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KLASIKA
PRIVĀTĀ VIDUSSKOLA
ANNO 1993

eTwinning

Learning- research Activity

WWW:

Wonderful Waterwork

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Private Secondary School KLASIKA

Research Days “Life on the Waves” 2016

Study and research area “WWW: Wonderful Waterwork”

AIMS:

- ❖ Development and formation of skills enabling students to use information technologies efficiently.
- ❖ Extension of practical use of mathematical knowledge and skills.
- ❖ Development of students ability to analyze the information and make decisions thus enabling them to independently “discover” the properties of mathematical objects.
- ❖ Practice of pair work.

TASKS:

- ❖ To extend students knowledge on the parabola by watching and discussing a presentation;
- ❖ To teach the students to use the technologies effectively to fulfil the tasks (smartphone, dynamic mathematical computer environment GeoGebra, work with documents in the Google Doc);
- ❖ Actualize the knowledge and skills in math and physics;
- ❖ To develop the skills to draw the conclusions and present the results in a group;
- ❖ To practice skills to use relevant terminology and present the results in English.

PROCEDURE:

1. Students are acquainted with the aims of research and the procedure:

1.1.Presentation “WWW: Wonderful Waterwork” in English where the parabola is defined in 3 ways (graph, curve, conic section) and the physical properties of the parabola are demonstrated;
1.2.The main objectives are formulated:
- is the trajectory of the water fountain a parabola;

- at what angle should the water fountain run so that the”flight distance” is the longest?

2. Students are provided with worksheets with a detailed description of the procedure. The procedure consists of 3 stages: 1) the experiment; 2) processing of results; 3) the analysis.

3. Students are divided into pairs to carry out the research.

Digital technologies and tools:

❖ water tap;

❖ hose connected to the tap;

❖ ruler, measuring tapes;

❖ smartphone or digital camera;

❖ computers with Internet connection and a dynamic mathematics computer environment GeoGebra;

❖ students must be provided with personal Gmail account.

