Sequence 5

To measure Earth's size

Introduction

At last! thisfinal sequence will crown the efforts and perseverance that were yours all along this project. COngratulations. It is time to use the data you have carefully collected, either through your own survey, or through the other classrooms by Internet. We are now going to measure together the length of your meridian and finally find the size of our planet.

Note : This last step is the most tricky in the project, that's why we opted for a progressive action, in order to lead your pupils to the final calculation.

Link with the syllabus of the primary school (BO N° 1 of the 14/02/02) of cycle 3 :

- *Experimental and technological sciences* :
    - The subject : horizontal and vertical plane : interest for some technical devices.
    - The sky and the Earth :
        - light and shadows
        - the apparent movement of the Sun.
- Mathematics :
    - Space and geometry :
        - use of maps and plans.
        - use of tools (rule, set square, compasses) and techniques (foldings, tracing paper, cross-ruled paper).
        - geometric relations and properties : alignment, perpendicularity, parallelism.
    - Magnitude and measures :
        - keeping track of time and duration (year, month, week, day, hour, minute, second) and their relations.
        - angles : comparison, reproduction.
    - Exploitation of numerical data :
        - questions of ratio.
        - use of data in lists and tables.
    - View of the world : compare the global representations of the Earth (globes, planispheres) and the world (maps).

Link with data files :

Data file N° 17 : Light and shadows.

Data file N° 19 : Apparent movement of the Sun.

Excerpt from the text about the application of the new experimental and technological sciences programs :

Specific skills :

? *To know that locations are relative : a place is east (or north) of another, but west (or south) of a third.*

? *To be able to imagine the apparent course of the Sun in the sky and its changes during the year. To know that it is at its shortest on the winter solstice (the Sun is then low on the horizon) and at its longest on the summer solstice (the Sun is then high in the sky)..*
? *To be able to use a calendar to determine the characteristics of each season and the dates that mark the beginning of each. (maths)? use a plane or a map to locate an object, anticipate or make a move, evaluate a distance*

Comments :
The measures taken by groups of pupils will give an opportunity to compare the results and tackle with the question of accuracy of a measure. It is not necessary to introduce the idea of uncertainty and you should certainly not use the according formalism. We only need, in link with the mathematical field about decimals, to suggest a thought about the number of figures it should be reasonable to use to express an experimental result.

Summary of the sequence:

First : Eratosthenes' measure

Our Greek scientist was at the end of his considerations, and he tells us the results of his experiment. Give the following text to your pupils :

> "Having measured the angle between the sunrays and the vertical -the obelisk in his city of Alexandria-, Eratosthenes drew on the ground a section of the Earth following a meridian. He put on it the cities of Syrene and Alexandria and drew the sunrays getting to these two cities. He compared the angles of these rays with the vertical in each of these towns and extended the sunray from Syrene to the centre of the Earth, and had the sudden idea to measure the circumference of our

planet.

Soon, he understood that something was necessary to complete his project : the distance between Alexandria and Syrene. He knew that caravans going through the desert were used to measure the distances between cities. Men who were called "bematists" walked beside the camels and counted their steps. Knowing the average length of a step, they deduced the distance they had walked by multiplying this length by the number of steps to make their trip! There was approximately one million steps between Alexandria and Syrene... That meant approximately 5000 Egyptian stadiums (a unit they used at that time).

Eratosthenes rapidly discovered after a few simple calculations that the circumference of the Earth was exactly 250 000 stadiums. He told that to his colleagues scientists and geographers and the news spread all around the Greek world that a scientist named Eratosthenes had for the first time measured the size of our planet."

Now, it's your turn ! Try to reproduce the figure taht made Eratosthenes famous in the whole world and discover how he could measure the circumference of the Earth. The, use your own measures and those of a partner school to find the size of our planet by yourself.

1- How did Eratothenes measure the circumference of the Earth

a - The vertical at the scale of the Earth

The pupils, having read the text in the classroom, are going to ask various questions. The first thing to make them understand is the notion of vertical in the two cities of Syrene and Alexandria. If they have understood the activities about the notions of verticality and horizontality, they will certainly have a good idea of their local vertical. But what happens when it comes to the scale of our planet?

Ask them the following question :

"When the gnomons are adjusted (see the part about this activity if necessary and the notes they had taken in their notebooks at that time), how are they with regard to the horizontal support?" They will readily answer that they are vertical and as such perpendicular to the horizontal ground.
"What would happen then for gnomons all around the Earth?"

They will discuss about that and note in their books their hypothesis, to be checked by an experiment. For that, they can use a simple strip of stiff paper on which they will stick small shafts or pins, perpendicular to the sheet. The gnomons as they see them around them when they are adjusted. Since they are convinced that the Earth is not flat, they curve the strip and see that the gnomons are not parallel anymore, but that the directions to which they point are changing. If they close the strip upon itself, they will see the gnomons radiate from the surface of the Earth. They can draw on their notebook this strange figure of Earth covered with shafts just like a hedgehog.

