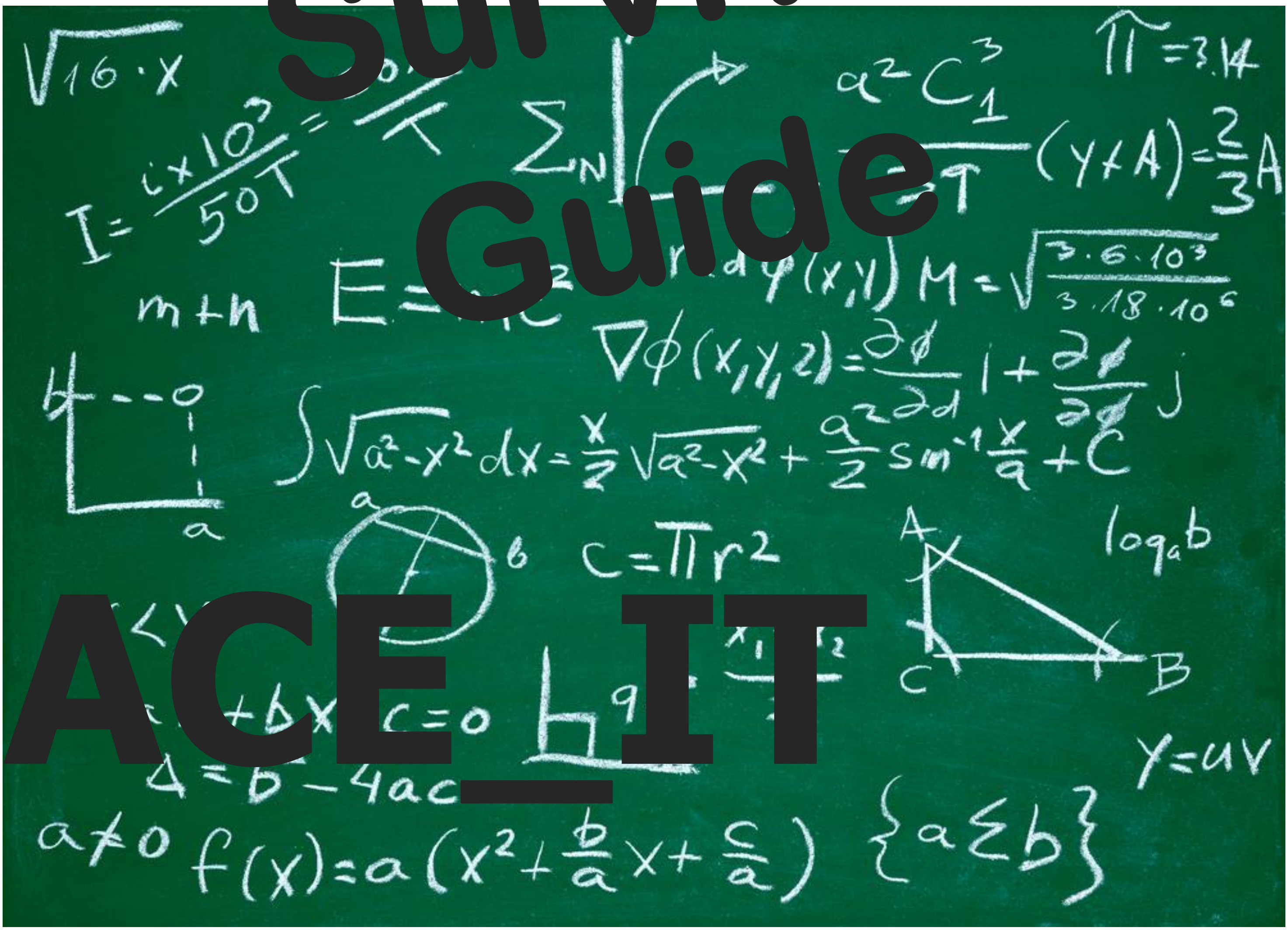
Maths Survival Guide



FACE_IT

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1 NUMBERS:

Cardinal numbers:

1	one	**11**	eleven	**21**	twenty-one	**70**	seventy
2	two	**12**	twelve	**22**	twenty-two	**80**	eighty
3	three	**13**	thirteen	**23**	twenty-three	**90**	ninety
4	four	**14**	fourteen	**24**	twenty-four	**100**	a/one hundred
5	five	**15**	fifteen	**25**	twenty-five	**200**	two hundred
6	six	**16**	sixteen	**26**	twenty-six	**1,000**	a/one thousand
7	seven	**17**	seventeen	**30**	thirty	**2,000**	two thousand
8	eight	**18**	eighteen	**40**	forty	**1,000,000**	a/one million
9	nine	**19**	nineteen	**50**	fifty	**1,000,000,000**	a/one billion
10	ten	**20**	twenty	**60**	sixty		

Ordinal numbers:

1st	First	Primero	21st	Twenty –first	Vigésimo primero
2nd	Second	Segundo	22nd	Twenty-second	Vigésimo segundo
3rd	Third	Tercero	23rd	Twenty-third	Vigésimo tercer
4th	Fourth	Cuarto	30th	Thirtieth	Trigésimo
5th	Fifth	Quinto	31th	Thirty-first	Trigésimo primero
6th	Sixth	Sexto	4oth	Fortieth	Cuadragésimo
7th	Seventh	Séptimo	41th	Forty-first	Cuadragésimo primero
8th	Eighth	Octavo	50th	Fiftieth	Quincuagésimo
9th	Nineth	Noveno	51th	Fifty-first	Quincuagésimo primero
10th	Tenth	décimo	60th	Sixtieth	Sexagésimo
11th	Eleventh	Undécimo	61th	Sixty-first	Sexagésimo primero
12th	Twelfth	Duodécimo	70th	Seventieth	Septuagésimo
13th	Thirteenth	Décimo tercero	71th	Seventy-first	Septuagésimo primero
14th	Fourteenth	Décimo cuarto	80th	Eightieth	Octogésimo
15th	Fifteenth	Décimo quinto	81th	Eighty-first	Octogésimo primero
16th	Sixteenth	Décimo sexto	90th	Ninetieth	Nonagésimo
17th	Seventeenth	Décimo séptimo	91th	Ninety-first	Nonagésimo primero
18th	Eighteenth	Décimo octavo	100th	Hundredth	Centésimo
19th	Nineteenth	Décimo noveno	101th	Hundred and first	Centésimo primero
20th	twentieth	vigésimo	200th	Two hundredth	Dos centésimas

Use ordinal numbers:

- to write fractions:
- in titles (names of kings and queens):

5/6 five sixths

2/3 two thirds

Charles II: Charles the Second

Henry VIII: Henry the Eighth

Number 0

Name	When we use it	Example
0 = zero	US English for the number In temperature	3 – 3 = 0 three minus three is zero -10º=10 degrees below zero
0= 0h	After a decimal point In bus or room numbers In telephone numbers in years	7.03 ·"seven point oh three" Room 201 = Room two oh one Bus 107 = Bus one oh seven 9130472 = "Nine one three oh four seven two." 1802 = "Eighteen oh two."
0 = nought	before a decimal point	0.06 = "Nought point oh six."
0 = nil	in football	Chelsea 2 Manchester United 0 = "Chelsea two Manchester United nil."
0 = love	in tennis	20 - 0 = "Twenty - love."

Large numbers in English:

Reading large numbers in English can seem complicated, but if you follow these simple steps, you will be able to read numbers in English quite easily.

Look at the following example. How would you read this in English?

897361524

When reading numbers in English, start from the right and add a comma (,) after every third number:

897,361,524

English numbers are always read in groups of three and from left to right. Each group of three numbers is read in the same way. So, **897** is eight hundred and ninety-seven, **361** is three hundred and sixty-one, and **524** is five hundred and twenty-four.

		8 9 7	3 6 1	5 2 4
trillion	billion	million	thousand	hundred

The complete number is read in the following way: **"Eight hundred and ninety-seven million, three hundred and sixty-one thousand, five hundred and twenty-four"**

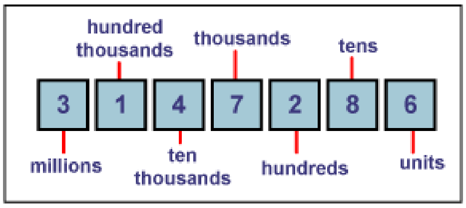
Another example:

12,045,235,003,134 → Twelve trillion, forty-five billion, two hundred and thirty-five million, three thousand, one hundred and thirty-four.

(Realize that in Spanish billion means 1,000,000,000,000)

Ordering numbers and place value:

Our numeral system is a **decimal** and a **place-value** system. It is decimal because it is formed with ten symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) and it is positional because the value of each digit depends on the position in the number.



In the number **3 147 286** (three million, one hundred and forty seven thousand, two hundred and eighty six), the **figure 2** has a value of 200 (two hundred), and the **figure 3** has a value of 3 000 000 (three million).

Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
M	HTh	TTh	Th	H	T	U
	4	0	5	0	0	0
		4	5	0	0	0

To compare two numbers, we can use these symbols:

$=$	$\neq$	$<$	$>$
EQUAL TO	NOT EQUAL TO	LESS THAN	GREATER THAN

Even number: número par

Odd number: número impar

OPERATIONS	SYMBOL	WORDS	EXAMPLE	
Addition or sum	+	**Plus or and**	8 + 3 = 11 Addend, Addend, Sum	Eight plus three is/equals eleven. Eight and three are eleven.
Subtraction or difference	−	**Minus or take away**	8 - 3 = 5 Minuend, Subtrahend, Difference	Eight minus three is/equals five. Eight take away three leaves/is five.
Multiplication or product	×	**Times or multiplied by**	6 × 3 = 18 Factor (or Multiplier), Factor (or Multiplicand), Product	Six times three is/equals eighteen. Six multiplied by three is/makes eighteen.
Division or quotient	÷	**Divided by**	16÷3=5 R 1 dividend, divisor, quotient, remainder	Sixteen divided by three is/equals five with a remainder of one.

When expressions have more than one operation, we have to follow rules for the order of operations. That is:

Brackets **O**ther **D**ivision and **M**ultiplication **A**ddition and **S**ubtraction (BODMAS)

All of these terms are fairly obvious except for **'Other'**, which are really just powers.