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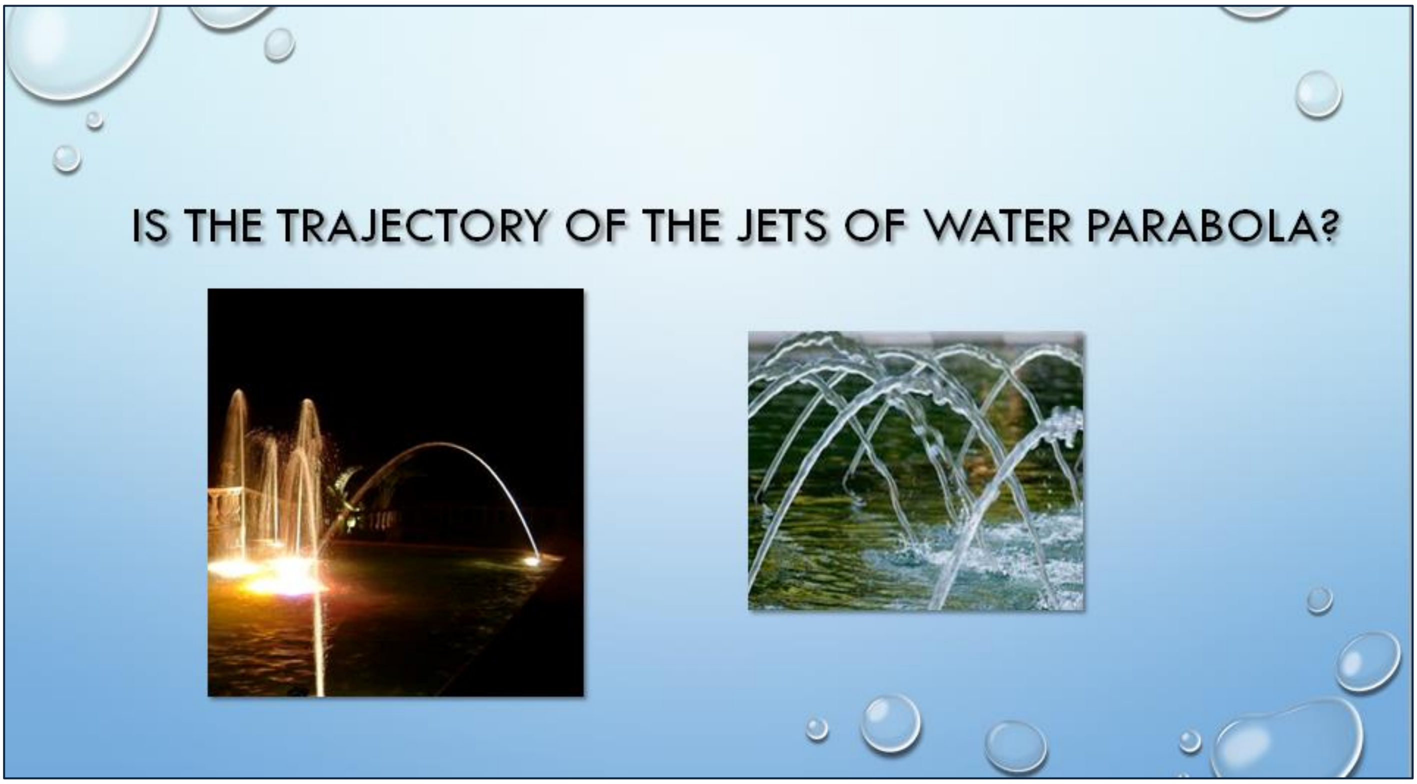
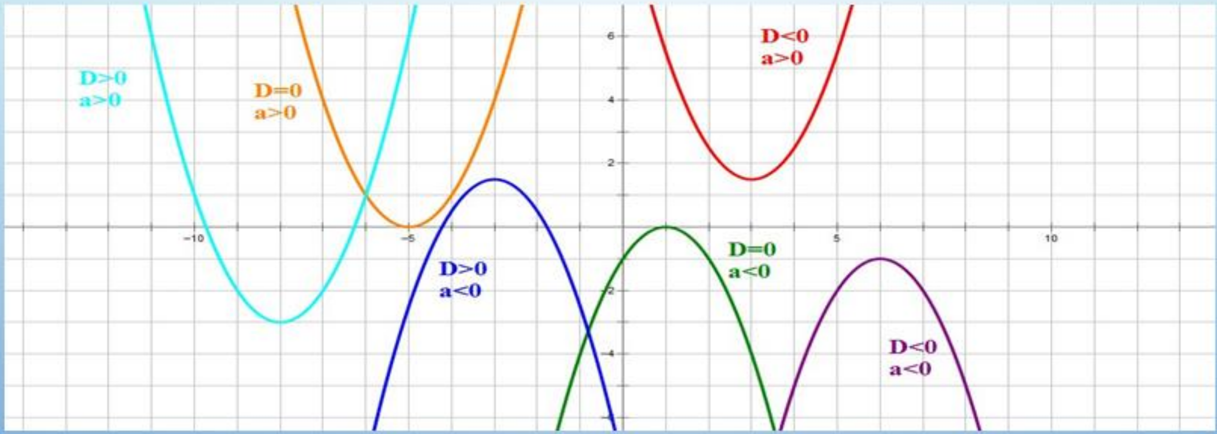
PRESENTATION

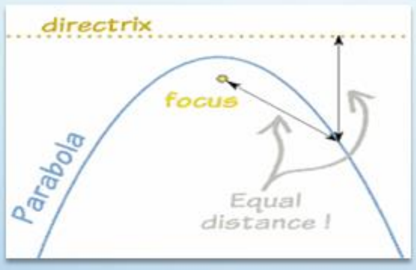


WHAT IS A PARABOLA?



- PARABOLA IS A GRAPH OF THE SQUARE FUNCTION $y = ax^2 + bx + c$

WHAT IS A PARABOLA?



A PARABOLA IS A CURVE WHERE ANY POINT IS AT AN **EQUAL DISTANCE** FROM:

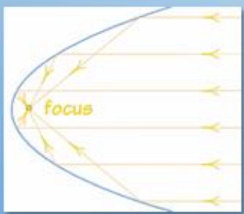
- A FIXED POINT (THE **FOCUS**), AND
- A FIXED STRAIGHT LINE (THE **DIRECTRIX**)

A PARABOLA HAS AMAZING PROPERTY:

ANY RAY PARALLEL TO THE AXIS OF SYMMETRY GETS **REFLECTED** OFF THE SURFACE STRAIGHT **TO THE FOCUS.**

SO THE PARABOLA CAN BE USED FOR:

- SATELLITE DISHES,
- RADAR DISHES,
- CONCENTRATING THE SUN'S RAYS TO MAKE A HOT SPOT,
- THE REFLECTOR ON SPOTLIGHTS AND TORCHES,
- ETC

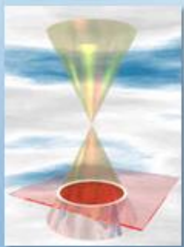


WHAT IS A PARABOLA?