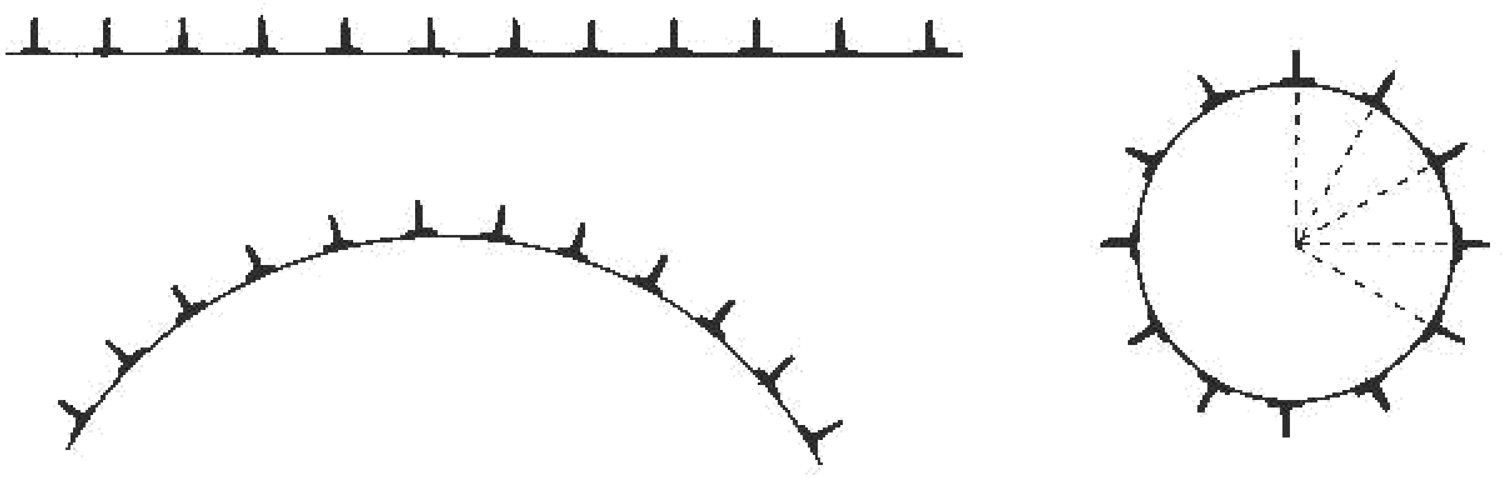


Figure 1

Seeing that all these gnomons are the vertical for each point of the Earth, ask them what happens when they extend by imagination all those shafts inside the Earth? They converge all to the centre of the Earth! It can be checked if you use the experiment, and replace the shafts by long needles or skewers.

They conclude that the vertical to each point of the Earth also points to the centre of our planet, and as such, the gnomons in two far away cities are not parallels but that their directions make an angle.

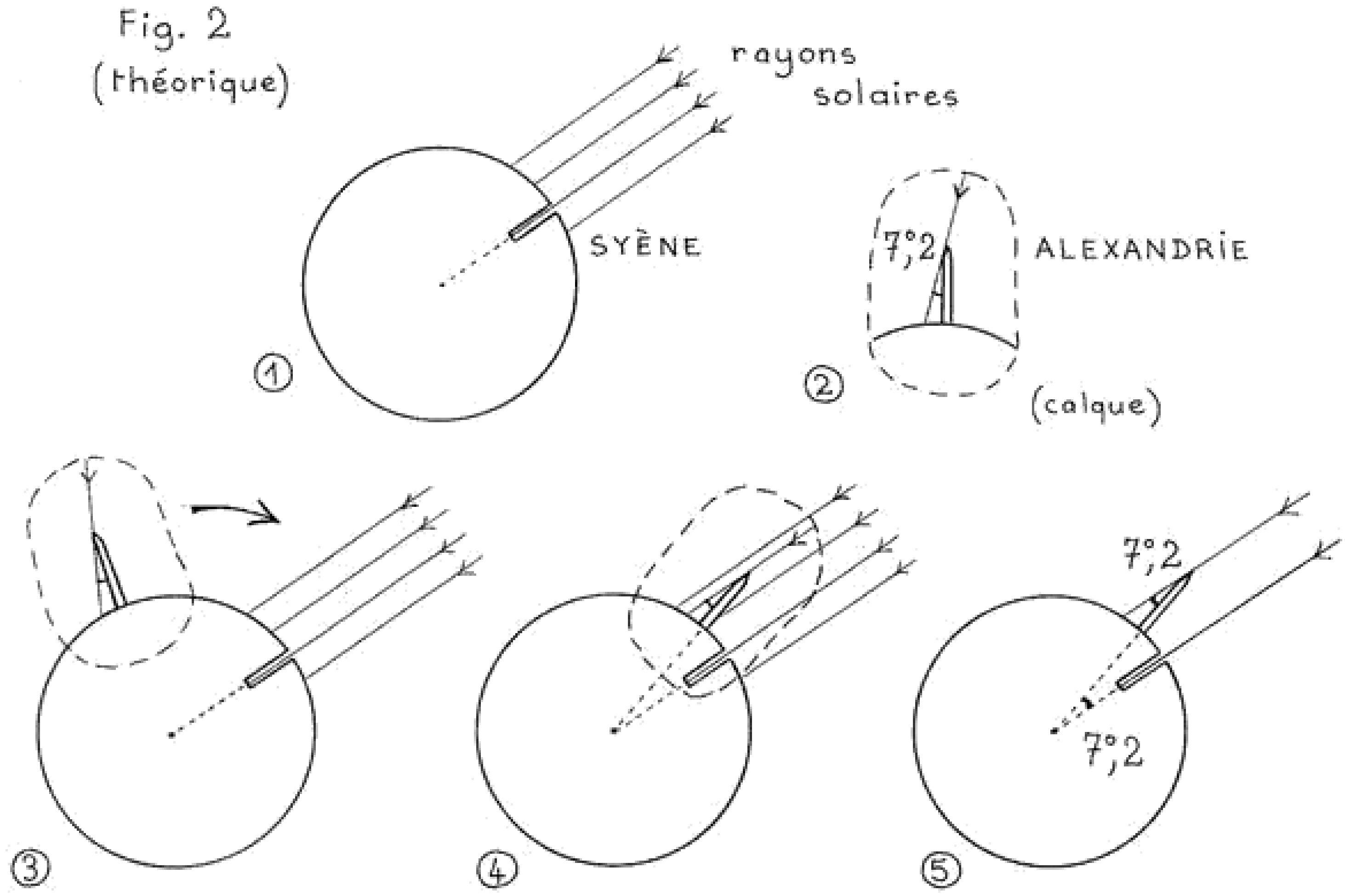
Then, draw on the blackboard a circle for the Earth and ask them how they could place on that figure the cities of Syrene and Alexandria, using what they know about Eratosthenes measures.

