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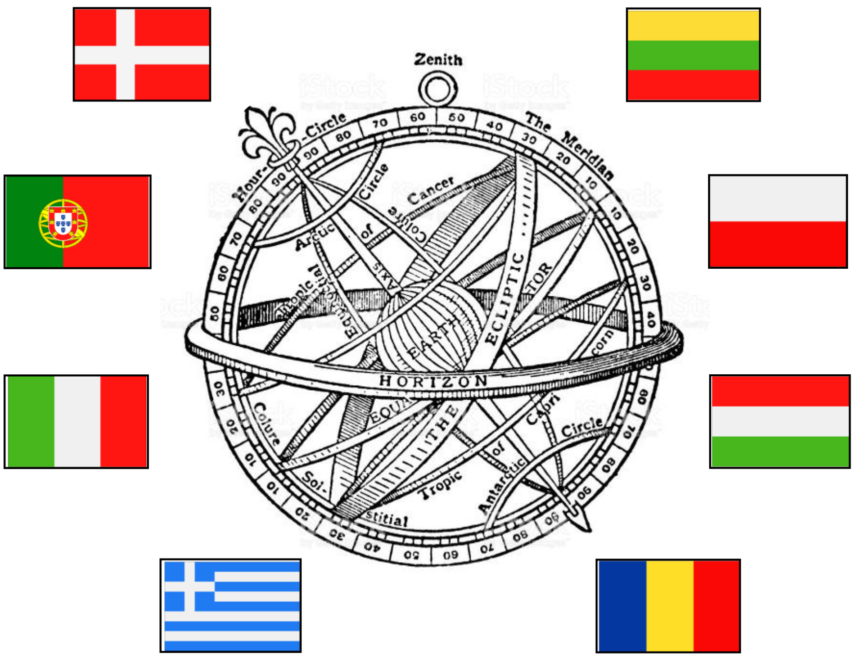
Math Around Us

Erasmus+KA2 Strategic Partnership for School Education
2015–2017



Math in Geography

Short-term exchanges of groups of Pupils &
Short-term joint Staff events
Almada, 8-12 May 2017

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Nelson Chibeles Martins, Mathematics teacher in Faculty of Sciences and Technologies (FCT), New University of Lisbon (UNL).

AGRUPAMENTO de ESCOLAS ANSELMO de ANDRADE

Agrupamento de Escolas Anselmo de Andrade is located in ***Almada***, in the crossing of the Tagus River (Rio Tejo), having the Atlantic Ocean facing Lisbon. Formed in 2008 and being itself an individualized pole and at the same time the pivotal point of contact with other spaces, our cluster integrates students from different study levels going from ranges as kindergarten to secondary education. Although they have specific plans, in which the different activities with the different classes and students from various ages are integrated, the three public schools that belong to our cluster share the same Educational Project "***Host, Guide and Integrate to Build the Future***".

Our population covers a wide range of different social ranks, with distinct academic expectations. We have a great diversity of students: socioeconomic diversity, geographical origin, children from immigrants, having Portuguese as a second language, as well as students with special educational needs supported by a ***Unit Support of Multidisability***, a ***Unit of Special Education*** and a ***Psychology and Guidance Service***.

The main and largest school, ***Escola Básica e Secundária Anselmo de Andrade,*** formed in 1971, offers a diversity of courses, basic and secondary grade, aiming to fit our offers to the students'needs and their families.

We offer the normal secondary courses (***Sciences and Technologies, Economics or Humanities and Arts***) and professional courses like ***ICT*** and ***Theater Performance***. Our students also learn ***English***, ***French*** or ***Spanish***, as a second foreign language, or ***Mandarin***, optional at secondary grade. In September 2016 we had around 960 students in this school, from the 5th to 12th grade, 400 of them in secondary classes.

We value the development of specific projects such as the methodology of consolidating apprenticeships, in constant interaction with the evolving environment, mainly with projects dealing with different areas such as environmental, artistic, civics, sports, education for health and school interchange (European schools), according to our Educational Project.

Having this in mind, we have established partnerships with academic and scientific institutions at a superior level (both national and international), with institutions of local power, enterprises from our community, with the parents' association, non-profitable social and cultural organizations, besides our training center which supports our structured professional training.