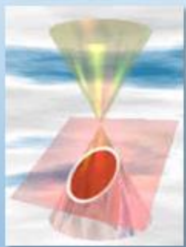
- IT IS ONE OF THE "CONIC SECTIONS“

- CONE

CIRCLE

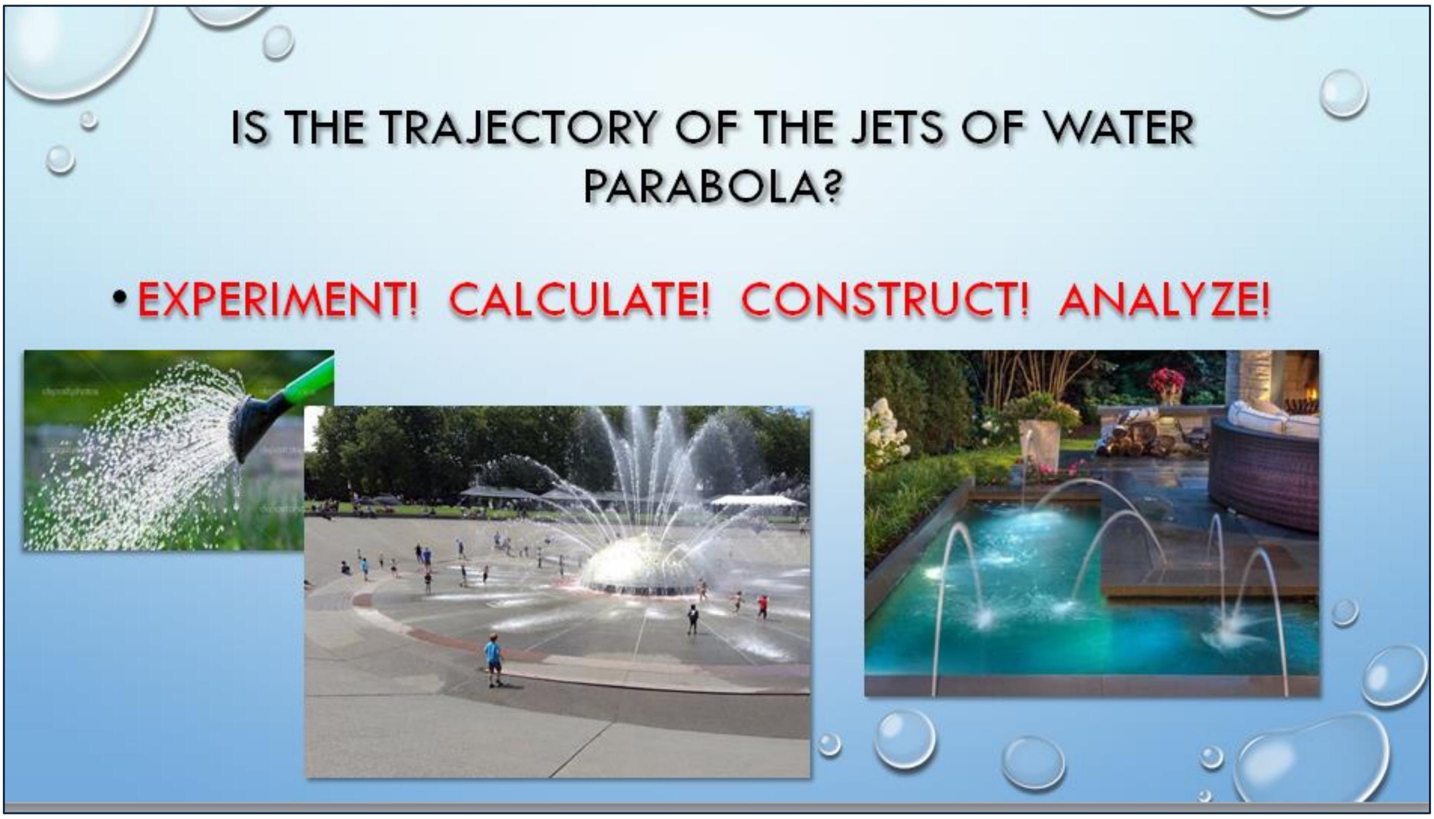


ELLIPSE



PARABOLA

HYPERBOLA


IS THE TRAJECTORY OF THE JETS OF WATER PARABOLA?
• EXPERIMENT! CALCULATE! CONSTRUCT! ANALYZE!

Private Secondary School KLASIKA

Study and research area “WWW: Wonderful Waterwork”

Name, surname, class ______________________________

While watching the water movement in the fountain it can be noticed that its shape resembles the graph of a square function – a parabola. Your task is to investigate whether the water particles of the fountain produced by the lawn watering device work under the law of free fall (i.e.whether the trajectory of how the water runs can be desribed by the square function $y = ax^2 + bx + c$ or to prove that air resistance and the peculiarities of how the water fountain runs do not allow to use this simple model to characterize the water fountain. To fulfil the task you`ll take photos of the water fountain.