b - To discover Eratosthenes' secret

The question is difficult andf you will help them find the answer that will lead them to the famous figure made by Eratosthenes!

The small historical texts told us that in Syrene, on the 21st of June at midday, the sunrays got down to the bottom of the wells and that vertical objects had absolutely no shadow. As such, they were perfectly vertical! On a great sheet (A4 or A3), they will draw a circle for the Earth just as the first sketch of figure 2. They draw several sunrays (parallel, of course), and one following that vertical.
(You can begin to draw on a great sheet of paper the parallel sunrays and then cut in a coloured sheet a circle for the Earth, put it on the sheet and pierce it with something to fasten it on its center, and make it turn until the sunrays fall vertically on Syrene).

How do you place Alexandria now? Ask them what Eratosthenes measure this same day, at the same time? "The angle between the sunrays and the obelisk... so the angles these rays make with the vertical !" They will need to recover the value of this angle (7.2 degrees), and draw a schema, just as explained in fig. 2. The pupils will use tracing paper, on which they will draw the angle for Alexandria and they will make it slide on their figures until the ray on the obelisk becomes parallel to the others.



Eratosthenes secret :
When they have written down with a black pen the position of Alexandria, they draw the vertical going through this city that goes to the centre of the Earth. Ask them what is the angle between this vertical and the one of Syrene? "It looks strangely equal to the angular sector, the one measured by Eratosthenes".

To check that, they return the tracing paper and superimpose the angle to the one at the centre of the Earth. It works! This would be Eratosthenes' secret ! Make them check for another angle (twice that value, for example), and they get the same result. They create a new Alexandria at an angle of 14 degrees, draw the vertical and measure the new angle at the centre of the Earth. They can also use the protractor to check these hypothesis.

Then, they reproduce on their notebooks the figure without its irrelevant lines, proud to have discovered that by themselves. They will also note the conclusion : the famous secret discovered by Eratosthenes : the angle measured between the sunrays and the vertical in Alexandria is exactly the angle between Alexandria and Syrene at the centre of the Earth. They will then see the "Z of Zorro" that will surely help them remember this incredible result!

One more question : what would happen if the Earth was turned so as to make sunrays no more vertical in Syrene? (turn the circle of Earth slightly counter-clockwise).

They try the experiment, draw the angles, compare them, and see that the angles are not equal anymore !!! A new angle has appeared in Syrene, changing everything.