We are an ***Eco School***, a project from ***ABAE*** (Blue Flag Association of Europe a Non-Governmental Organization for the Environment), a project being part of the priority axis ***"Low Carbon Economy"*** promoting good environmental practices through the collection of recyclable waste; in the same line of action we planted a ***Biological and Mediterranean Garden*** including aromatic plants. Our students also embraced the challenge of ***"Taking Portugal to the World"***, a project from ***Emepc*** **(**Mission Structure for the Extension of the Continental Shelf of Portugal) a platform of knowledge and connection to the ocean.

Every year our Art students organize an exhibition of their work at the Culture Center in ***Almada***, ***"Ver de Fazer"*** and our school is one of the secondary schools that participate in the ***"Higher***, ***Secondary and Professional Education Show"***, an event organized by the Department of Education of the Municipality of Almada, to promote the educational offer of all the schools in the municipaliy among the community, and help to clarify the students about the options of choice available to their pursuit of studies.

ALMADA CITY

Almada, one of the main Arab military squares south of the Tagus, was conquered by the Christian forces of ***D. Afonso Henriques***, our first king, with the help of English crusaders in 1147. In 1190, ***D. Sancho I***, the second king of ***Portugal***, granted ***Almada*** the first Charter (*Foral*). ***Almada*** obtained the category of city in 1973.

Consolidating its role as protagonist in the ***Lisbon Metropolitan Area***, ***Almada*** has unique indicators that contribute to the quality of life of those who live or intend to live here. In 2015 the municipality had 169,914 inhabitants, the main municipality, in population terms, of the South Bank of the Lisbon Metropolitan Area; 62.9% of the population is between 25 and 64 years, (working-age).

Almada City

Located just a few minutes away from ***Lisbon***, ***Almada*** is distinguished as a municipality with a wide network of parks and gardens, cultural and sports facilities, modern and well-equipped schools and a university campus of excellence.

Almada assumes itself as an educating and knowledge city, focusing on teaching as a strategic pillar of local development: belongs to the ***International Association of Educating Cities*** since 1997, one of the 477 cities of 37 countries that signed the ***"Educating Cities Declaration of Barcelona, 1990"***.

The municipality has an extensive network from pre-school to higher education institutions, including vocational and senior education and night schools.

In 2015 there were 129 establishments in the municipality of Almada, about 47% of which (68) belong to the public network, for a total of ***39,396 students***: 61 schools from kindergarten to secondary grade (29,329 students) and 7 colleges (10,057 students).

Geographical Science

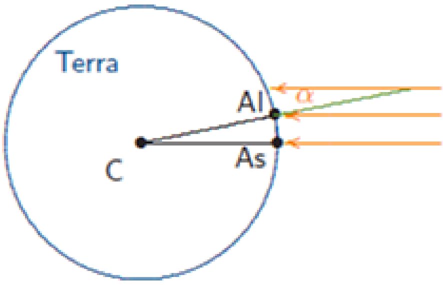
Geography, a science and a subject of teaching. As a science of society and nature, it is a branch of knowledge necessary for the training of teachers. It is present in the daily life of every citizen, either by the eagerness to know the world, or by the challenges posed by the environment for the demands of territorial planning, tourism, or simply as school tasks of basic education.

Geography, is the science of "WHERE", the science of locations, spatial structures and spatial processes, of the distributions of phenomena in a territorial space. The "***space"***, geographic reality by excellence, is where the geographical variable measured by distances is subscribed in order to measure distances and record the results; geographers use a wide number of techniques, such as ***direct proportionality***, which enables the ***spatial representation and analysis***.

The map is one of the most important tools of a geographer's work and designing maps implies the use of concepts of ***distance*** between places (how many kilometers go from one place to another) and direction (where the place is located).

The earliest records of geographical knowledge can be found in the ancient Greeks. Specialists in philosophy like ***Pythagoras*** and ***Aristotle*** had a belief that the Earth was round. ***Eratosthenes of Cyrene*** (284-194 BC) was the first to use the word ***Geography***. This word appears for the first time in a work of public domain entitled ***Geography***; ***Eratosthenes*** has calculated the circumference of the Earth with gigantic approximation: 250,000 stadiums (1 stadium = ±168 meters) or ±42,000 Km.

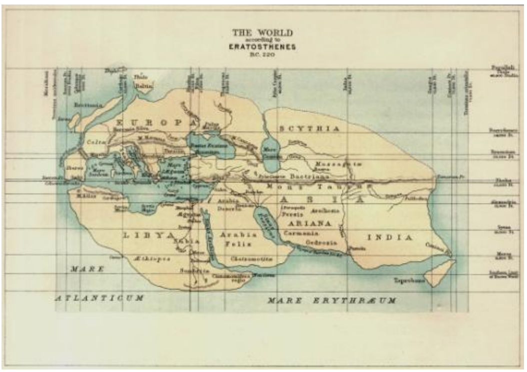
The real perimeter of the Earth is 40,072 km (Pict.1).



