FRACTAL GEOMETRY, A NEW DIMENSION OF HUMAN KNOWLEDGE

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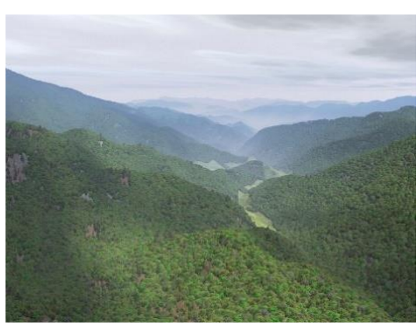
„The scientific study of natural phenomena aims to take a mathematical form and this study is complete when the mathematical form has been found. Born in parallel with the Greek art, mathematics has kept in its internal fabric a certain affinity with art.” *Gheorghe Țițeica*

Fractal geometry is a new language used to describe, analyse and model the complex shapes in nature. If the traditional elements of the language of Euclidean geometry are forms visible as lines, circles, spheres, the new language elements are algorithms that can be converted into shapes and designs only with the help of computers. The algorithm is a powerful descriptive tool. For example, if such a language should be formalized - say experts - we could describe the formation of a cloud as simple and accurate as an architect who describes a house using traditional geometry elements.

Mountains, clouds, or woods are natural fractals that can be easily modeled on a computer using a recursive algorithm.

One way to make such a landscape is to apply a random center shift algorithm. According to this algorithm, a square is divided into four equal squares, the center point of each is vertically raised by a certain random variable. The process is repeated on the four new squares, and so on, until the desired level of detail is reached. The square can be replaced with a triangle.



Images made by the technique of generating fractal landscape

We will further illustrate the generation of natural elements with the help of fractal geometry, as well as the definition of fractals as attractors of systems of iteration functions. We need some theoretical preliminaries.

The Hutchinson operator

Let (X, d) be a complete metric space.

Definition: The set H (X) consists of non-empty compact subsets of X.

Definition: Let x™X and B™H(X). d(x, B) = min {d(x, y)/ y™B} represents the distance from x to B.

Definition: Let A and B from H(X). d (A, B) = max {d(x, B)/x™A} represents the distance from A la B.

h (A, B) = max {d (A, B), d (B, A)}

It can be shown that h defines a metric on H(X), called the Hausdorff -Pompeiu metric.

Theorem: (H(X)) is a complete metric space.

The metric space H(X) is the space where the fractals are found.

Definition: Let (X,d) be a metric space. The function f: X→X is named a *contraction function* if there exists k∈[0,1) so that d(f(x),f(y))≤ kd(x,y), for every x, y∈X.

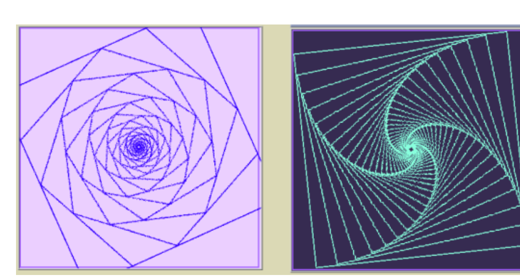


Figure 1. The successive transformations of a square of sides 1 after some contractions

Contraction principle (Banach)

Theorem: Let (X, d) a complete metric space and f: *X → X a k contraction function.* Then:

1) f has a unique fixed point u and
2) for any $x_0 \in X$, the sequence $f^{(n)}(x_0)$ converges to u.

Hutchinson Operator

We consider R^2 (the Euclidean plane) as a complete metric space with the usual distance (Euclidean). Let n be a fixed natural number (not zero) and let for any $j \in \{1,2,...,n\}$, a contraction $W_j : R^2 \to R^2$ having the contraction factor k_j. If *A* is a subset of R^2, we note with $W_j(A)$ the image o the set *A* by the function W_j.

Definition: *We define the application (Hutchinson operator):* $H : H(R^2) \to H(R^2)$, $H(A)=W_1(A) \cup W_2(A) \cup ... \cup W_n(A)$.

We will note: $H = (W_1, W_2, ..., W_n)$. *Also (* R^2, $W_1, W_2,...,W_n$ *) is called iterative function system (IFS).*

Theorem: *Hutchinson's operator is a contraction on the complete metric space of the compact plan parts* $H(R^2)$ *with the Hausdorff distance. In addition, the contraction factor* ***k*** *is the largest element of the set* $\{k_1, k_2, ..., k_n\}$.