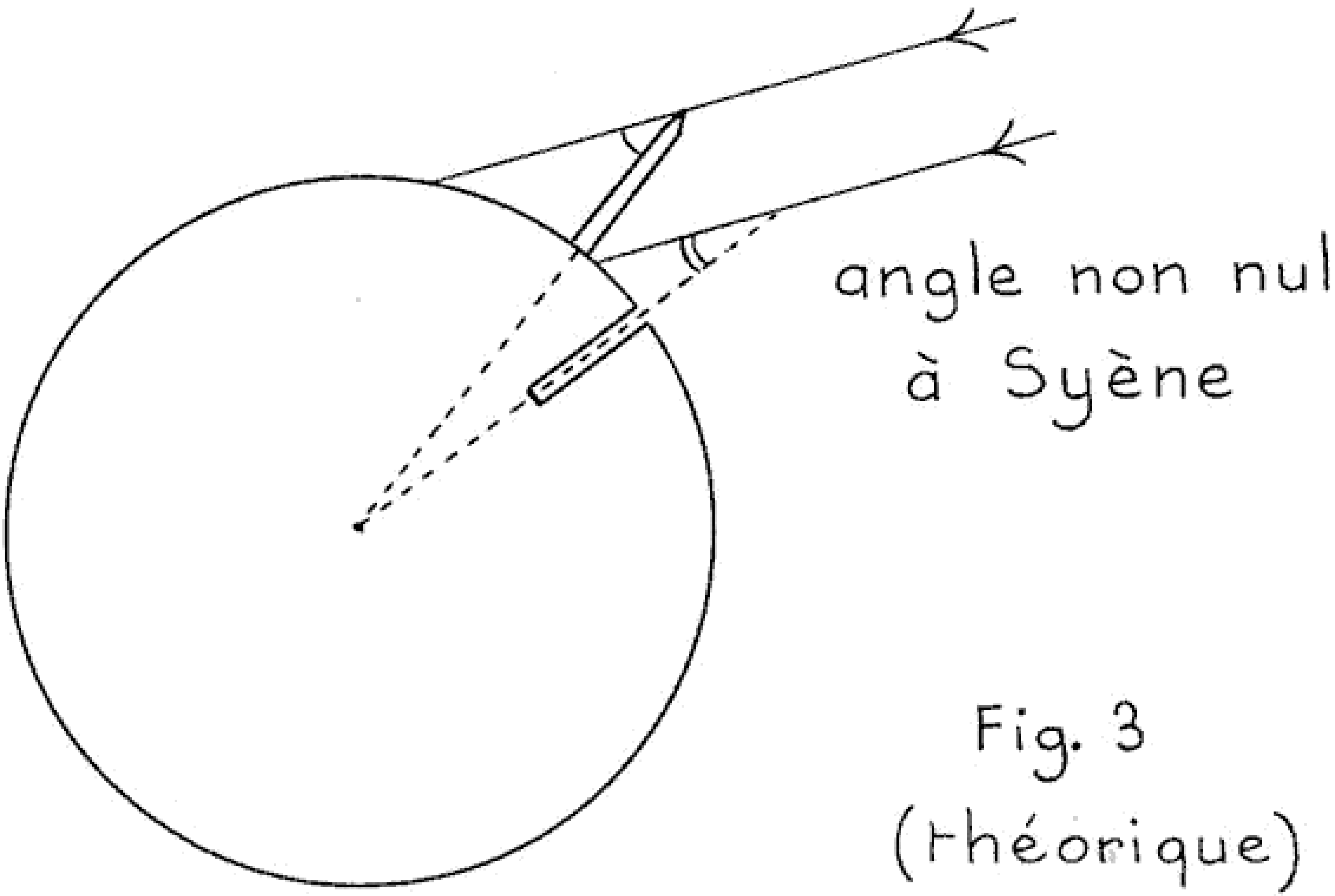


Fig. 3
(théorique)

Tracing the new angles appeared in Syrene and Alexandria between the rays and the vertical, maybe they will discover after a few tries that the angle between the two cities at the centre of the Earth (see the first model) is equal to the difference of the angles measured in the two cities between the sunrays and the vertical (they can also measure them with a protractor and look for the relationships between the three angles : in Syrene, in Alexandria and in the centre of the Earth between the two cities). It can also be easily seen with the tracing papers.

They just have extended the conclusion to the cases (the most current!) when sunrays do not fall vertically. They will write down this discovery on their notebook, because it will be useful to reproduce Aratosthenes' figure with their own measures. (Note that this conclusion also applies ont the 21st of June between Syrene and Alexandria, but that one of the angles is null... The difference is then equal to the angle measured in Alexandria.)

Now, your pupils are ready to measure the circumference of the Earth in any case!

c- To measure the length of the meridian passing through Syrene and Alexandria

To make them understand the rule of three (or rules of proportions), a rule they will need absolutely to measure the meridian, tell them to think about the following scenario :

Imagine that Eratosthenes measured a different angle in Alexandria. Imagine that Syrene and Alexandria are in fact on an Earth similar to a pie cut for example in 8 equal pieces, the two cities being just as on figure 4. If you know the length of the rim of a piece of the pie, how could you find the length of the circumference of that pie?