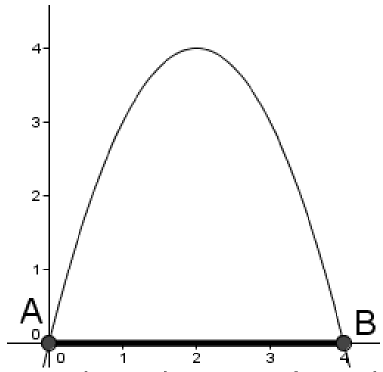
To find out whether the trajectory of how the water particles move is a parabola you will have to define the coefficients (constants) of the square function y = $y = ax^2 + bx + c$ describing the parabola. To fulfil the task you will use your photos. Also you will use the photos to define the angle formed by the water fountain and the horizontal surface (or the vector of initial velocity at the place where the water starts running with the horizontal surface).

At what angle should the water fountain run so that the “flight distance” is the longest?

PROCEDURE

The experiment step by step

1. Take photos of the water fountain in relation to the ruler or any other object with a definite length placed strictly horizontally in the plane of the water running out of the water pipe. Take several photos at different angles of the running water fountain. In the photo there must be point A placed at the same horizontal level with the opening slit of the water pipe and point B where the water fountain touches the ground (see the Figure).



By changing the angle of the water pipe find the angle at which the water fountain's “flight” distance is as long as possible and take another photo. Measure the distance AB between the water pipe's opening slit A and point B where the water fountain touches the ground for each variant.

2. Direct the water fountain vertically upwards and take a photo.

3. Hand out the photos you have taken among your team members for further analysis and fill in the chart.

Photo	Distance AB (cm)	Responsible for the processing results
1.		
2.		
3.		
4.		
5.		

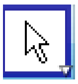
RESULT PROCESSING

1. Download your water fountain photo!

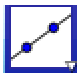
2. Open the dynamic mathematical computer environment **Geogebra** where the view **ALGEBRA** is so that coordinate axes can be seen on the screen.

3. Using the command **Edit->Insert Image from -> File** put the photo in the working field.

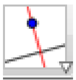
4. Click the button of the mouse on the right on the photo and choose the command **Object Properties** in the context command card. In the window **Preferences** mark the choice square **Background Image** and close the window so that the photo is placed under the coordinate axes.

5. Activate the tool **Move**  and using the moving dots in the lower corners place the photo so that the slit of the water pipe seen in the photo coincides with the starting point of coordinates while the point where the water fountain touches the ground is on the positive direction of the Ox axis.

6. Fix the moving dots of the photo: click the mouse button on the right on each of the dots and in the context command card choose the command **Object Properties**. In the window **Preferences** mark the choice square **Fix Object**, then close the window.

7. In the group of tools **Line through Two Points**  choose the tool **Segment between two Points** and join the starting points of coordinates with the point on the photo, where the water fountain touches the ground.

8. Measure the length of segment: activate the tool **Move**, put the arrow on the segment, click the button of the mouse on the right and in the context command card choose the command **Object Properties**. In the window **Preferences** mark the choice square **Show Label: Name&Value**, close the window. On the photo there will appear the sign of segment "a" and its length (cm).

9. To construct the perpendicular through the midpoint of the segment, in the group of tools **Perpendicular Line**  activate the tool **Perpendicular Bisector** and click on the segment.

10. To measure the height of the water fountain in the photo “h”, mark the segment which join the fountain's highest point with the crossing point of the perpendicular through the midpoint of the segment and Ox axis. Measure the length of the obtained segment (see points 7 and 8).

11. Close the view of the spreadsheet and write the measurements in the chart (see the example).

Experiment number	Distance of "flight" of the jet **XF**(cm) in the photo	The height of the jet **h** (cm) in the photo	Distance of "flight" of the jet **XD** (cm) in reality	The height of the jet **H** (cm) in reality
1.				
2.				
3.				
4.				
5.				

12. To measure the height H of the fountain in nature take into account that the values are directly proportional:

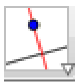
$$\frac{XF}{h} = \frac{XD}{H}; \quad H = ?$$

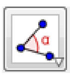
Obtain H from the value of proportion and calculate its value in the chart made in the view of the spreadsheet by introducing the formula into appropriate cell.

Taking into account that the trajectory of the water fountain is close to a parabola – the graph of the square function $y = ax^2 + bx + c$ using 3 point coordinates:

1. the point where the opening slit of the water pipe is - (...;...),
2. the point where the water fountain touches the ground - (...; ...),
3. the highest point of the water fountain - (...;...),
   put these coordinates in the formula and calculate the numerical values the coefficients a,b and c.

