



Scenarios for Teaching Mathematics

Scenario Fact Families (Grade 5)

Context: One of the more efficient ways for students to learn their addition and subtraction facts is to learn them through working with Fact Families. Using addition and subtraction fact families helps students understand that subtraction undoes addition and vice versa. Learning the fact families for multiplication and division also supports students in thinking about the relationships between multiplication and division. It also provides a more efficient way to remember multiplication facts. Once you know these facts you can apply the inverse to solve simple division problems. In this example the teacher of a fifth grade class suddenly gets confused about how to present fact families when working with the fact family of 2 and 1.

Scenario: Mrs. is teaching a regular fifth grade class and does an introductory lesson using fact families. She asked the students to give her a suggestion for a fact family they would like to show. J..... suggests the Fact Family for the number 2.

Mrs. writes on the board the following:

$$2 \times 1$$

$$1 \times 2$$

$$2 \div 1$$

Then Mrs. suddenly freezes and is unable to come up with the missing expression. (She may have been thinking that 1 divided by 2 is $\frac{1}{2}$ and she didn't want to show a fraction. The missing fact is $2 \div 2 = 1$).

The teacher then tells the students that this was a hard one, so let's do a different number.

Mrs. writes:



$$22 \times 10 = 220$$

$$10 \times 22 = 220$$

$$220 \div 10 = 22$$

$$220 \div 22 = 10$$

and completes the Fact Family for 220 instead.

Based on this scenario on fact families, what may the students have understood or not understood about the lesson?

Based on this scenario on fact families,

what may the students have understood or not understood about the lesson?

What is the relationship of this lesson to the Common Core Standards?

Domain:

Cluster Heading:

Cluster Standard:

What Standards for Mathematical Practices are evident in this scenario?

Scenario Fact Families (Grade 5)



Analyses

Context: One of the CCSS Grade 5 Critical Areas is developing fluency with whole number and decimal operations. Students develop understanding of why division procedures work; analyze fluency with multi-digit addition, subtraction, multiplication and division; and apply their understanding to decimals and fractions. In this example the teacher of a fifth grade class suddenly gets confused about how to present fact families when working with the fact family of 2 and 1.

Although this scenario is based on a Grade 5 classroom, the concept being taught is found in the Grade 3 standards. Many students learn their addition and subtraction facts through working with Fact Families. Using addition and subtraction for fact families helps students understand that subtraction undoes addition and vice versa.

Learning the fact families for multiplication and division also supports students in thinking about the relationships between multiplication and division. It also provides a more efficient way to remember multiplication facts and apply the inverse to solve simple division problems.

Scenario: Mrs. is teaching a regular fifth grade class and does an introductory lesson using fact families. She asked the students to give her a suggestion for a fact family they would like to show. Jasper suggests the Fact Family for the number 2. Mrs. Winters writes on the board the following:

$$2 \times 1$$

$$1 \times 2$$

$$2 \div 1$$

Then Mrs. Winters suddenly freezes and is unable to come up with the missing expression. (She may have been thinking that 1 divided by 2 is $\frac{1}{2}$ and she didn't want to show a fraction. The missing fact is $2 \div 2 = 1$). The teacher then tells the



students that this was a hard one, so let's do a different number. Mrs. Winters writes:

$$22 \times 10 = 220$$

$$10 \times 22 = 220$$

$$220 \div 10 = 22$$

$$220 \div 22 = 10$$

and completes the Fact Family for 220 instead.

What the students may be understanding, or not understanding: Students are used to having three numbers in their fact family. In the first example of 2, only two numbers are involved which is unusual. Most fact families have three numbers and students and the teacher became confused when there were only two numbers involved. Suggested Alternative Strategies: Writing a Complete Equation: In the first example, Mrs. only wrote numerical expressions for the fact families. Writing out the full equation, including products, quotients, and equal signs, would have allowed the teacher and students to see the missing fact. Doubles: When working with any doubles there will only be two equations in the fact family, for example,

$$4 \times 4 = 16 \text{ and}$$

$$16 \div 4 = 4.$$

Division by Zero: Zero is the mathematical representation of the concept of "nothing". This might cause some misconceptions in division (or sharing). If you share "nothing" among any number of people, each person gets "nothing" or zero. This is represented as $0 \div \text{each person involved} = 0$. A fact family with zero might look like this:

$$4 \times 0 = 0$$

$$0 \times 4 = 0$$

$$0 \div 4 = 0 \text{ (dividing nothing among four people, the four people get nothing)}$$



$4 \div 0 = \text{undefined}$ (four apples divided by nothing, the four apples are still there)

To further see this concept, we can think about multiplication undoing division:

If $3 \times 4 = 12$, then $12 \div 4 = 3$.

If $0 \times 4 = 0$, then $0 \div 4 = 0$.

If $4 \times 3 = 12$, then $12 \div 3 = 4$.

If $4 \times 0 = 0$, then $0 \div 0$ is NOT equal to 4, and is undefined.

Relation of Scenario to the Common Core Standards:

Domain:

Operations and Algebraic Thinking (Third Grade) Cluster Heading:

Multiply and Divide with in 100.

Cluster Standard: 7.

Fluently multiply and divide within 100 using strategies such as the relationship between multiplication and division (example: knowing that $8 \times 5 = 40$, $40 \div 5 = 8$) or properties of operations.

By the end of third grade, know from memory all products of two one-digit numbers.

Standards for Mathematical Practices connected to this Grade 5 case scenario:

1. Make sense of problems and persevere in solving them. Students in Grade 5 should solve problems by applying their understanding of operations with whole numbers, decimals, and fractions, including mixed numbers. They ask themselves whether they can solve problems in different ways.

2. Reason abstractly and quantitatively. Grade 5 students should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals.



3. Construct viable arguments and critique the reasoning of them. Students in Grade 5 may construct arguments using objects, pictures, and drawings. They explain calculations based models, properties of operations, and rules that generate patterns. Students communicate with each other by asking questions like: “how did you get that?” and “why is that true?”.

4. Model with mathematics. Students in Grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, and making charts, lists, graphs, or creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

5. Use appropriate tools strategically. Grade 5 students who are mathematically proficient consider the available tools (including estimation) when solving a problem and decide when certain tools might be helpful. They may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions, graph paper to accurately create graphs and solve problems, or make predictions from real world data.

6. Attend to precision. Grade 5 students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and when expressing their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids.

7. Look for and make use of structure. Students in Grade 5 closely look to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply, and divide with whole numbers, fractions, and decimals.

8. Look for and express regularity in repeated reasoning. Students in Grade 5 use repeated reasoning to understand algorithms and make generalizations about patterns. They connect place value and their prior work with operations to understand algorithms and to fluently multiply multi-digit numbers. Students in this



grade level explore operations with fractions with visual models and begin to formulate generalizations.