to underline the fact that the linear scale may vary within a map,especially for small maps**.** The variations depends on the sphericity of the surfaces it represents. On maps of larger scales such distortions are negligible**.**When maps decrease in scale, and distortions inherent to their projection of the spherical surface increase, the measurements of distances may be less accurate (Fuechsel, n.d.).

The most common world maps (see figure 1) are often made with help of a projection-method called Mercators. The difficult part is to transform the planets spherical surface to a flat two

dimensional map. At a globe, the latitudes goes from north to south. Mercators projection makes the latitudes parallel. That leads to a problem; the longer the distance from the equator the more the latitudes pulls from each other compare to how they really goes. That makes the countries far from the equator look much bigger than what they actually are (Melin 2015).

There are other projection-methods too. One method calls Gall-Peters projection (see figure 2). There are some divided opinions weather this method is better than Mercators but according to Vagabond, this method gives a better picture of how the countries sizes relate to each other (Vagabond n.d.).

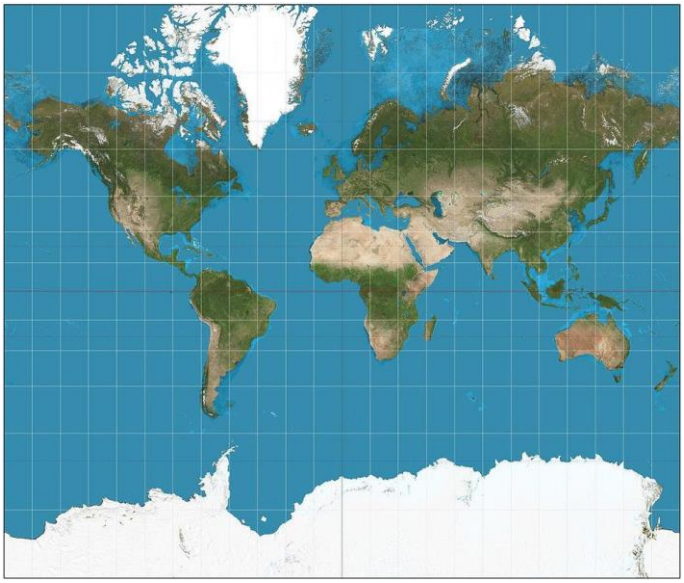


*Figure 1: The most common scale of a world map. Made with Mercators projektion Source: (Vagabond n.d.).*

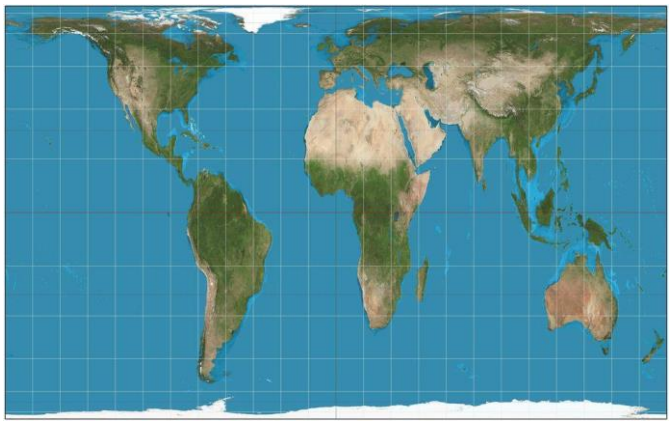


*Figure 2: A map after Gall Peters projection. This map shows the countries more proportional than the most common world map does. Source: (Vagabond n.d.).*

## Methods

To answer our research questions we have utilized digital sites such as Wolfram Alpha and Google Earth and also mathematical methods as spherical trigonometry and scale calculation. This study required the usage of wolfram alpha, google earth, atlases, calculators and a ruler to measure the distances.

# Results

**How long is the distance between the swedish city Söderköping and the german city Bad Harzburg, when using digital methods?**