Terra - Earth
Al – Alexandria
As –Aswan (Syene)
C - Earth's center

Picture 1

From here ***Eratosthenes*** constructed a grid with several meridians and parallels, one of the first maps in the history of cartography (Pict.2).



Picture 2

The word ***cartography*** was introduced by the Portuguese historian Manuel Francisco Carvalhosa, in a letter dated from December 8, 1839, from Paris, and addressed to the Brazilian historian Francisco Adolfo de Varnhagen, becoming internationally established by its usage.

During the 15th century, at the beginning of the ***Great Age of Discoveries***, cartographers were more than ever needed to record the new continents and oceans to be discovered.

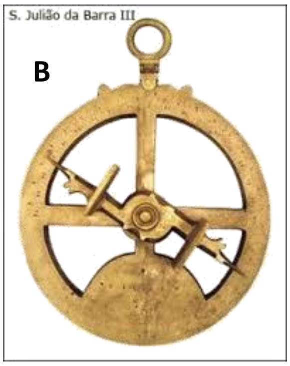
The Portuguese discoveries were the set of achievements accomplished by the voyages and maritime explorations between 1415 and 1543 that began with the conquest of ***Ceuta***, in Africa.

The discoveries made an essential contribution to outlining the ***world map*** and were responsible for important advances in ***nautical technology*** and ***science, cartography*** and ***astronomy***❈, developing the first ships capable of safely navigating to the open sea in the ***Atlantic Ocean*** (Pict.3).

Picture 3 – Cantino’s planisphere, the oldest Portuguese nautical chart known, 1502.

❈ The ***Quadrant*** and the ***Astrolabe,*** adapted to nautical instrument, were very probably the first instruments of astronomical navigation that the ***Portuguese pilots*** used; ***Diogo Gomes,*** navigator of the ***Infante D. Henrique***, referring to a voyage he carried out, around 1462, states to have "observed the height of the arctic pole with the quadrant” (Pict.4).

A



B

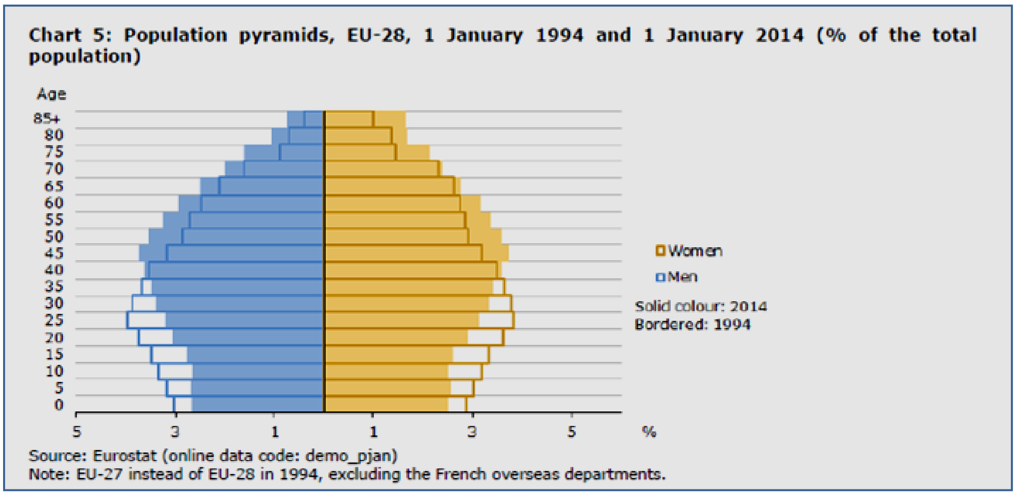
Picture 4- The Quadrant (A) and Astrolabe (B)

In the second half of the 20th century, with Anglo-Saxon neopositivism came up ***Quantitative Geography***, an empirical science whose basis lays on the ***logical-deductive method***, definition and use of ***spatial models*** resulting from human activity and lastly the need for ***statistical treatment*** of large amounts of data in the search for ***probabilistic*** and ***non-causal*** relations links this ***New Geography to mathematical language***.The notion of ***relative space*** where relative distances impose on absolute distances, giving rise to a new cartography, the one of ***distance time*** and ***distance cost***.