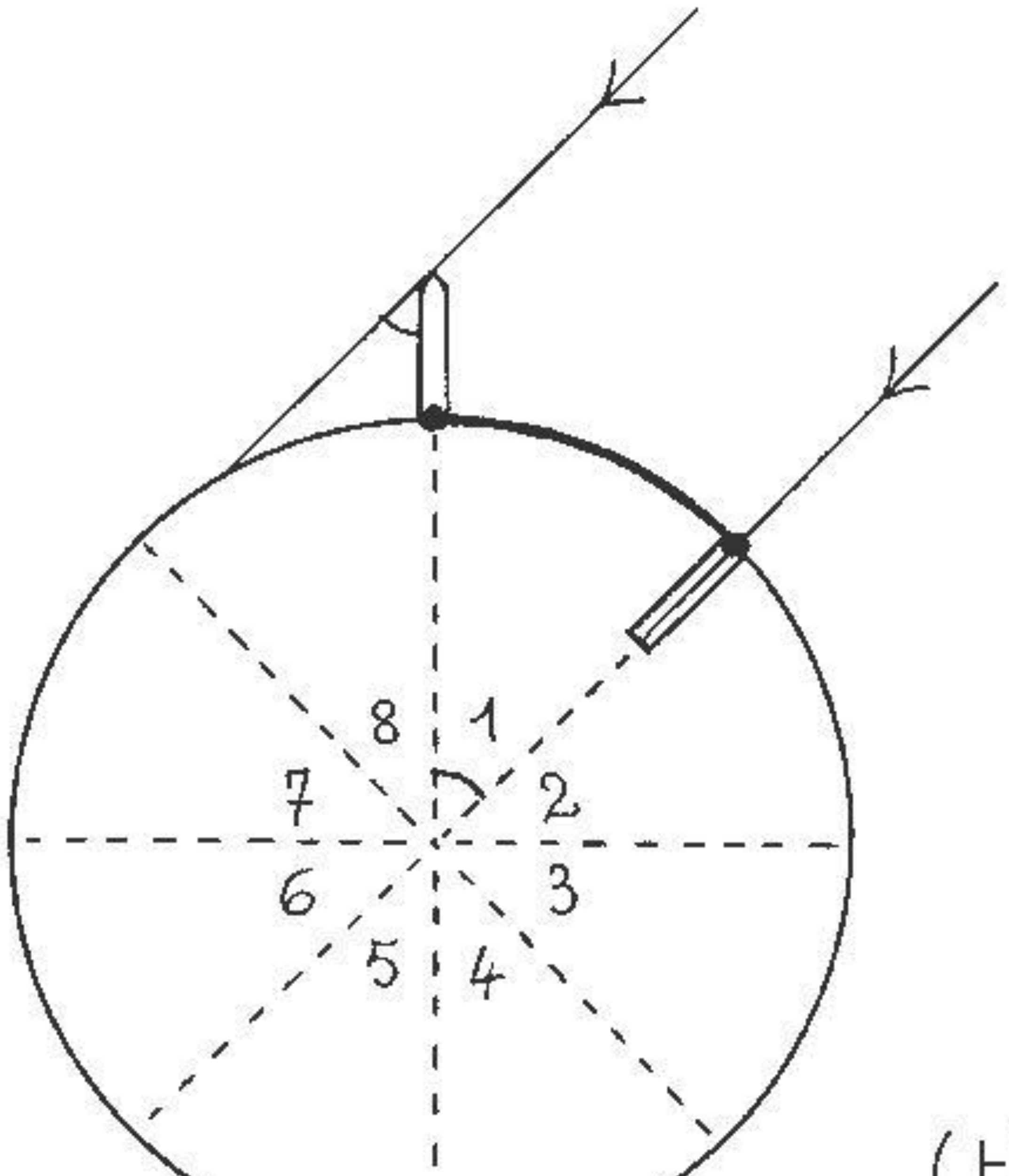


Fig. 4
(théorique)

"It's easy, you only need to multiply the length by 8!" Are you sure? You can tell them to check : make a great circle, divide it in 8 equal sections and measure with two threads the length of the rim of a piece and the length of the entire circumferen. They will find a difference of 8 to 1 between the lengths of the two threads.

That's exactly what Eratosthenes told himself, but how many pieces are there in the "pie"?

They can suggest several experiments to try to discover it. You can divide the pupils in groups, in order to try all the proposals :
- they can use the angle of 7.2 degrees and make it turn around the centre of the Earth to see how many "pieces" they need to fill the Earth (or half the Earth, and then multiply the result by two).
- They can use a thread and compare the length of the rim for the "piece" Syrene-Alexandria (on the real figure made before) and compare it to the length of the circumference of the Earth.
- Those who prefer could divide 360 degrees (a whole circle) by 7.2 degrees (the angle at the centre).

They will find a factor of 50 precisely (through calculation, at least !).
They only need to use the rules of proportionality since Eratosthenes tells us that the distance between Syrene and Alexandria is of 5000 Egyptian stadiums. Multiply by 50 and find : 250 000 exactly !!! Just as the great Greek scinetist. The puzzle is now solved.

But what was the Egyptian stadium distance value in the metric system? Final quest that will take them to encyclopedias or internet search engines. They will find the following answer :
1 Egyptian stadium = 157.5 meters, which gives a circumfere of 39 375 km for the Earth : Compare it to the values found in your dictionaries and you will be astonished by the preciseness of this measure.

2- To use the surveys of another school to measure one's own meridian

Foreword : we suggest you, if you have time enough for that, to have a glance at the optional activities that could introduce notions of parallels and meridians on the surface of the globe, especially by comparing the measures of your angles with the ones published by the other schools. If you don't have time for that, you can decide to explain these notions in the classroom, with a globe and a planisphere (optional session : To get one's bearings on the surface of the earth.*)*

a- To Choose your partner

Let's remind you in a few words the requirements for the success of this last step :

You must have measured the shadow of your gnomon at the time of solar midday (a 10 minutes delay can be accepted, but not more!) and deduced from that measure the specific value of the angle of sunrays with the vertical of your place.

You must have published your angle measure (s) in the table given on the site La main à la pâte (http://www.mapmonde.org/eratos/measures.php?lang=en), in order for the other classrooms to make use of your indispensable surveys

You must have chosen with the pupils a partner-school who made their measure at the same date on solar midday (a two-days difference can be possible, if you are close to June). This partner must have at least 3 or 4 degrees of difference in latitude with you. Note where the shadows point, because some schools see the Sun at its highest to the North on midday and so the shadow points to the South, unlike most classrooms on English territory.

b- Make your own " Eratosthenes' figure "

Now, you are going to make Eratosthenes' figure with your own city and that of the partner school chosen according to the data published in the table. The pupils make the schema of Figure 5 for a specific day, according to the same principle.