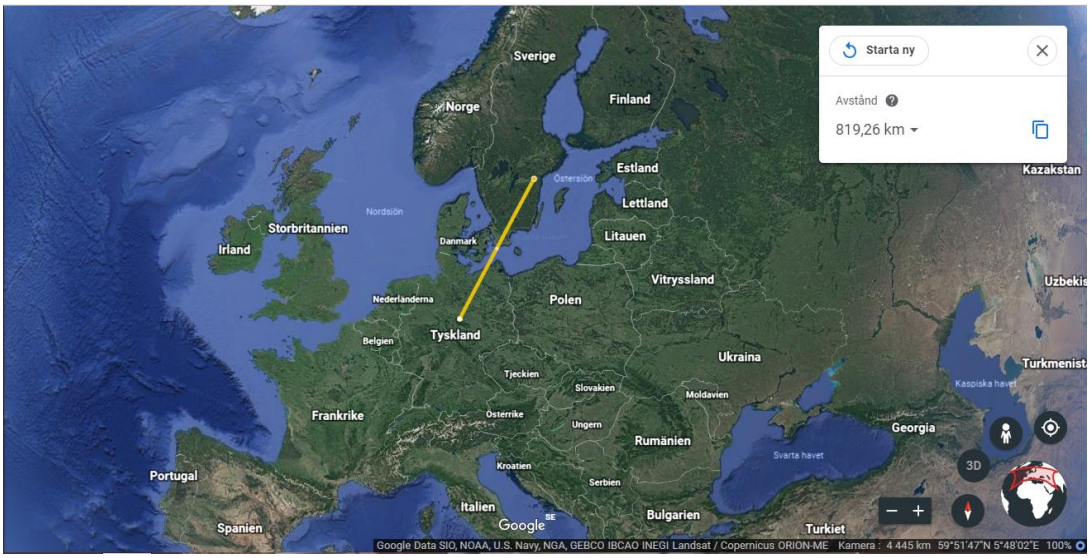



*Figure 3: Distance between Bad Harzburg and Söderköping (819.26 km)*
*Source: (Google Earth, 2019)*

*Figure 4: Distance between Bad Harzburg and Söderköping (813.6 km)*
*Source: (Wolfram alpha, 2019)*

Results from Google Maps shows that the distance between Bad Harzburg and Söderköping is 816.26 km. According to Wolfram alpha the distance is 813.6 km.

**How long is the distance between the swedish city Söderköping and the german city Bad Harzburg, when using an atlas?**

*Table 1: The distance between Söderköping och Bad Harzburg when atlases have been used.*

| Scale | Distance in cm | Distance in km |
|---|---|---|
| **1: 13 000 000** | **6.00** | **780** |
| **1: 15 000 000** | **5.00** | **750** |

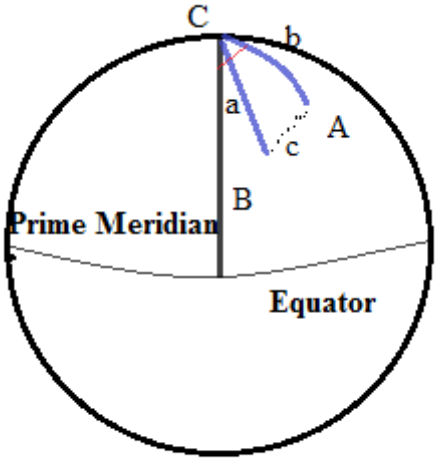
The table above shows the distance between the cities by using two different atlases with different scales. The results varies a lot in comparison to the digital results.

**How long is the distance between the swedish city Söderköping and the german city Bad Harzburg, when using mathematical calculations?**

For us to able to measure the distances using this method, we must know the latitudes and longitudes of the cities as well as the angles in our spherical triangle.

Söderköping:
Latitude: 58,5° N
Longitude: 16,3° E (Georg, n.d.).

Bad Harzburg:
Latitude: 51. 9°N
Longitude: 10. 6°E (Georg, n.d.).




A= 58.5°. Latitude of Söderköping

B= 51.9°. Latitude of Bad Harzburg
C = 16.3°- 10.6°= 5.7°. Longitude of Söderköping minus the longitude of Bad Harzburg
AC = b = 90°−58.5° = 31.5°
BC = a = 90°- 51.9°=38.1°

$$\cos c = \cos 38{,}1\ (38.1) \cdot \cos 31{,}5 + \sin 38{,}1 \cdot \sin 31{,}5 \cdot \cos 5{,}7 \Rightarrow \cos c = 0.99177$$

Which implies that $c = 7{,}3559°$

The Earth´s radius: 6371 km
The formula of the arc length is $2\pi r \cdot \frac{\alpha}{360°}$ and will in this case estimate the distance between our two cities.
The distance can now be calculated as $2\pi \cdot 6371 \cdot \frac{7{,}3559°}{360°} = 817{,}32$ km

**How long is the distance between the swedish town Söderköping and Peniche, the portuguese city, when using digital methods?**


*Figure 5: Distance between Peniche and Söderköping (2805.45 km)*
*Source: (Google Earth, 2019).*

*Figure 6: The distance between Peniche and Söderköping (2803 km)*
*Source: (Wolfram alpha, 2019)*

Results from Google Maps show that the distance between Peniche and Söderköping is 2805.45 km. 2803 km is the distance according to Wolfram alpha.