Math in Geography

The Earth Geometry

Cartography



Statistics applied to Geography: demography rates

Geography is dependent on other subjects to obtain basic information, especially from certain specialized branches; data from other subjects like chemistry***, mathematics***, physics, astronomy, anthropology and biology are used and these data are related to the studies of populations and their environment.

High areas of research for graduates in Geography by Portuguese Colleges:

a) Environment and natural resources - in this case, it is important to mention potential participation in environmental impact studies, make use of ***mathematical tools such as Input-Output Analysis and Integral Calculus***. See, for example, the calculation and handling of Functions like Marginal Social Damage, Private Marginal Damage;

b) Spatial Planning and Management - here we should mention that currently the perspective focuses on the so-called "New Territory Management" that calls, among other issues, to a solid domain of denominated ***Financial Mathematics***. See, for example, the Net Present Value and the Internal Rate of Return;

c) Geographic Information Systems and Remote Sensing – here the essential issue is the ability to ***elaborate and interpret algorithms***, in order to allow a better and more complete valuation of the knowledge of the graduates in Geography;

d) Development - in this context, issues related to cluster analysis, economic geography, complex development indicators are worth highlighting. It is fundamental the domain of tools associated ***to Differential Calculus - Derivatives and Partial Derivatives, Differential Equations and Equations to Differences, Integral Calculus and Function Studies.***

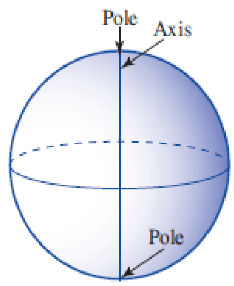
Math in Geography: Practical Exercises

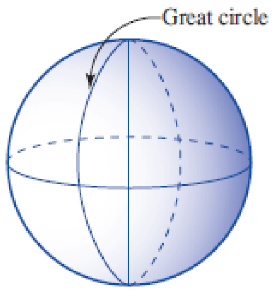
1 - The Earth Geometry

The Earth has the form of a sphere so, spherical geometry, basic trigonometry, acute angles, perimeter, circumference, arc lengths, great circles and small circles, distances on the Earth's surface, ***latitude and longitude***, are mathematic subjects we need to work with.

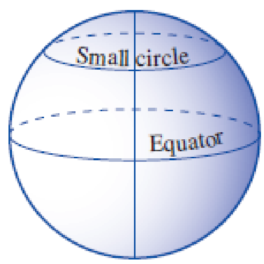
1.1 Great circles and small circles

Consider the sphere drawn on the right. The axis of the sphere is a diameter of that sphere. The ends of the axis are called the ***poles***. If we draw any lines around the sphere passing through both poles, a ***great circle*** is formed. A great circle is the largest possible circle that can be drawn around the sphere.

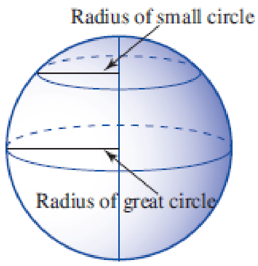




Now consider a circle drawn perpendicular to the axis of the sphere. Only one circle, called the ***Equator***, will be a great circle. The centre of the equator will be the centre of the sphere as shown on the right. Other circles that are perpendicular to the axis of the sphere will be smaller than a great circle and are called ***small circles***.



To calculate the ***length around a circle***, either great or small, we need to know the ***radius***. The small circle will have a radius smaller than that of the great circle, as shown in the picture on the right.



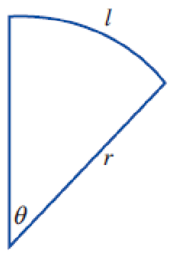
The length of a circle is found using the formulas for the circumference of a circle:

$C = \pi d$, where d is the diameter of the sphere

$C = 2\pi r$, where r is the radius of the sphere

1.2 Arc lengths

An arc is a portion of the circumference of a circle. The length of an arc can be calculated as the fraction of the circle determined by the angle subtended by the arc at the centre, as shown in the picture on the right.



