## Math Formulas to Evaluate the Impact of The Wheel Radius onto Velocity.

The goal is to determine what would happen if the size of the wheels of a car were increased to the size of wheels of a bicycle...

Car engines deliver power over a much larger range of speeds than cyclist's legs do. Thus it is necessary to take into account the two types of transmissions between car and bicycle. In this workshop we will calculate gear ratios on a bicycle and car gearbox ratios in order to compare them as a final conclusion. Finally we will evaluate the impact of wheel diameters on speed driven by the car or by the bicycle.

Documents provided for this task are firstly instructions, secondly answer sheets.
Let's experience and calculate with formulas in mathematics.....

Bicycle front chainrings and rear sprockets set


Car gearbox system


Connection between engine and gearbox

extracted from http ://www.fiches-auto.fr/articles-auto/fonctionnement-d-une-auto/s-1566-demultiplication-des-rapports-de-boite.php

## Part A - Metres of Development of a Bicycle.

Contemporary bicycles have multiple gears and thus multiple gear ratios.

1. The gear ratio is based on Front/Rear measurement using two numbers (for example $53 / 19$ ) where the first is the number of teeth in the front chainring, and the second is the number of teeth in the rear sprocket.

The fraction obtained this way is called the gear ratio, which is a pure number, independant of any units of measurement.
2. The metres of development corresponds to the distance (in metres) traveled by the bicycle for one rotation of the pedals, which uses the wheel diameter $D$ in the formula gearratio $\times \pi D$.

Fill the answer sheet grid with results of the following questions.

1. Count the number of teeth for each front chainring and three rear sprockets, the smallest sprocket, an intermediate one and the largest one.
2. Calculate the corresponding gear ratios.
3. Calculate the metres of development rounded to one decimal place.
4. Quote the gears as very high, high, medium, low, very low.

## Part B - The Power of your legs on Bicycle.

Whereas any car displays the revolution frequency used from the engine in revolution per minute, shortly written as "rpm", the goal of this workshop is to evaluate the corresponding revolution frequency produced by your body when riding the bicycle.

Fill the answer sheet grid with results of the following questions.

1. Measure the distance $d$ around the school yard.
2. Select the gear obtained by the largest front chainring and the smallest rear sprocket. Ride the bicycle on this distance, measure the time needed at constant speed.
3. Give to calculate the (linear) output speed $V_{1}$ in meter per second. Calculate.
4. Measuring the wheel diameter calculate the wheel radius $r$.
5. Calculate the gear ratio $R$ of the gear selected above.
6. Using $V=\omega \times r$, calculate the OUTPUT angular velocity $\omega_{1}$ in radian per second.
7. Using $\omega_{1}=R \times \omega_{0}$ where $R$ denotes the gear ratio, calculate the INPUT angular velocity $\omega_{0}$ in radian per second.
8. Finally using $\omega_{0}=\frac{2 \pi N}{60}$, calculate the revolution frequency $N$ in rpm produced by your body taking the role of the engine set in this gear.
9. Comment on this final result.

## Part C - The Gear Ratio of the Car.

Let's say that automobile gearbox consists of a set of teethed wheels of different sizes interacting with each other. The same principle as for the bicycle drivetrain works with fractions of number of teeth leading to gear ratios.

1. Ask the teacher for the revolution frequency from the engine $N$, in revolution per minute (rpm) displayed by the car and the speed driven as well as the selected gear.
2. Using $\omega_{0}=\frac{2 \pi N}{60}$, calculate the INPUT angular velocity $\omega_{0}$ in radians per second corresponding to the selected gear from the engine.
3. Convert the speed given in km per hour into meter per second.
4. Measuring the wheel diameter give the wheel radius $r$.
5. Using $V=\omega \times r$, calculate the OUTPUT angular velocity from the engine $\omega_{1}$ in radian per second.
6. Moreover, in reality because of the axles of the wheels, calculate OUTPUT angular velocity of this gear transmission and the differential $\omega_{2}$ in radian per second given by $\omega_{2}=3.3 \times \omega_{1}$.
7. Using $R=\frac{\omega_{0}}{\omega_{2}}$ calculate the gear ratio $R$ of this selected gear.
8. Comment on your findings. As an example, 2004 Chevrolet Corvette C5 Z06 with a six-speed manual transmission has

1 st gear ratio $=2.97-2$ nd gear ratio $=2.07-$
3 rd gear ratio $=1.43-4$ th gear ration $=1-$
5 th gear ratio $=0.84-6$ th gear ratio $=0.56-$
reverse ratio $=-3.38$.

## Part D - Final Conclusion.

Therefore the car's tires can almost be considered as a third type of gearing.
Calculate the new ratio if the car is equipped with wheels of the size of bicycle wheels.
Conclude.

## Answer Sheet A - The Bicycle's Metres of Development.

- comment among very high, high, medium, low, very low

Wheel diameter $D=$

| Front chainring <br> Number of teeth | Rear sprocket <br> Number of teeth | Gear ratio <br> $R$ | Metres of <br> Development | Comment <br> on the gear |
| :--- | :--- | :--- | :--- | :--- |
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## Answer Sheet B-The Power of your legs on the Bicycle .

| Quantity | Units | Formulas | Results | Calculations |
| :---: | :---: | :---: | :---: | :---: |
| Selected Gear Ratio |  |  |  |  |
| Distance $d$ | metres |  |  |  |
| Time $t$ | seconds |  |  |  |
| Linear speed V | $m s^{-1}$ |  |  |  |
| Wheel radius <br> $r$ | metres |  |  |  |
| OUTPUT <br> angular <br> velocity $\omega_{1}$ | $r a d s^{-1}$ |  |  |  |
| INPUT <br> angular <br> velocity $\omega_{0}$ | $r a d s^{-1}$ |  |  |  |
| Revolution frequency $N$ | rpm |  |  |  |

## Answer Sheet C - The Gear Ratio of the Car - Selected Gear

| Quantity | Units | Formulas | Results for THIS <br> gear | Calculations |
| :--- | :--- | :--- | :--- | :--- |
| Linear speed <br> $\ldots . . k^{-1}$ | ms $^{-1}$ |  |  |  |
| Revolution <br> frequency $N$ | rpm |  |  |  |
| INPUT <br> angular <br> velocity $\omega_{0}$ | rads |  |  |  |$\quad$| Wheel radius $r$ | metres |  |  |
| :--- | :--- | :--- | :--- |
| OUTPUT <br> angular <br> velocity $\omega_{1}$ | rads |  |  |

## Answer Sheet D - Car Equipped with Bicycle Wheels

| Quantity | Units | Formulas | Results for THIS <br> gear | Calculations |
| :--- | :--- | :--- | :--- | :--- |
| Linear speed <br> $\ldots . . k^{-1}$ | ms $^{-1}$ |  |  |  |
| Revolution <br> frequency $N$ | rpm |  |  |  |
| INPUT |  |  |  |  |
| angular |  |  |  |  |
| velocity $\omega_{0}$ |  |  |  |  | rads | WHEEL |  |  |  |
| :--- | :--- | :--- | :--- |
| RADIUS $r$ | metres |  |  |
| OUTPUT <br> angular <br> velocity $\omega_{1}$ | rads |  |  |