**How long is the distance between the swedish city Söderköping and the portuguese city Peniche, by using an atlas?**

*Table 2. The distance between Söderköping and Peniche when atlases have been used.*

| Scale | Distance in cm | Distance in km |
|---|---|---|
| **1: 13 000 000** | **20.9** | **2717** |
| **1: 15 000 000** | **18.3** | **2745** |

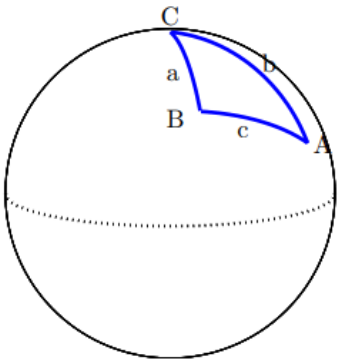
The results when using two atlases is close to our digital results.

**How long is the distance between the swedish town Söderköping and the portuguese city Peniche when using mathematical calculations?**

For us to able to measure the distances using this method, we must know the latitudes and longitudes of the cities as well as the angles in our spherical triangle.

Söderköping:
Latitude: 58,4759° N
Longitude: 16,3234° E (Georg, n.d.).

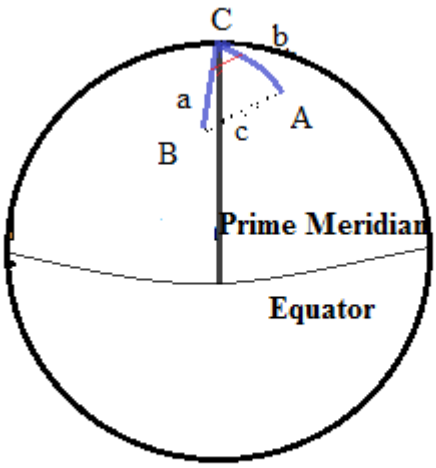
Peniche:
Latitude: 39,3558° N
Longitude: -9,38112° E (Georg, n.d.).




C-A= b
C-B=a
B-A= c

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

$$c = \frac{2r\pi V}{360°}$$




A= 58.5°. Latitude of Söderköping
B= 39.4°. Latitude of Peniche
C = 16.3°-(-9.4°) = 25.7°. Longitude of Söderköping minus longitude of Peniche
AC = b = 90°−58.5° = 31.5°
BC = a = 90°- 39.4° = 50.6°

$$\cos c = \cos 50{,}6 \cdot \cos 31{,}5 + \sin 50{,}6 \cdot \sin 31{,}5 \cdot \cos 25{,}7$$
$$c = 25{,}17°$$

The Earth´s radius: 6371 km
The formula of the arc length is $2\pi r \cdot \frac{\alpha}{360°}$ and will in this case estimate the distance between our two cities.
Distance is $2\pi \cdot 6371 \cdot \frac{25{,}17°}{360°} = 2797$ km

## Conclusions

Why did the digital methods show different results? Because when we used Google maps we measured the distance from our school Nyströmska to the specific city. Google maps gives you the opportunity to decide from which geographical point you want to measure the distance from, while Wolfram alpha measures the distance from the centre of the chosen cities. However the results differs with very few margins and this makes both of the digital sites useful and trustworthy. In google earth's information box we are informed that the dimensions may not be 100% accurate. The dimensions do not take into account height changes. However these only applies to Google Earth and not Google Earth Pro.

Why did the atlas results vary from our digital results? Is using an atlas a useful method when measuring distances? In this case an atlas works quite well. Both Peniche and Bad Harzburg are small cities that are not placed on the map. But the reason why the results on Bad Harzburg differed more than the results on Peniche is because it is harder to identify and locate where the german city is. Peniche is located around the coast of Portugal and much easier to detect. This makes it easier to place the ruler right on the map and get a better result. To get a more precise measurement one can use an atlas when the two distances are clearly shown.

When using the formula of spherical trigonometry we managed to get reasonable measurements between the cities. The difference between the distance from our digital method and mathematical methods were small. This makes the mathematical calculation a reliable and useful method.

Söderköping- Bad Harzburg= 813.6 km (Wolfram Alpha 2019).
Söderköping- Bad Harzburg= 817.32 (Spherical trigonometry 2019).

Söderköping- Peniche = 2803 km (Wolfram Alpha 2019).
Söderköping- Peniche = 2797 km (Spherical trigonometry 2019).

Which procedure is most efficient then? Well that would be using a digital method. When using a mathematical method it is easy to miss a step and calculate something wrong. Spherical trigonometry requires many steps and a basic knowledge of trigonometry. Not everyone is capable of understanding each step of the calculation which makes a simple misstep more likely to occur. This will affect the results. In conclusion it is much easier and time rewarding to use a digital method such as Wolfram Alpha or Google Earth.

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