13. In the cell of the chart E1 write the title “Function . In the cell E2 write the formula of the function you have obtained and in the working field of the photo there will appear a graph of a square function.

14. To define the angle of the water fountain at the starting point construct a tangent of the parabola through the opening slit of the water pipe. From the group of tools **Perpendicular Line**  choose the tool **Tangents** and click first on the starting point, then on the parabola.

15. To calculate the angle from the group of tools **Angle**  use the tool **Angle**.

16. In the cell of the chart write the title “Angle” while in the cell F2 write the result of your measurement in degrees.

ANALYSIS

To write the minutes of the research as a team use the offer to complete the Google document **WWW – Wonderful Waterwork**, which you will find in your e-mail.

The research is carried out as a part of an International project “ICT World” therefore we kindly ask to fill in the minutes in English.

1. Use the tool **Snipping Tool** to cut out the photo with the constructed on it parabola from the **GeoGebra** working field, place it in the document.

2. In the chart for results fill in the line corresponding to the number of your experiment.

3. Get acquainted with the results obtained by your team mates.

4. In the part **SUMMARY** describe your observations during the experiment and make the conclusions, i.e. answer the questions, asked in the beginning. Try to justify your answers!

6. Close the windows of all the programmes!

7. Get ready to present your results orally!

Research work WWW - Wonderful WaterWork
The Minutes

08.09.2016

Photo of experiment №1
Names of students: Ekaterina Yurachkovskaya and Dana Svirchenkova