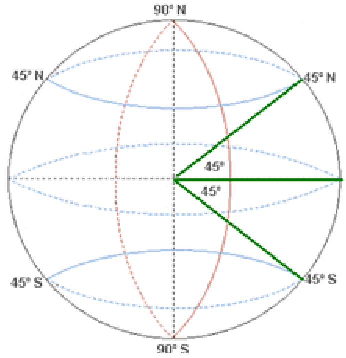
The arc length l can be calculated using the formula:

$\mathrm{l} = \frac{\emptyset}{360} 2\pi r$, where $\emptyset =$ number of degrees in the central angle

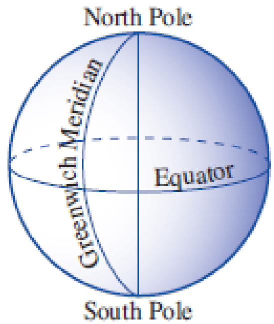
1.3 Latitude and longitude

As the Earth is a sphere, great circles and small circles on the surface of the Earth are used to locate points on the surface. Consider the axis of the Earth to be the diameter joining the ***North Pole*** and the ***South Pole***. The only great circle that is perpendicular to this axis is the ***Equator***. The angular distance either north or south of the equator is the ***latitude.*** So the ***Equator*** is located at ***0°latitude***.

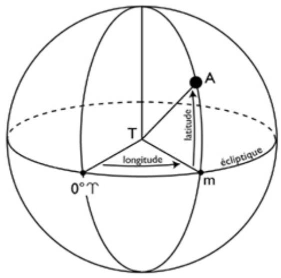
Small circles parallel to the equator are called ***parallels***. These small circles are used to describe how far north or south of the equator a place is. The maximum latitude for any point on the Earth is ***90° N*** (North Pole) and ***90° S*** (South Pole).



To locate a place on the globe in an east–west direction, the line of reference is the ***Greenwich Meridian***. The Greenwich Meridian is half a great circle running from the North to the South Pole. All other places on the globe are located by the half great circle on which they lie. These half great circles are called ***semimeridians***.



Each semimeridian is identified by the angle between it and the Greenwich Meridian, the ***prime meridian*** is the line of ***0°longitude*** for measuring distance both east and west of Greenwich. ***The International Date Line*** is the ***semimeridian opposite*** to the ***Greenwich Meridian***. The International Date Line has ***180° of longitude***, either east or west.



Any location, on a map or globe, have a pair of coordinates: the first is the parallel of latitude that it lies on, the second one is the meridian of longitude, for example ***Lisbon coordinates, 38° 42´N and 9° 8´W*** (a degree is divided into 60 minutes).

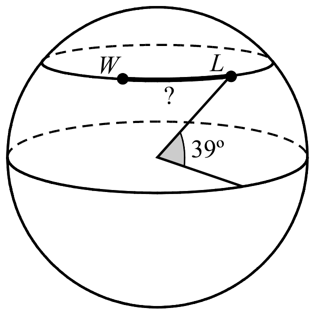
Exercise 1:

Consider:

- $\pi = 3.1416$
- The earth is a sphere with a radius of 6,371 km.

The geographical coordinates of ***Lisbon***, the capital of Portugal, are approximately, 39° latitude N, 9°longitude W. The capital of the United States, ***Washington***, D.C. has the same latitude as Lisbon but its longitude is approximately, 77° W.

What is the distance, in kilometers, from Lisbon to Washington, D.C. measured along the parallel which goes through the two capitals? (Pict. 5)



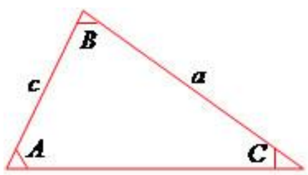
Picture 5

NOTE: apply to basic trigonometry to calculate the parallel radius that crosses Lisbon and Washington D.C. and after use the arc length formula.

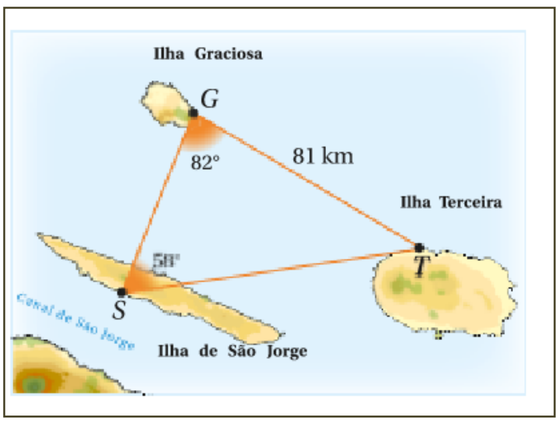
Exercise 2:

In picture 6 it is represented part of the map of the ***Azores Archipelago***. A helicopter flew from ***Terceira (T)*** to ***Graciosa Island (G)*** in a straight line travelling the distance of 81 km. Then it travelled to a fixed location in ***S. Jorge Island (S)*** and came back to the same point in Terceira Island. Calculate the travelled distance between Graciosa and S. Jorge Island. Calculate the outcome in kilometers rounded to the nearest unit.

