

European Path (e) Motion, Physics Textbook by Greek school

Work Sheet 1: Watermill

Name : Date:

A. Useful Knowledge

Watermills through History

A watermill is a structure that uses a water wheel to drive a mechanical process such as ground flour or lumber production. The ancient Greeks and Romans are thought to be the first to use water to power their mills. In the early 1st century BC, the Greek epigrammatist Antipater of Thessalonica referenced a waterwheel which was effectively used to grind grain and reduce human work. The Romans built some of the first watermills outside of Greece for grinding flour and spread the technology for constructing watermills throughout the Mediterranean.

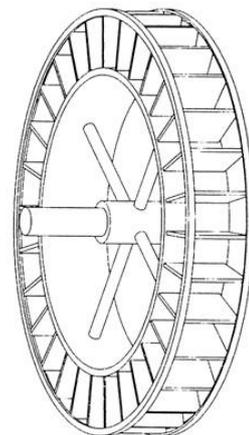
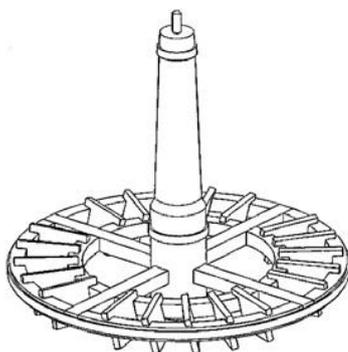


How Does it Work?

A watermill works by diverting water from a river or pond to a water wheel, usually along a channel or pipe. The water's force drives or pushes the blades of the wheel which then turns or rotates an axle that drives whatever machinery is attached to it. After turning the waterwheel, the water exits the watermill.

Horizontal or Vertical?

Watermills that use water wheels with a horizontal wheel and vertical shaft are sometimes called "Greek Mills." A "Roman Mill" refers to a watermill that uses a vertical wheel (on a horizontal shaft). Greek style mills are the older and simpler of the two designs, but they require high water velocity to work well. The Roman style mills are much more complicated in their component parts and require gears to transfer the power from a shaft with a horizontal axis to one with a vertical axis.



B. Concepts

Gravitational Potential Energy (U) : is the energy that is stored in an object that is located at a height above the surface of the earth. It can be expressed mathematically as follows :

$$U = \text{mass} \times g \times \text{height}$$

Where g is the acceleration due to gravity. At sea level, $g=9,81\text{m/s}^2$. For example if a pond of water is located on a hill 20m above a house the water in the pond has stored energy. Can you estimate this gravitational potential energy of water ?.....

Kinetic Energy(K) : is the energy an object has because of its motion; any object that is moving has kinetic energy. The kinetic energy depends on both mass and velocity (v) and can be expressed mathematically as follows :

$$K = \frac{1}{2}mv^2$$

Work (W) : is force acting over a distance and can be expressed as :

$$\text{Work} = \text{Force} \times \text{distance}$$

If a force of 10 Newtons was used to move an object for 20 meters, we can determine the work involved.

$$\text{Work} = \dots\dots\dots$$

A joule (J) is the unit of work and is the same thing as Newton meter.

Power(P) : is work done over a period of time and has units of Watts (W), or Joules/sec. It is defined by the following relationship :

$$P = \frac{W}{t}$$

An illustration of power is described in the following example. Two piles of bricks need to be moved across a street. Mary carries two bricks at a time and takes two days to complete the job, while Elizabeth, who is driving a backhoe, does the job in 15 minutes. Mary and Elizabeth both do exactly the same amount of **work**. However, the amount of **power** expended is much different. If this job requires 20.000 Joules of work, you can calculate the power of backhoe :

$$P_{\text{Elizabeth}} = \dots\dots\dots$$

On the other hand :

$$P_{\text{Mary}} = \dots\dots\dots$$

Thus, if more power is generated, the quicker the work is performed.

C. Watermills

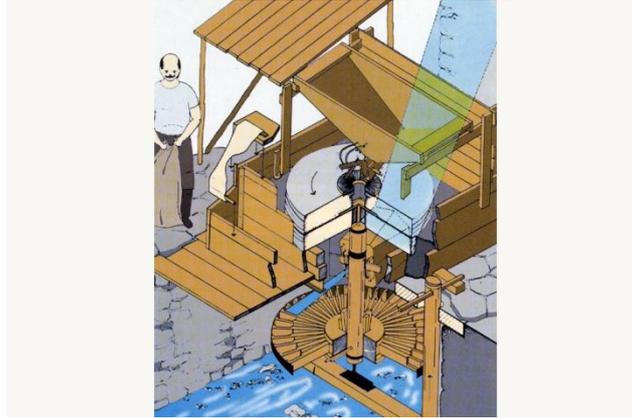
The simplest and most common type mill has an **overshot waterwheel**. An overshot mill is a potential energy device in which the water falls a vertical distance. This causes the waterwheel to rotate which changes the potential energy of the water into work inside the mill.