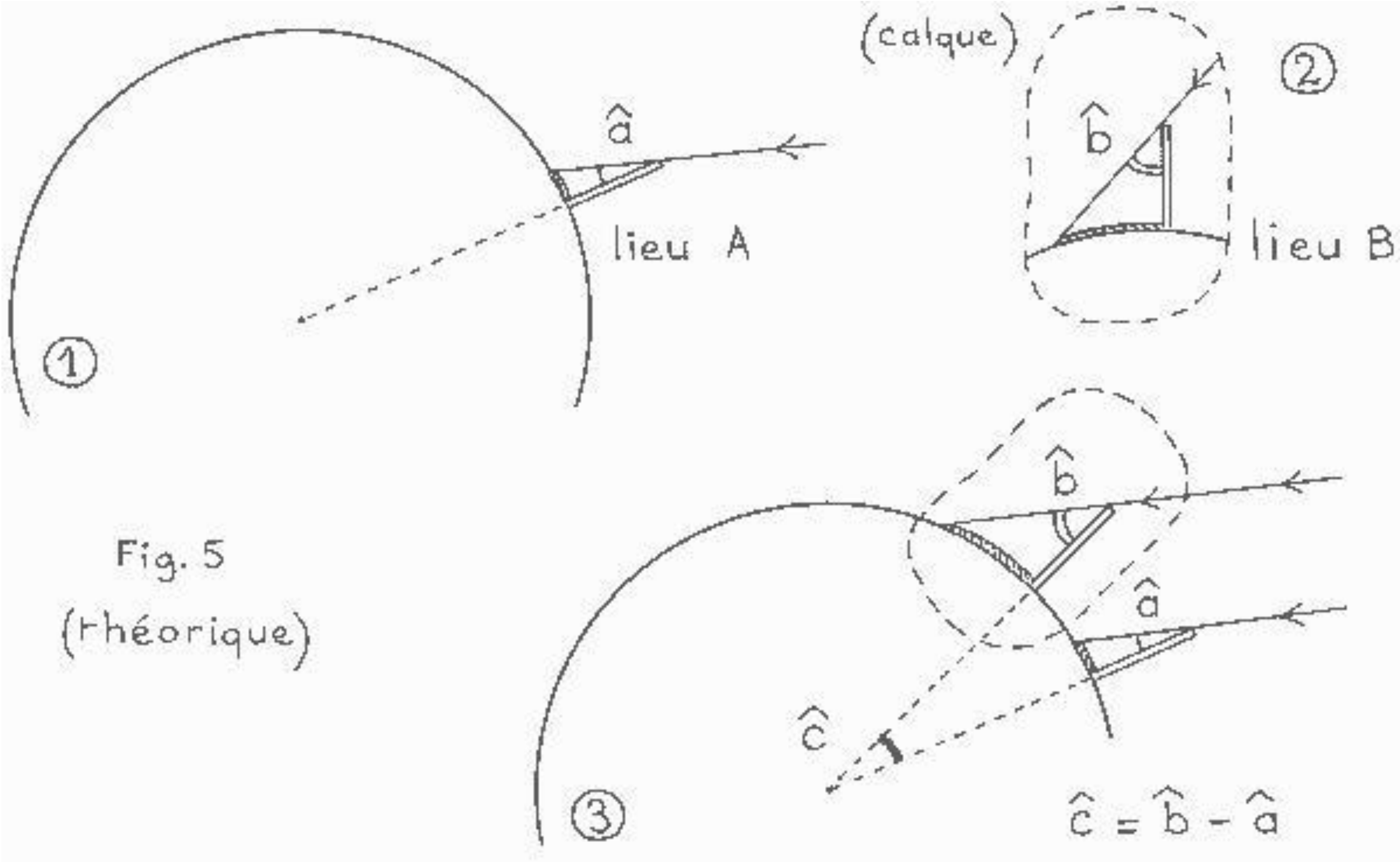
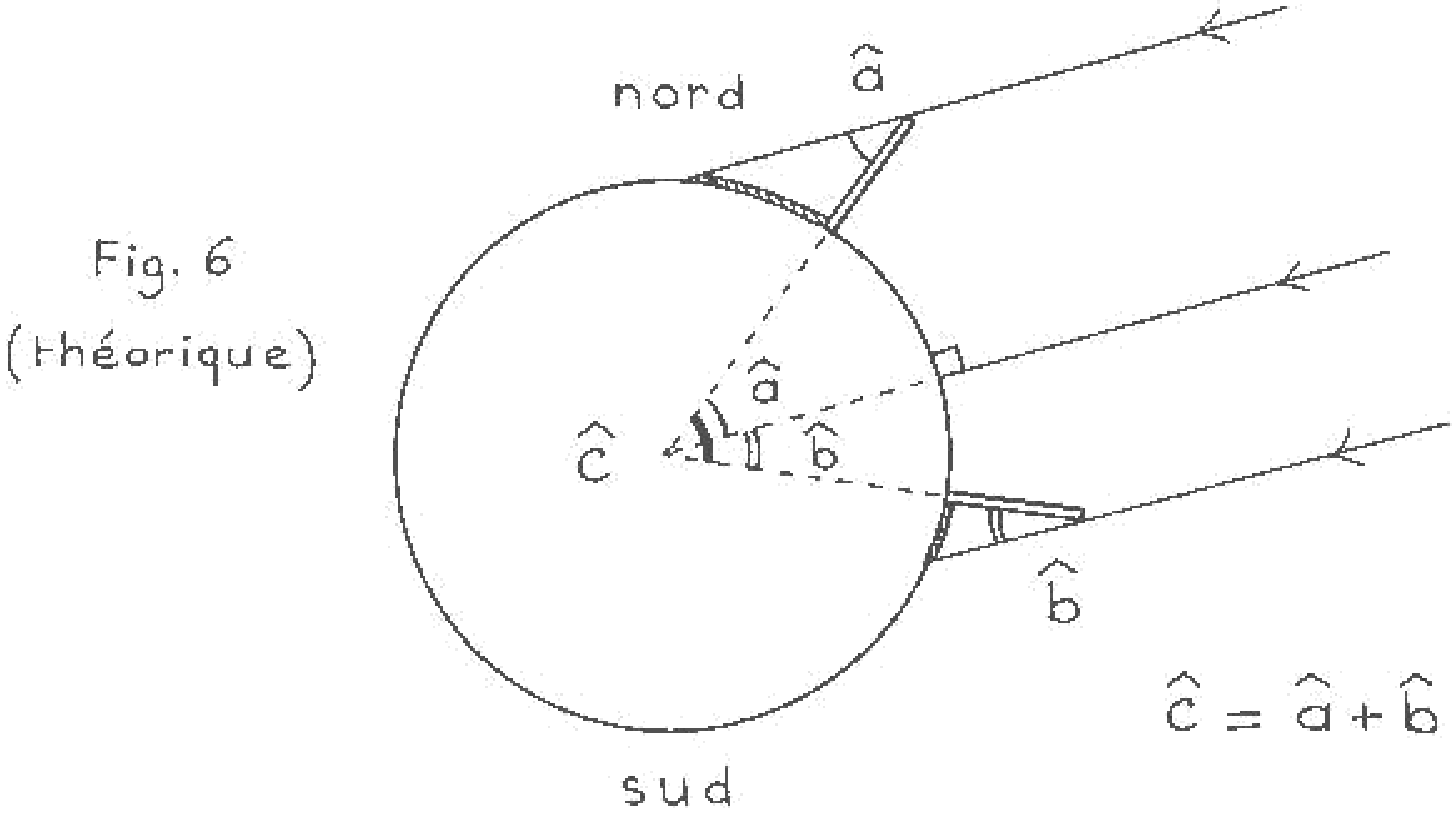