Divisibility:

A number is divisible by	If
2	The last digit is even (0,2,4,6,8)
3	The sum of the digits is divisible by 3
5	The last digit is 0 or 5
6	The number is divisible by both 2 *and* 3
10	The number ends in 0
11	If you sum every second digit and then subtract all other digits and the answer is: • **0**, or • **divisible by 11**

A **prime number** only has two factors: the number one and itself. For example: 2, 3, 5, 7,…

A **composite number** has more than two factors. For example: 4, 9, 15, 32,…

The **GCD (greatest common divisor)** is the biggest of the common divisors of two or more numbers. It is the result of multiplying the entire prime common factors with the smallest exponent.

The **LCM (least common multiple)** is the smaller of the common multiples of two or more numbers. It is the result of multiplying the entire prime factors with the highest exponent.

Powers:

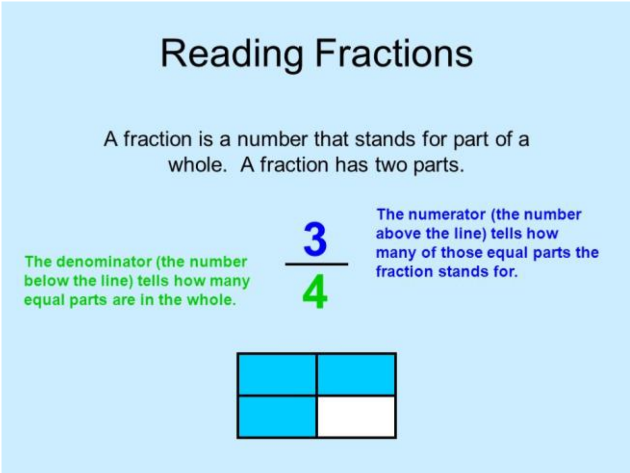
A **power** is an abbreviated way of writing a product of equal factors:

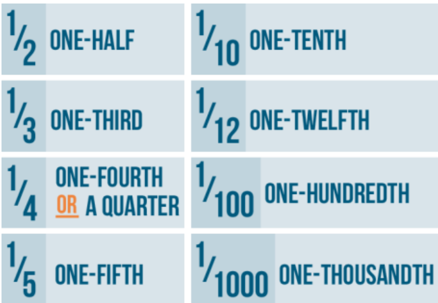
We say:

- Four to the power of 2.
- Four raised to the second.
- **Four squared.**

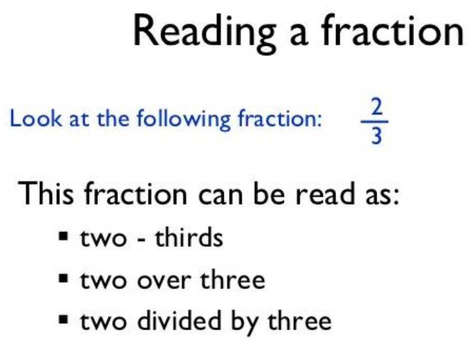
Property	Rule with exponent	Rule with words
Power of One a^1	a	Always base
Negative Power a^{-m}	$\frac{1}{a^m}$	Flip sign of exponent, Flip base
Power of Zero a^0	1	Always 1
Power of a Product $a^m \times a^n$	a^{m+n}	Keep base add exponents
Power of a Power $(a^m)^n$	$a^{m \cdot n}$	Keep base multiply exponents
Power of a Quotient $\frac{a^m}{a^n}$	a^{m-n}	Keep base subtract exponents

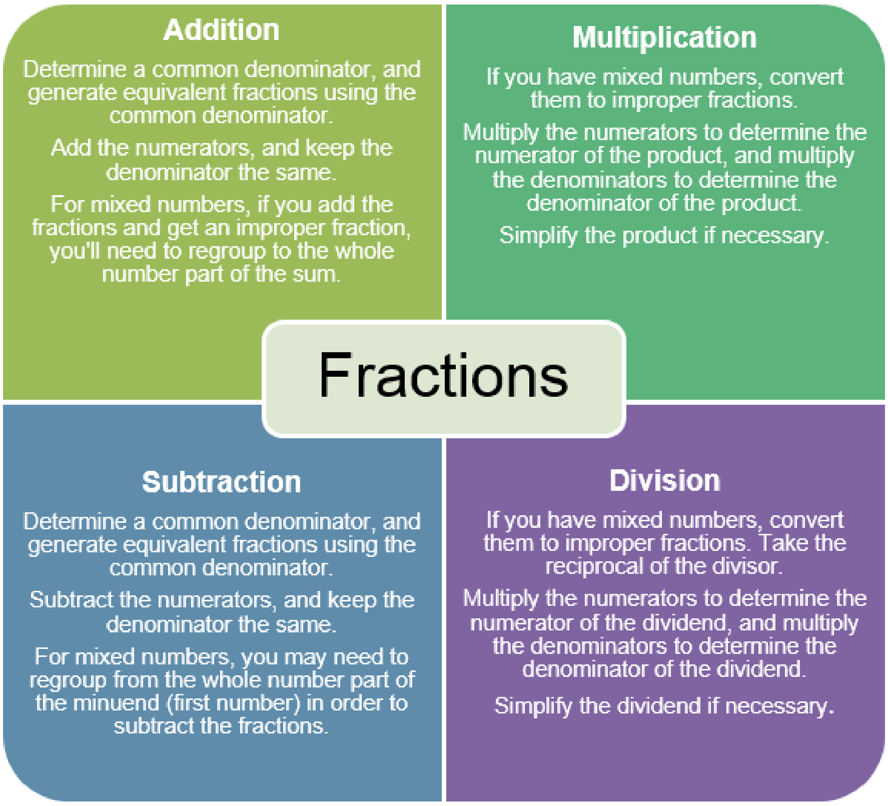
Fractions:





$^1/_2$	ONE-HALF	$^1/_{10}$	ONE-TENTH
$^1/_3$	ONE-THIRD	$^1/_{12}$	ONE-TWELFTH
$^1/_4$	ONE-FOURTH OR A QUARTER	$^1/_{100}$	ONE-HUNDREDTH
$^1/_5$	ONE-FIFTH	$^1/_{1000}$	ONE-THOUSANDTH





Decimal Places:

Keep your eye on the 9 to see where the decimal places fall.

millions	9,000,000.0
hundred thousands	900,000.0
ten thousands	90,000.0
thousands	9,000.0
hundreds	900.0
tens	90.0
ones	9
tenths	0.9
hundredths	0.09
thousandths	0.009
ten thousandths	0.0009
hundred thousandths	0.00009

Types of decimal numbers:

An ***exact* or *terminating*** decimal is one which does *not* go on forever, so you can write down all its digits. For example: 0.125

A ***recurring decimal*** is a decimal number which do not stop after a finite number of decimal places, but where some of the digits are repeated over and over again.

For example: 0.1252525252525252525... is a recurring decimal, where '25' (called the period) is repeated forever.

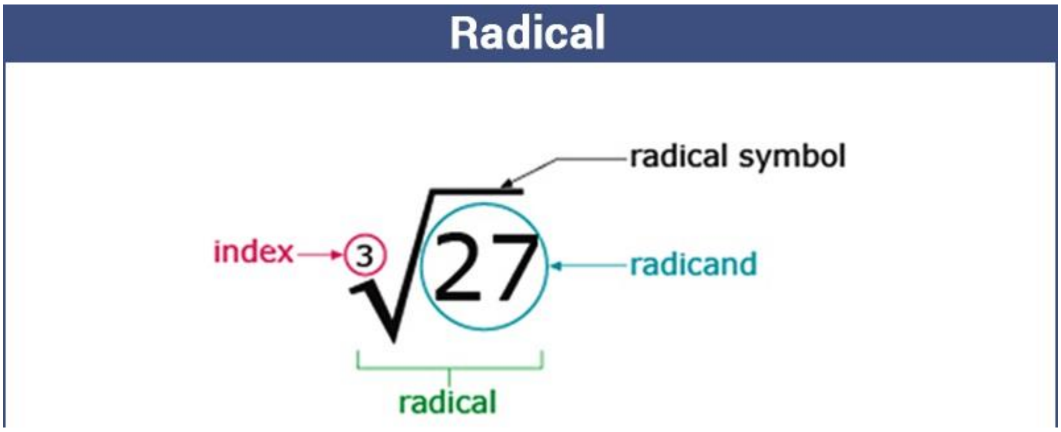
There exists two types of recurring decimals:

- Pure recurring decimal: It becomes periodic just after the decimal point. Ex. 1.3535... (35 is called the period). It is usually expressed as $1.\widehat{35}$
- Eventually recurring decimal: When the period is not settled just after de decimal point. Ex. 1.457777... or $1.45\hat{7}$

Scientific notation:

Scientific notation is a way of writing numbers that are too big or too small to be conveniently written as ordinary numbers. Scientific notation is commonly used in calculators and by scientists, mathematicians and engineers. In scientific notation all numbers are written in the form of $a \times 10^b$(***a*** times ten raised to the power of ***b***), where the **exponent *b*** is an integer, and the **coefficient *a*** is a decimal number between 1 and 10. It gives the number's **order of magnitude.** If the number is negative then a **minus sign** precedes ***a*** (as in ordinary decimal notation).

Standard decimal notation	Scientific notation
2	2×10^{0}
3800	3.8×10^{3}
450000	4.5×10^{5}
0.2	2×10^{-1}
0.000 000 007 51	7.51×10^{-9}



Properties of Radicals

$$\sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$$

Product property: The root of a product is the product of the roots

$$\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

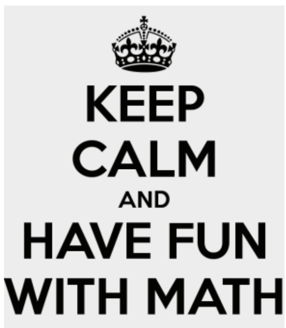
Quotient property: The root of a quotient is the quotient of the roots