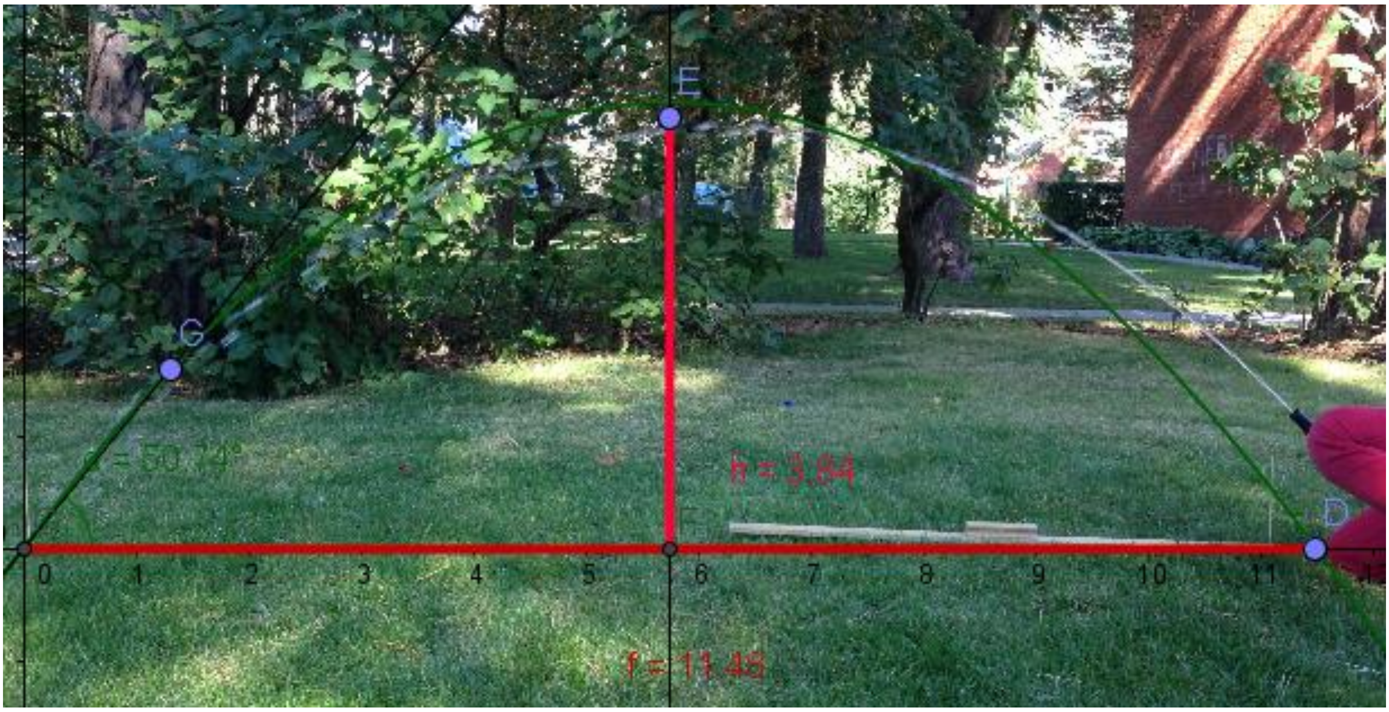


Photo of experiment №2
Students names: Dayana Vershinska and Evelīna Priedīte

Photo of experiment №3
Students names: Arturs Jermaks and Luka Fendins

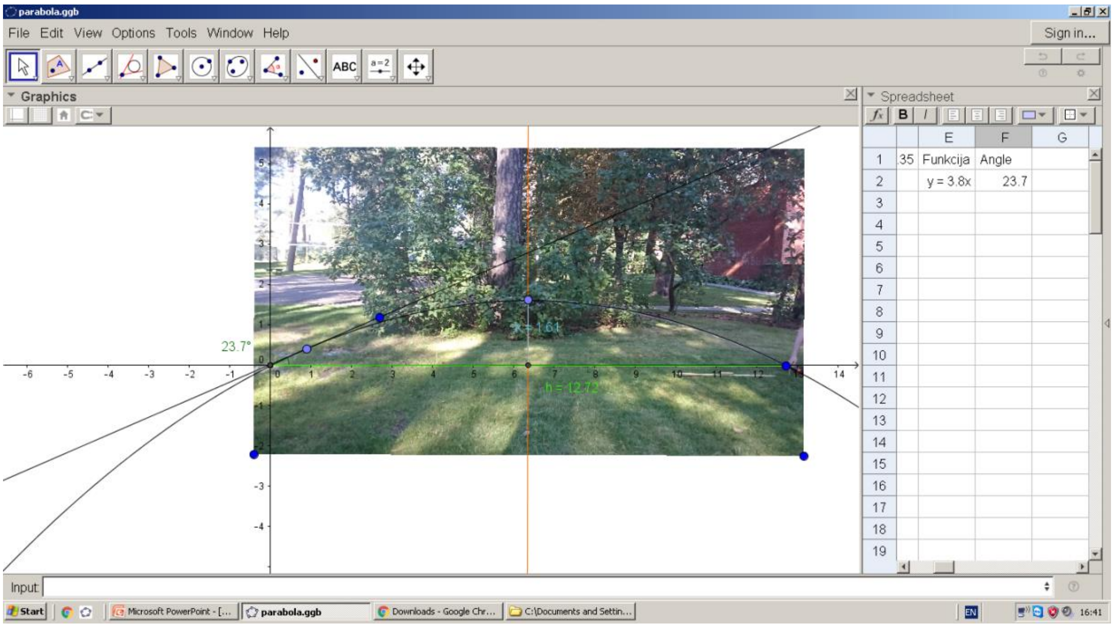


Photo of experiment №4