Fig. 5
(théorique)

Tell them to make the tracing paper slide upon the section of the Earth. It is important for it to be made of a really great circle (or half-circle), because the two angles will be different by only a few degrees if both partners are in England.

Be careful! some schools in the intertropics belt or in the southern hemisphere will see the shadow point to the South, and the Sun will be at its highest to the North. If so, they will absolutely have to tell in the table in which direction the shadow was pointing, because it changes the calculation : use the last explanations given on Eratosthenes' figure and put a city in the southern hemisphere on the same meridian than Alexandria. The shadow being reversed, ask the pupils to draw the angle between the sunrays and the vertical for this city (or measure it) and the one between this city and Alexandria from the centre of the Earth. The idea is to find the relationship between these angles : you should, that time, add the values of the angles to discover the angle between the two cities at the centre of the Earth.



To be sure, you can draw the sunray that falls between these two cities and goes through the centre of the Earth : it has the role of Assouan, since the rays follow the vertical. It splits the angle at the centre of the Earth in two between the two cities. If you apply the principle of equality of angles discovered with the couple Alexandria and Assouan on these two "sub-angles", you find that the first angle (superior) is equal to the one measured in the northern city and the second to the angle measured in the southern city. You should add these two "sub-angles" to find the whole angle.

In a few words : if the shadows of the partner schools point to the same direction, (both to the north or to the south), you must substract the angles measured between the sunrays and the verticals to get the angle at the centre of the earth. If the shadows point to opposite directions, you must add them.

c- To measure the distance between the partner cities

Just as in the case of Eratosthenes we talked about before, you only need to apply the rule of proportions to find the length of the circle of the Earth, with the distance between the two cities. You must know teh distance between the two cities. Eratosthenes had opted for two cities on the same meridian (more or less), but in your case, your partner will probably not be on your meridian (on the same longitude). the pupils will have to admit this time that they should measure the distance between the parallels of the partner cities and not the direct distance between the two cities.

(To be convinced, ask yourself what imaginary city on your meridian will measure the same angle as your partner : it ois the city placed on the same parallel. The difference? It will see the Sun at its own midday -at your hour-, and you have the right to place it on your drawing since we remind you that it is a section of the Earth along a meridian : your meridian.