NOTE: apply to the Law of Sines



$$\frac{a}{senA} = \frac{b}{senB} = \frac{c}{senC}$$

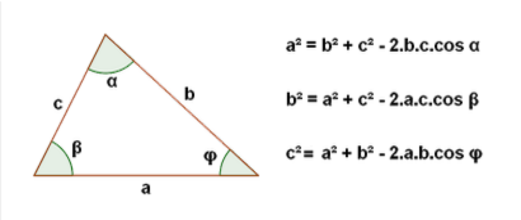


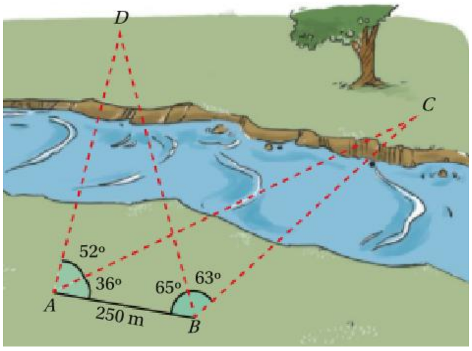
Picture 6

Exercise 3:

Using two points, ***A and B***, situated on one of the river banks, with the distance of 250 meters from each other, measurements were made to calculate the distance between point ***C and*** point ***D*** on the other bank. From the measurements given in picture 7, calculate the distance between point C and point D, with the outcome in meters, rounded to the nearest unit.

NOTE: apply to the Law of Sines and the Law of Cosines





Picture 7

Exercise 4:

A scout walks 10 km in the direction 30°NE. His movement towards East is:

(A) 5 km

(B) less than 5 km

(C) more than 8 km

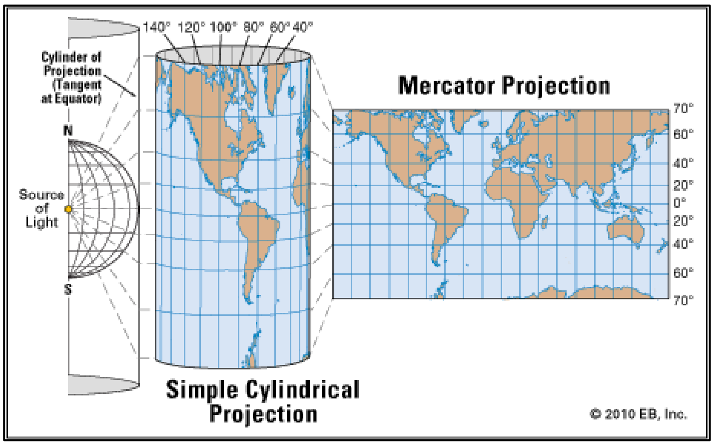
(D) between 6 and 7 km

2 – Cartography

Cartography is the art and science of map making, practiced by cartographers. Modern practitioners of cartography have the advantage of computers and other equipment to assist them, making their maps more precise. ***A map of the world reflects an immense mathematical and aesthetic challenge***, that of translating the globe to a two dimensional surface.

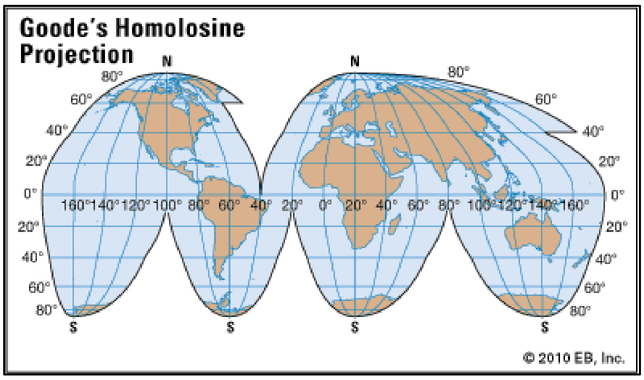
Many cartographers have struggled with this issue over the centuries, striving to project the features of the globe accurately and effectively.