Definition: *The fixed point F ™ H(R^2) of the Hutchinson operator* (*it exists and it is a unique according to the contraction mapping principle) is called the attractor of the iterative function system (deterministic fractal) and it is the limit of the string H_n(A), for every A ™ H(R^2).*

These notions, which may seem too technical for students in secondary education, can be understood because they are richly illustrated with programs made in the LabVIEW environment, a software package that we have called **Fractall.**

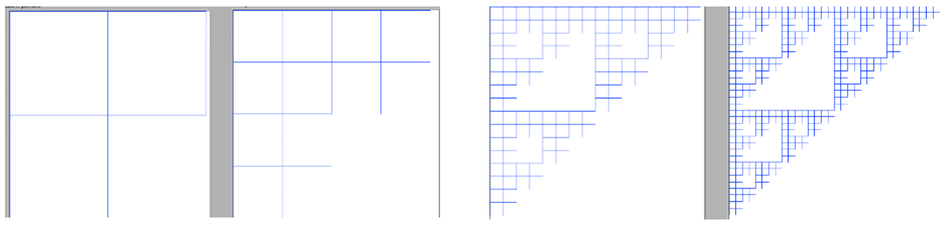


*Figure 2.**The Sierpinski triangle** as attractor of an iterative system*

The four geometrical transformations which applied successively to a square lead to a fern are:

$$f(x,y)=\begin{bmatrix}0.00 & 0.00\\ 0.00 & 0.16\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix};\ f(x,y)=\begin{bmatrix}0.85 & 0.04\\ -0.04 & 0.85\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+\begin{bmatrix}0.00\\ 1.60\end{bmatrix};$$

$$f(x,y)=\begin{bmatrix}0.20 & -0.26\\ 0.23 & 0.22\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+\begin{bmatrix}0.00\\ 1.60\end{bmatrix};\ f(x,y)=\begin{bmatrix}-0.15 & 0.28\\ 0.26 & 0.24\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+\begin{bmatrix}0.00\\ 0.44\end{bmatrix}.$$

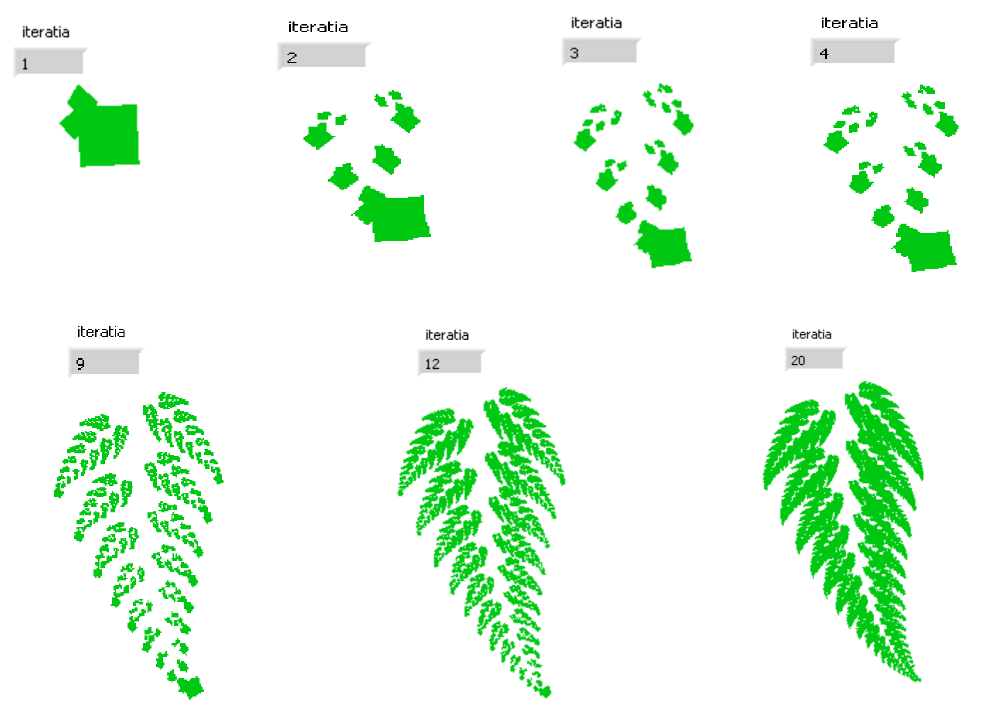


Figure 3. The fern as attractor

Fractal geometry, a newer approach of the phenomena from nature

In the description from the beginning of the writing “Poor Dionis”, Mihai Eminescu accentuates what has been discovered hardly after a century: “*The acorn contains the forest. In the piece exist the whole… the drop is the ocean, because once it takes contact with the ocean, it can’t be distinct from this.”*

This is one of the fractal geometry’s principles, a domain that allows, with the help from some very simple repeated rules, to create complex mathematic objects,that can grant a better design of the nature.

> *“Just one move in the universe. The life of the person is just a fraction of the same unit. But if in a succession of unknown fraction, one piece is known, then the hole succession become known. This is obvious the humanity’s first purpose has been concentrate in this fraction, that the natural rules of the mathematic and the logic. The laws of the piece and the whole have been the first things studied with accuracy.” Mihai Eminescu*

Nowadays, the basic ideas about nature in science need radical revisions. These continuous changes in the knowledge paradigms started half a century ago and are developing more and more. Science can not evolve without change, and today, this change occurs in its fundamental philosophy. We have the opportunity to witness a profound transformation in understanding how many areas of science are grounded. A part of the scientific community recognizes the universality of fractals, regardless specialty: physics, astronomy, geography, but also social sciences, linguistic domain, psychology. The Fractal Geometry allows finding a model and a harmonious ordering of some systems where it seems to be an absolute chaos of the various empirical facts.