Get a road map or an atlas to decide, from their scale, the distance between the two parallels. The maps of the atlas and some IGN maps show some parallels and meridians, they will be very useful. Be careful because of the projection : on the planispheres, they distort the continents and do not respect the scales for each zone of the globe !

d - To measure the length of your meridian

You have now all the elements to make your calculation, you only need to find the multiplying factor that will make you pass from the distance between the parallels to the total circumference of the meridian. You can, if you wish, use the methods devised in part 1 to discover Eratosthenes' results or make a simple rule of three (or proportion rule) if your children have understood the principle of that method.

e - To find the diameter of the Earth.

Nothing easier : you have measured the circumference of our planet, you only need to divide by the famous Pi to find the diameter of our Earth, and you are done !

In his time, Eratosthenes had found 250 000 stadiums exactly for the circumference. This round figure shows that he did not want to be precise, but to get an idea of the Earth's size. And it was really successful, since it gives a result of a little more than 39 000 km and therefore a diameter of approximately 12 500 km.

The children will be able to compare their results to those of Eratosthenes and seek in dictionaries or on Internet the values recently found by the scientists (you will discover that Earth is not perfectly roun like a ball, but that it is flattened on the poles and that its polar diameter is slightly less than its diameter at the Equator. They will also be able to communicate with theiur partner school through e-mail to discuss with their comrades of their results and their clever calculations.

You can try it again as many times you have synchronous measures with other schools. If no measure has been published for the same date of your own, you could use measures made on the day before or after (between May and June) because the angle you have measured doesn't change much from one day to the other.

If here are great differences exist between your result and reality, try to find with your pupils the possible origin of the error :
Inaccurate angle measure, measure of the distance between the partners inaccurate, etc. Publish your results on our site and discuss with your partners and with ourselves via Internet.

f- To publish your measuresin the great results table

The measures table made for your school in the workspace Eratosthenes (http://www. mapmonde. org/eratos/measures.php?lang=en)now has a column for the measure of the meridian (in km). You only need to write down the result of your calculation in that column to make it appear in the measures table and show it to the other classrooms of the project. You can get to the workspace's user's guide (http://www.mapmonde.org/eratos/help.php?lang=en) to get more details on the methods of publication.

g- To send a postcard to all the members via our site

Like Emmanuel di Folco during his trip in Egypt, the classrooms can send one or more electronic postcards in memory of the project. Once printed, these cards can be cut and stuck in order to get a real postcard. This activity gives your pupils an opportunity to leave a trail of their journey in the footsteps of Eratosthenes and tell the other classrooms the great steps of their discoveries.

User's guide :

The postcard is made of four elements you will have to prepare before connecting to the site :

A text (a maximum of 10 lines and 100 words)

An image (jpg or gif, a maximum of 450 pixels wide and 400 pixels high)

A date : day/ month/ year

A legend (a maximum of 7 words)

Once these documents are ready, get to the workspace Eratosthenes.

Three options are open to you :

Add a postcard

Read or change the postcards

User's guide

The first option gives an access to the form made for you to publish your postcard. Copy the text, the date and the legend in the good windows and select the image on your computer. You only need to click "send" to save your work and make it visible to everyone! The name of your classroom will appear automatically in the signature of the postcard.

To see the result of your work, use the second option. The postcards your classrooms have made appear with a link "delete" and "modify". These links can only be read with someone with your password and your logoin. Be careful : you can change only the text of the postcard. If you wish to change the image, you have to delete the postcard and add a new one.

Advice :

On screen, the postcard is divided in two parts, one for the text, the other for the image and the legend. Tell your pupils to illustrate their postcard with a picture of the classroom and the gnomon, or pictures of the classroom in specific activities that pleased them.

Precautions :

Any publication on the Internet is under the laws of publications. Its author, editor and distributor are responsible for it. You must have a written authorization of the parents if you want to publish photographs where their children could be recognized. We therefore ask you to take the necessary precautions with the parents of children on your photographs or avoid photographs where the children can be easy to identify.

3- Last survey on the 21st of June

The 21st of June is a very special day since it is the one where Eratosthenes made his first measure. Furthermore, we know that in that summer solstice day in the northern hemisphere, the Sun gets exactly to the zenith in Assouan at solar midday, Assouan being on the Cancer Tropic. Like Eratosthenes, we encourage you to put an end to this project with a measure on the 21st of June (if the Sun wants!). Numerous fairs happen on that day, it is an opportunity to make a stall "Direct measure of the size of the Earth"!

You have probably seen that these days, during your surveys, the angle changes less and less from one day to the other as we get closer to the solstice. In the week around it, this variation will be negligible, so any measure made between the 17nth of June and the 25th of June will be valid! So as not to be very disappointed if the day of the solstice the Sun does not show, make a survey as soon as you get to the 18nth (and up to the 25th if necessary).

THis will give the opportunity for all the classrooms to take some part to this historical measure !

You will then be sure to get a first-rate partner since the angle of the sunrays with the vertical to Assouan is null on that day. Our Egyptian partners in Alexandria and Assouan promised us to join the party and make a simultaneous measure. It is the opportunity to make this historical survey with your pupils. You will only need to measure the distance between your city and the parallel on which Assouan is placed, so your distance to the Cancer Tropic. Furthermore, your angular difference with the tropic (the angle you measure this day) is high enough for you to have a precise measure that will easily give you a good measure of the diameter of the Earth. Try this great experiment on the 21st of June !