$$\sqrt[n]{a^b}=(\sqrt[n]{a})^b$$

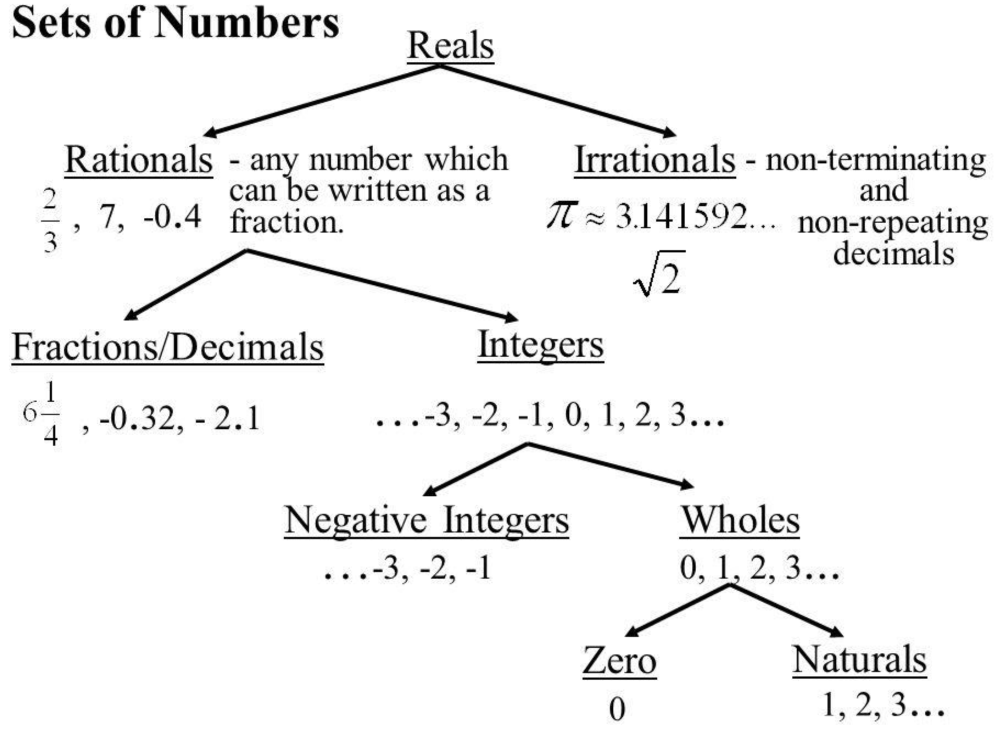
Power property: The root of a power is the power of the root

$$z^{\frac{5}{3}}=\sqrt[3]{z^5}$$

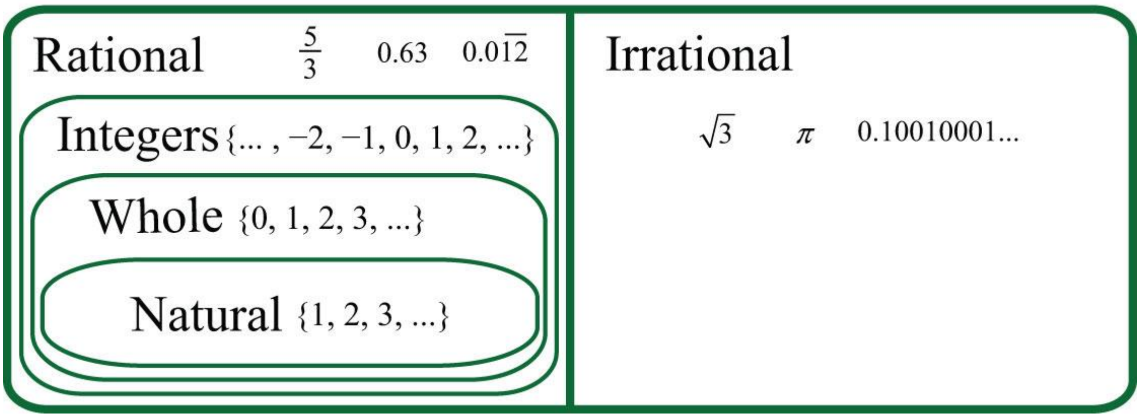
"Fractional Exponents & Radicals"



Sets of Numbers



Real Numbers



Proportionality and percentages:

The **rate** of two numbers a and b is the fraction $\frac{a}{b}$

For example, supposing one has 4 oranges and 3 lemons in a bowl of fruit, the ratio of oranges to lemons would be $\frac{4}{3}$ (4 out of 3) the ratio of lemons to oranges would be would be $\frac{3}{4}$ (3 out of 4)

Additionally, the ratio of oranges to the total amount of fruit is $\frac{4}{7}$ (4 out of 7)

A **proportion** is the equality of two rates:

Ex $\frac{4}{7} = \frac{12}{21}$

To calculate the unknown number in a proportion we use the definition of equivalent fractions:

$\frac{x}{7} = \frac{12}{21}$ (x is to 7 like 12 is to 21) → x · 21 = 12 · 7 → $x = \frac{12 \cdot 7}{21} = 4$

With **direct proportion**, if you multiply or divide two corresponding values by the same number you get another pair of corresponding values:

Quantities	Numbers	Numbers
A	$a \rightarrow$	b
B	$c \rightarrow$	x

If quantities A and B are directly proportional, these numbers satisfy:

$$\frac{b}{x} = \frac{a}{c}$$

We work out: $x = \frac{b \cdot c}{a}$

With **inverse proportion**, if you multiply (or divide) one of the values of a quantity by a number, the corresponding value is divided (or multiplied) by that number:

Quantities	Numbers	Numbers
A	$a \rightarrow$	b
B	$c \rightarrow$	x

If quantities A and B are inversely proportional, these numbers satisfy:

$$\frac{b}{x} = \frac{c}{a}$$ (Only quantities *c* and *a* are swapped)

We work out: $x = \dfrac{a \cdot b}{c}$

Direct - Direct	Inverse - Inverse	Inverse - Direct
Direct Pr. Direct Pr. a ----- b ----- c a′ ----- b′----- x	Inverse Pr. Inverse Pr. a ----- b ----- c a′ ----- b′----- x	Inverse Pr. Direct Pr. a ----- b ----- c a′ ----- b′----- x
$\frac{c}{x} = \frac{a}{a'} \cdot \frac{b}{b'}$	$\frac{c}{x} = \frac{a'}{a} \cdot \frac{b'}{b}$	$\frac{c}{x} = \frac{a'}{a} \cdot \frac{b}{b'}$

A **percentage** is a number or ratio as a fraction of 100. It is often denoted using the percent sign, "%", or the abbreviation "pct."

For example, 45% (read as "forty-five percent") is equal to 45/100, or 0.45.

Simple Interest (SI) Formula

$$SI = \frac{Principal \times Interest\ Rate \times Time}{100}$$

where
– SI is the total simple interest payable
– principal is the sum of money on which the interest to be earned
– interest rate is the percentage at which interest accrued over time
– time is the length of period in years

The language of algebra:

When we use algebra, we work with numerical relationships where one or more quantities are unknown or indefinite. These unknown quantities are called **variables** or **unknowns** and they are represented by letters.

When we translate the terms in a problem into algebraic language, we get **algebraic expressions.**

There are many different kinds of algebraic expressions:

- **Monomials:** $7x^3$, $-\frac{3}{2}x$, $4\pi r^2$ (area of a sphere)
- **Polynomials:** $5x^3 + x^2 - 11$, $2\pi rh + 2\pi r^2$ (total area of a cylinder)

Some algebraic expressions include the symbol '=':

- **Identities:** $5(x + 4) = 5x + 20$. We get the second part of the equality by working with the first part.
- **Equations:** $5(x + 4) = x + 44$. The equality is only true for one of the values of the unknown x. In this case, for $x = 6$.

A monomial is the product of a number multiplied by one or various letters (variables).

In a monomial, *the letters* (the **literal part**) *represent numbers with an unknown value*. They conserve all the properties of the numbers and their operations.

- The **coefficient** of a monomial is the number that multiplies the literal part.
- The **degree** of a monomial is the total number of factors that make up its literal part.

  Numbers are monomials with a degree of zero, since $x^0 = 1$.
- Two **monomials** are **similar** when their literal part is identical.

- The **sum** of similar monomials is another monomial. It is similar to the first two, and its coefficient is the sum of their coefficients.

  For example: $7x^5 + 11x^5 = 18x^5$

  If two monomials are not similar, their sum can not be simplified, and it has to be left as it is. The result, therefore, is not a monomial.

  For example: $7x^5 + 11x^3$ can't be simplified.

- **Subtracting** is similar to adding.

  For example: $3abx^2 - 8abx^2 = -5abx^2$
- The **product** of two or more monomials is another monomial with a coefficient that is the product of their coefficients. Its literal part is the product of the literal part of the factors.

  For example: $(3x^2ab) \cdot (5xac) = 15x^3a^2bc$
- The **quotient** of two monomials is the result of dividing their coefficients and their literal parts. It may or may not be a monomial.

  For example, $\dfrac{3x^5y}{6x^2y} = \dfrac{1}{2}x^3$ is a monomial, but $\dfrac{3x^5y}{6x^2y^4} = \dfrac{x^3}{2y^3}$ is not.

A **polynomial** is the sum of two or more monomials. Each of the monomials that make up the polynomial is called a **term.**

A monomial can be considered a polynomial with only one term.

- If a polynomial has similar monomials, we simplify the expression and find the polynomial in **reduced form.**
- The **degree** of a polynomial is the highest degree of its monomials once it has been reduced.

  We need to reduce the polynomial before we decide its degree, because its largest monomials might be simplified and disappear.
- The **numerical value** of a polynomial for $x = a$ is the number you get when you substitute the x with a. For example, the value of $2x^3 - 5x^2 + 7$ for $x = 2$ is $2 \cdot 2^3 - 5 \cdot 2^2 + 7 = 2 \cdot 8 - 5 \cdot 4 + 7 = 3$.
- If the numerical value of a polynomial for $x = a$ is 0, then we say that a is a **root** of this polynomial.

3.1 Adding and subtracting polynomials

To add two polynomials, we group their terms and add the similar monomials. To subtract two polynomials, we add the minuend and the opposite of the subtrahend. For example, with $A = 6x^2 - 4x + 1$ and $B = x^3 + 2x^2 - 11$:

$$\begin{array}{rcr} A & \rightarrow & 6x^2 - 4x + 1 \\ +B & \rightarrow & x^3 + 2x^2 \qquad - 11 \\ \hline A + B & \rightarrow & x^3 + 8x^2 - 4x - 10 \end{array} \qquad \begin{array}{rcr} A & \rightarrow & 6x^2 - 4x + 1 \\ -B & \rightarrow & -x^3 - 2x^2 \qquad + 11 \\ \hline A - B & \rightarrow & -x^3 + 4x^2 - 4x + 12 \end{array}$$

3.2 Product of a monomial times a polynomial

To multiply a monomial by a polynomial, we multiply the monomial by each term in the polynomial and add the results. For example:

$$(3x^2) \cdot (x^3 - 2x^2 - 1) = 3x^2 \cdot x^3 - 3x^2 \cdot 2x^2 - 3x^2 \cdot 1 = 3x^5 - 6x^4 - 3x^2$$

3.3 The product of two polynomials

To multiply two polynomials, we multiply each monomial from one of the factors by all the monomials in the other factor. Then, we add the similar monomials in the result.