Students names: Ilja Beluss and Ariana Kornejeva

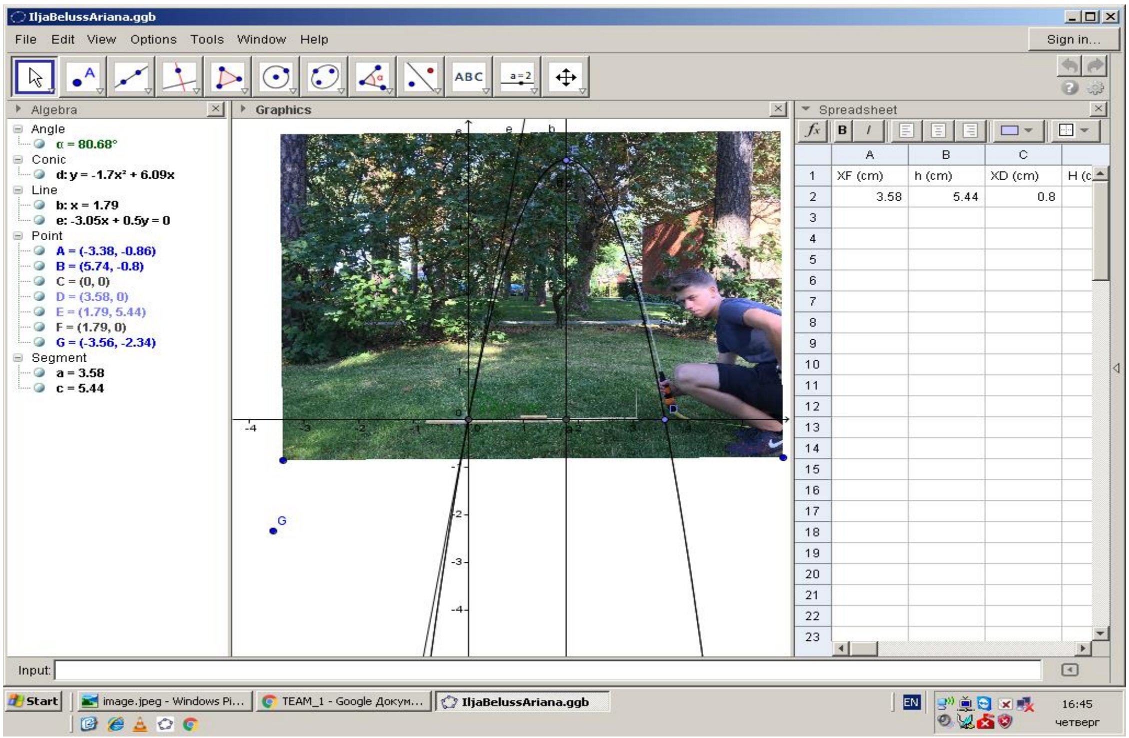


Photo of experiment №5
Students names: Rihards Zvingēvics-Dreimanis and Rihards Allens

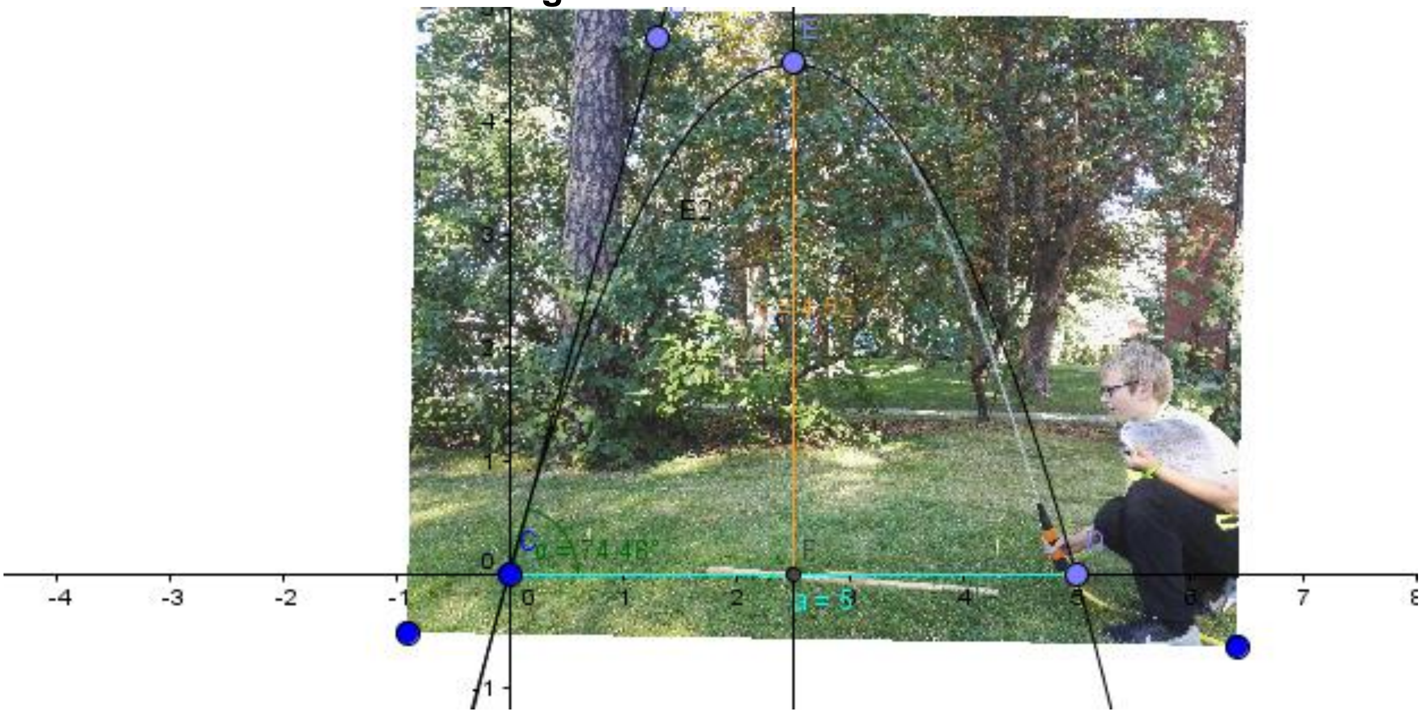
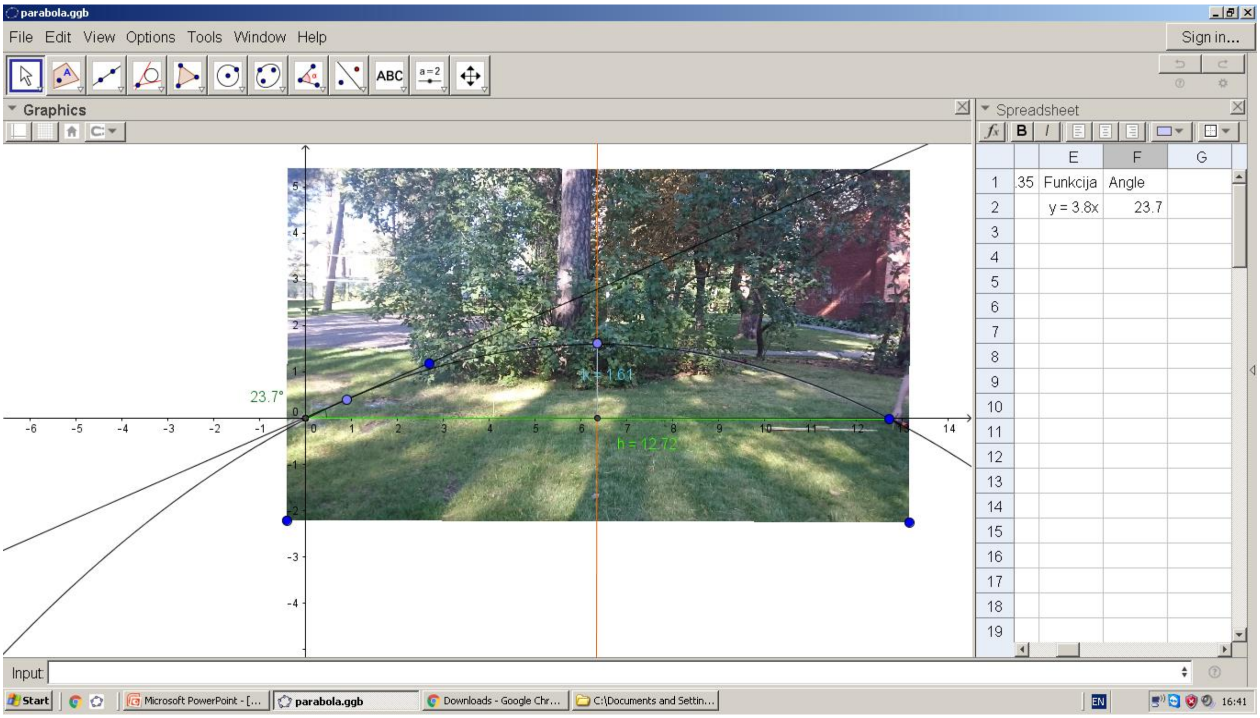