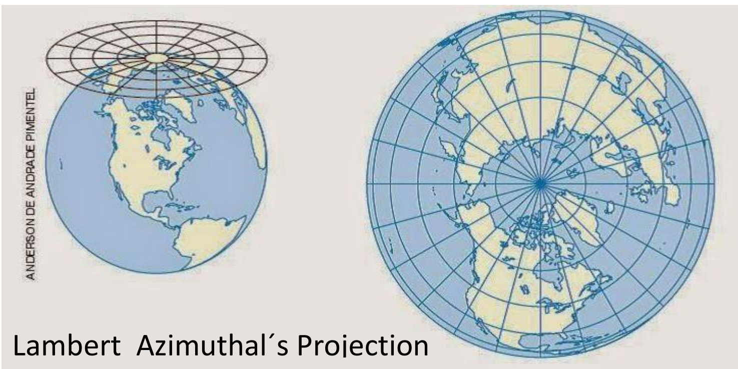
Numerous approaches have been taken to solve this problem, including the ***Mercator Projection*** (Pict.8), a map which distorts geological features north and south of the Equator in order to fit the globe into a neat rectangle.



Picture 8

Other maps portray the globe in sections, reducing the amount of distortion necessary (Pict.9)



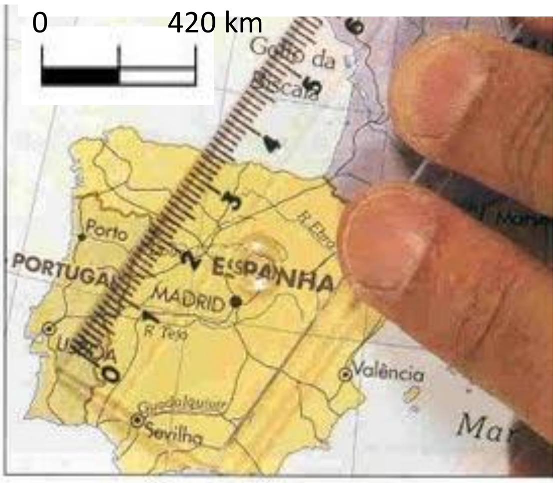


Picture 9 – Goode and Lambert's Projection

A map represents a portion of the Earth's surface. Since an accurate map represents the land, each map has a ***"scale"*** that indicates the relationship between a certain distance on the map and the distance on the ground. The map scale is usually located in the legend box of a map, which explains the symbols and provides other important information about the map.

A ratio or representative fraction indicates how many units on the earth's surface is equal to one unit on the map. It can be expressed as 1/21,000,000 or 1:21,000,000. In this example, one centimeter on the map equals 21,000,000 centimeters (210 kilometers) on the earth´s surface, this is a ***numerical scale*** (Pict.10A); a ***graphic scale*** it is a simply line (ruler bar) marked with the correspondence between the distance on the map and the distance on the earth´s surface (Pict.10B).

Picture 10A – Map of the Iberian Peninsula at a numerical scale: 1:21,000,000;



Picture 10B – The same map of the Iberian Peninsula at a graphic scale:

1 __ 420Km

The maps also allow locating one place relative to another, on the earth's surface, using the ***Compass Rose*** (Pict.11). For example Spain is located ***east*** of Portugal.

Picture 11

To calculate distances on a map, using the scales, it is necessary to use the concept of ***direct proportionality***: two quantities **x** and **y** are directly proportional if the ratio between them is constant:

$\frac{x}{y} = k; x = ky; y = \frac{x}{k}$, where **k** is a nonzero real number called a constant of direct proportionality.

Exercise 1:

On a map of Europe, with a numerical scale 1: 10,000,000, the distance between ***Almada*** (Portugal) and ***San Benedetto del Tronto*** (Italy) is 20 cm, in a straight line. Calculate the real distance, in kilometers, between the two cities.

NOTE: Use the rule of three simple: $\frac{1}{20} = \frac{10,000,000}{X}$

Exercice 2:

The distance between ***Almada*** (Portugal) and ***Copenhagen*** (Denmark) is approximately 2,970 Km. Calculate the distance in centimeters, in the ***EU-28*** map (Pict.12), between the two cities. The map scale is:

0 500 km

Picture 12

Exercise 3:

The distance between ***Almada*** (Portugal) and ***Budapest*** (Hungary) is approximately 3,000 Km, and the distance between this two cities, on a Europe map, is 30 cm. Calculate the numerical scale of the map indicated in centimeters.