You can see the waterwheel and the water flume which supplies the water to the top of the waterwheel. The water is supplied by a small lake back in the trees. The water is released on the wheel just slightly forward of the vertical center of the wheel. The weight of water does cause an unbalanced torque on the wheel. Because this torque is not balanced by any other torques it causes the wheel to rotate. This rotation carries the water from a higher level to a low level and reduces the potential energy of the water. The potential energy removed from the water will be converted into work inside the mill. So,

$$U = W \rightarrow mgh = Fd$$

Undershot Mills use the principles of kinetic energy. Kinetic means to be in motion. Therefore this source of energy is the energy of moving water. An undershot mill uses the velocity of the water to turn the waterwheel. The water gains its kinetic energy as part of a fast moving stream.



The water slows down upon striking the waterwheel therefore the kinetic energy of the water is also less. The kinetic energy lost by the water is converted to work inside the mill. If the water comes nearly to rest then all of the kinetic energy of the water has been converted into work. So,

$$K=W \rightarrow \frac{1}{2}mv^2 = Fd$$

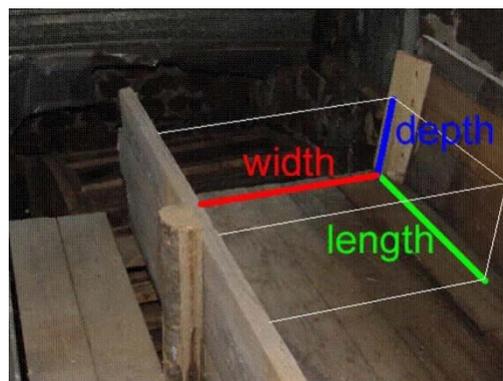
Then, the power is

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv^2}{t}$$

Finding the mass per time, a rate, is still a measurement that is not easily determined. We need to convert the mass of the water into the volume of the water. To accomplish this we will use the density of water which is a constant for fresh water and is defined to be mass per volume.

To determine the volume of the water we need to understand how the water was delivered to the waterwheel. This was usually done by a wooden flume or sometimes by a metal pipe. The water might travel to the flume via of an earthen or stone mill race. The wooden flume is the most common delivery device and we will only analysis this device for our purposes. The wooden flume is a rectangular shaped device as can be observed in the image below. The volume of the flume can be calculated by the following relationship :

$$V = lwd$$



The volume is defined for the flume as length (l) times the width (w) times the depth (d). So, the mass will be given :

$$\rho = \frac{m}{V} \Rightarrow m = \rho V = \rho lwd = \rho lA$$

The width times the depth is the cross sectional area and will be represented with an A. Then, the power will become :

$$P = \frac{\frac{1}{2}\rho lA v^2}{t} = \frac{1}{2}\rho \frac{l}{t} A v^2$$

Velocity for the water is defined as the distance the water travels in a given amount of time. The relationship for velocity is defined as :

$$v = \frac{l}{t}$$

The l will represent the distance the water travels (length) and t is the time period in which the distance was measurement. So, the power will become finally :

$$P = \frac{1}{2} \rho A v^3$$

Velocity of water is measured as it travels in the flume. To determine this velocity today we have devices that can be placed in the water. In the days of the water powered mill they would observe a leaf floating in the water or determine how much water flowed down the flume in a given amount of time and knowing the size of the flume they could determine the velocity.

D. The watermill of San Giorgio village.

The watermill is next to the springs of river Louros, very close to a special site, the Roman aqueduct of ancient Nicopolis. The water mill was built in 1832.



The springs of Saint George flow from underground with great velocity and water amount which reaches 2 m³/s on average. The water is stored in a tank under the central building and through a stream it is led to the watermill.



The watermill is situated in the basement of the mill, built under the millstones. The wheel (1 – 1,40m diameter) rotates horizontally and accepts high water velocity. At the above and back part of the mill there is a big tank which is filled with water. Then, there is a special stone water flume of 0,6 – 1,3m depth, 0,5 width and 10m length.



In order to start working the lever should be turned on, the water falls with great speed on the wheel and the above millstone starts rotating.

If the water density is $\rho = 1000\text{Kg/m}^3$ and the cross sectional area of the stream is $A = 50\text{cm} \times 60\text{cm}$, estimate the velocity of the water in the flume and calculate the Power of the mill with the help of the last equation.

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E. Working with watermills

- Using the same logic as done with the undershot watermill you can calculate the Power of an overshot watermill.

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For example, If the water for the mill comes from 13 m height with velocity about 2 m/s and the cross sectional area of the flume is $0,05 \text{ m}^2$ you can estimate the Power of the mill :

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- What advantages does the watermill have as a renewable source of power?

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- What drawbacks does the watermill have as a reliable source of power?

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