Photo of experiment №6

Students names: Raitis Zēvalds and Ernsts Čerņavskis



The table of results

Experiment number	Distance of "flight" of the jet **XF**(cm) in the photo	The height of the jet **h** (cm) in the photo	Distance of "flight" of the jet **XD** (cm) in reality	The height of the jet **H** (cm) in reality	**Formula of the function**	Slope of the jet at the starting point
1	**11.48**	**3.84**	**228**	**76.26**	**$y=-0.12x^2+1.38x$**	**50.74**
2	**12**	**6**	**129**	**64.5**	**$y=-0.17x^2+2x$**	**63.43**
3	**12**	**3.52**	**275**	**80.67**	**$y=-0.09x^2+1.12x$**	**48.24**
4	**3.12**	**5.43**	**83**	**125.73**	**$y=-0.03x^2+1.5x$**	**80.38**
5	**5**	**4.52**	**153**	**138.31**	**$y=-0.72x^2+3.6x$**	**74.48**
6	**12.72**	**1.61**	**540**	**68.35**	**$y=-0.04x^2+3.8x$**	**23.7**

Summary

WATER JET – THE PARABOLA! But it can be argued that this jet is only an approximate parabola: we have ignored air resistance, interaction between the droplets, and water dispersion. However, applied mathematics is about modelling real life situations; we have tried to make the mathematics as close as posible to the real thing.

The parabola's equation is $\boldsymbol{y = ax^2 + bx + c}$. The two constants a and b are determined by the initial speed and the angle at which the jet of water comes out of the tap. We have calculated the two constants a and b, c = 0 and have graphed it using GeoGebra. We can conclude that the fountain in the photo is very close to the graphed line of the parabola. Theoretically we learned from the Internet the the greatest distance of the jet flight should be when the slope of the jet at the starting point is 45 degrees. Unfortunately. In our experiments the velocity of the jet was not constant, therefore we cannot make the same conclusion.