NOTE: Use the rule of three simple: $\frac{1}{20} = \frac{10,000,000}{X}$

3 - Statistics applied to Geography: demography rates

Quantitative and ***qualitative*** analysis are important. Statistics and measurement are used commonly in our lives like making home purchase decisions, setting up investments and weather variations are expressed as probabilities

Geographers use statistics to describe and summarize data, make generalizations concerning complex spatial patterns, estimate likelihoods of outcomes for events at particular locations, use sample data to make inferences about a larger set of data (a population), wish to compare or associate (correlate) patterns of distributions.

Basic Terms and Concepts: ***data element*** - basic element of information which we measure, ***data set*** - groups of data (commuting sheds of industries), ***observations-cases-Individuals*** - elements of phenomena under study, ***variable*** - property or characteristics of each observation that can be measured, classified or counted; values may vary among set of observations: rainfall, per capita income, years of schooling.

Exercise 1:

Based on the data given in table 1, with some demographic figures from the eight partner countries in 2015, calculate the ***Natural Growth Rate*** for the countries in the table below:

Table 1

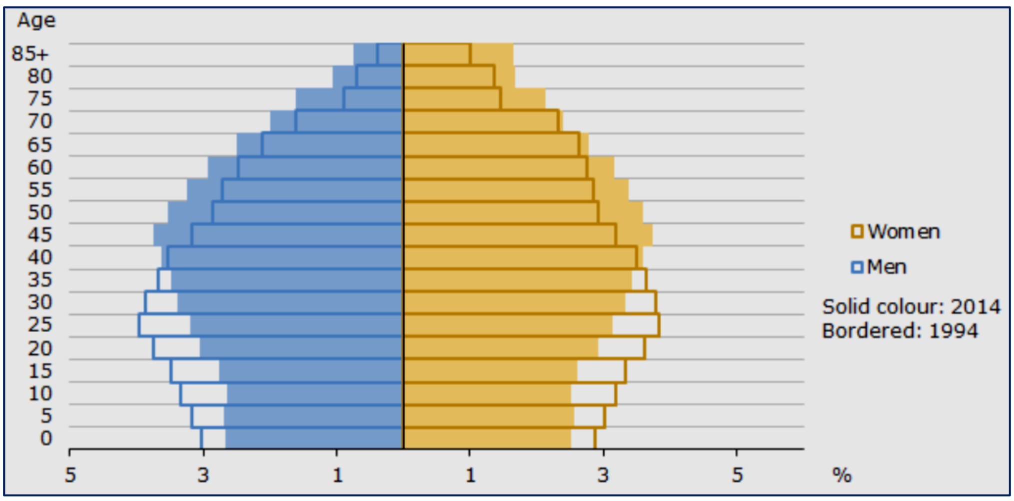
Countries	Resident Population (RP)	Total Births (TB)	Total Deaths (TD)
Denmark	5,683,483	58,205	52,555
Greece	10,820,883	91,847	120,844
Hungary	9,843,028	92,135	131,575
Italy	60,730,582	485,780	647,571
Lithuania	2,904,910	31,475	41,776
Poland	37,986,412	369,308	394,921
Portugal	10,358,076	85,500	108,511
Romania	19,815,481	197,491	260,661

NOTE: NGR = $\frac{TB - TD}{RP}$ x 1000 (‰), the Natural Growth Rate (NGR)

Exercise 2:

The ***Population Pyramids***, ***EU-28***, 1 January 1994 and 1 January 2014 (% of the total population), show that the proportion of older people (≥ 65) increased between 1994 and 2014 (Pict.13), the top of the 2014 age pyramid is broader. Which conclusion can be taken according to the evolution of the proportion of young people (0-15) in the same period of time?

Picture 13



Exercise 3:

Life expectancy at birth is increased by the reduced probability of dying; one of the most significant changes in recent decades has been the ***fall in infant mortality rates***. Around 19 thousand children died before reaching one year of age in the ***EU-28*** in 2013; the infant mortality rate fell during the 20 years from 1993 to 2013, from 8.7 to 3.7 deaths per 1000 live births.

Based on the data given in table 2, with some demographic figures from the eight partner countries in 2015, calculate the ***Infant Mortality Rate*** for the countries in the table below:

NOTE: $IMR = \frac{TID}{TB} \times 1000$ (‰)

Table 2

Countries	Total Births (TB)	Total Infant Deaths (TID) $(\leq 1 year\ old)$
Denmark	58,205	216
Greece	91,847	364
Hungary	92,135	383
Italy	485,780	1,398
Lithuania	31,475	132
Poland	369,308	1,476
Portugal	85,500	250
Romania	197,491	1,500

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