For example: $P = 5x^3 - 2x^2 - 1$, $Q = 6x - 3$

$$\begin{array}{rcl} 5x^3 - 2x^2 \qquad - 1 & \longleftarrow & P \\ 6x - 3 & \longleftarrow & Q \\ \hline -15x^3 + 6x^2 \qquad + 3 & \longleftarrow & \text{product of } -3 \text{ times } P \\ 30x^4 - 12x^3 \qquad - 6x \qquad & \longleftarrow & \text{product of } 6x \text{ times } P \\ \hline 30x^4 - 27x^3 + 6x^2 - 6x + 3 & \longleftarrow & P \cdot Q \end{array}$$

When there are few terms, you do not need to use the above method. You can find the product directly:

$$(2x^2 - 1)\,(3x + 4) = 6x^3 + 8x^2 - 3x - 4$$

3.4 Notable products

We use these names for the three following equalities:

I. $(a + b)^2 = a^2 + b^2 + 2ab$	SQUARE OF A SUM
II. $(a - b)^2 = a^2 + b^2 - 2ab$	SQUARE OF A DIFFERENCE
III. $(a + b) \cdot (a - b) = a^2 - b^2$	SUM TIMES A DIFFERENCE

Equations:

1.1 Equations and their solutions

An **equation** is an equality that has a letter (**unknown**) whose value we want to find out.

The **solution** of an equation is the value of the unknown that makes the equality true.

For example, $2x^2 - \frac{10}{x} = 3$ is an equation.

The value $x = 2$ is the solution, because $2 \cdot 2^2 - \frac{10}{x} = 3$.

1.2 What is solving an equation

Solving an equation is finding the solution (or solutions) or finding out that there is no solution. You already know processes for solving some types of equations methodically. But if we find the solution any other way, it is also valid.

For example, let's find the solution of $x^2 - 5x + 6 = 0$:

- Is the solution $x = 0$? $0^2 - 5 \cdot 0 + 6 = 6 \neq 0 \rightarrow$ NO
- Y $x = 2$? $2^2 - 5 \cdot 2 + 6 = 0 \rightarrow$ YES

Frequently, the equations we have to solve look complicated. For example:

$$\frac{3x-1}{20} - \frac{2(x+3)}{5} = \frac{4x+2}{15} - 5$$

Let's see what steps we have to take to isolate x (on the right, you can see how we solved the equation from the example following these steps):

1. Remove the denominators, if there are any. To do so, you multiply the two members of the equation by a multiple that is common to the denominators, preferably, their least common multiple (LCM).
2. Remove the parentheses, if there are any.
3. Move the terms with x to one member and move the numbers to the other member.
4. Simplify each member.
5. Isolate the x. This is how you get the solution.
6. Check: substitute the solution in each member of the initial equation to make sure the results are the same.

A second-degree equation looks like this:

$ax^2 + bx + c = 0$, where $a \neq 0$

To isolate the x, we follow a long and complicated process that we will not discuss now. The final result is this formula:

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The double sign ($\pm$) means that there can be two solutions:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The two solutions can be reduced to one or none, depending on the case.

Equations of the type $ax^2 + c = 0$

To solve equations of the type $ax^2 + c = 0$, you do not need to apply the quadratic formula, because it is easy to isolate x.

$$x^2 = -\frac{c}{a} \rightarrow x = \pm\sqrt{-\frac{c}{a}}$$

Equations of the type $ax^2 + bx = 0$

To solve equations of the type $ax^2 + bx = 0$ you do not need to apply the quadratic formula, because you can isolate the x as a common factor and make one of the factors equal to zero.

$$x \cdot (ax + b) = 0 \rightarrow \text{Solutions: } x_1 = 0,\ x_2 = -\frac{b}{a}$$

Systems of linear equations:

Two equations form a **system of equations** when our aim is to find a common solution.
If both equations are linear, it is called a linear system.

$$\begin{cases} ax + by = c \\ a'x + b'y = c' \end{cases}$$

2.1 Solving systems of linear equations

SUBSTITUTION METHOD

We isolate an unknown in one of the equations and substitute it in the other. This way we obtain an equation with a single unknown. We solve it. Its solution is substituted in the first equation. For example:

$$\left.\begin{aligned} 3x - 5y &= 1 \\ x + 2y &= 15 \end{aligned}\right\} \to x = 15 - 2y \to 3(15 - 2y) - 5y = 1 \to \dots \to y = 4 \to$$

$$\to x = 15 - 2 \cdot 4 = 15 - 8 = 7$$

Solution: $x = 7,\ y = 4$

EQUALISATION METHOD

We isolate the same unknown in both equations and equalise the results. As in the method above, we obtain an equation with a single unknown. For example:

$$\left.\begin{aligned} 3x - 5y &= 1 \\ x + 2y &= 15 \end{aligned}\right] \begin{aligned} &\to x = \frac{1+5y}{3} \\ &\to x = 15 - 2y \end{aligned}\Bigg\} \to \frac{1+5y}{3} = 15 - 2y \to y = 4$$

$$x = 15 - 2 \cdot 4 = 7$$

Solution: $x = 7,\ y = 4$

REDUCTION METHOD

Both equations are prepared (multiplying by the appropriate numbers) so that one of the unknowns has the same coefficient in both. When we subtract them, we obtain an equation without that unknown. For example:

$$\left.\begin{aligned} 3x + 5y &= 76 \\ 4x + 2y &= 6 \end{aligned}\right\} \begin{aligned} &\xrightarrow{1^{st} \cdot 4} 12x + 20y = 304 \\ &\xrightarrow{2^{nd} \cdot 3} 12x - 6y = 18 \end{aligned}$$

Subtracting: $26y = 286 \to y = 11$

$3x + 5 \cdot 11 = 76 \to x = 7$

Solution: $x = 7,\ y = 11$

Inequations:

Sometimes the statements that result in an algebraic expression do not say 'is equal to', but 'is greater than', 'is less than'. For example, the following expressions are **inequations:**

$2x + 4 > 0$ $\qquad$ $10 - 5x \le 15$

An **inequation** is an algebraic inequality. It has two sides with one of the following symbols between them: $<$, $\le$, $>$, $\ge$.

Any value of the unknown that makes the inequality true is called a **solution.** Solving an inequation is giving all of its solutions.

4.1 Solving first-degree inequations

To solve an inequation, we follow a series of steps: remove the parentheses, remove the denominators, move any x to one side and the numbers to the other…

All of these are valid, and exactly the same, for inequations, except one:

If both sides of an inequation are multiplied or divided by a negative number, the inequality changes direction.

Worked example

Solve the following inequations:

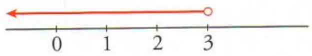
a) $2x + 1 < 7$

b) $7 - 5x \le 12$

a) $2x + 1 < 7 \rightarrow 2x < 6 \rightarrow x < 6 : 2 \rightarrow x < 3$

Solution: x can be any number less than 3.

Set of solutions: $(-\infty, 3)$

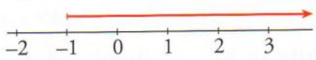


b) $7 - 5x \le 12 \rightarrow -5x \le 12 - 7 \rightarrow -x \le 5 : 5 \rightarrow -x \le 1 \rightarrow x \ge -1$

(As the sign changes, the direction of the inequality also changes).

Solution: x can be -1 or any number greater than it.

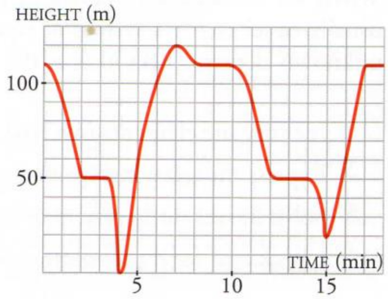
Set of solutions: $[-1, +\infty)$



Functions:

A team of naturalists are observing an eagle. It leaves its nest, hunts a rabbit, returns to its nest, leaves again, hunts a pigeon, and once again returns to its nest.

If we look closely at the graph on the right, we can see a lot of information: the height of the nest, the height the eagle tends to fly at to look for prey, the moment it decides to attack its prey, etc.



TWO VARIABLES, TWO AXES

The graph that describes the flight of the eagle relates two variables:

- The time that has passed since the observation started, t.
  This is the **independent variable.**
- The height of the eagle, a. This is the **dependent variable.**
  This has been represented on a Cartesian axis.
- The time, t, is shown on the **horizontal axis** or **abscissa.**
- The height, a, is shown on the vertical **axis**, or **ordinate**.

Each point on the graph represents a time and a height, and means that at that moment in time the eagle was at that height.

If we analyse the graph, we can see when the eagle ascends and descends during its flight, and we can describe these movements in detail.

SCALES

Each axis has a **scale.**

- On the horizontal axis, 1 square represents 1 minute.
- On the vertical axis, 1 square represents 10 metres.

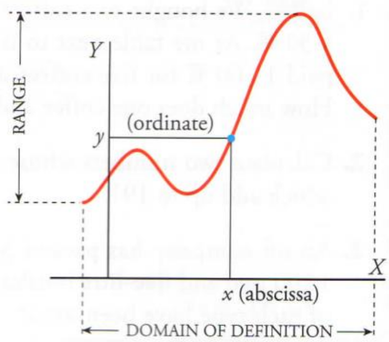
The scales of the axes allow us to give a qualitative description of the behaviour of the eagle, as well as to quantify it. For example, the maximum height the eagle reached while it was being observed is 120 m, and this happened in the 7th minute.

DOMAIN OF DEFINITION AND RANGE

The graph showing the flight of the eagle goes from minute 0-18. We only have information on the behaviour of the eagle during this period of time.

Interval 0-18 is called the **domain of definition** of the function.

The height of the eagle ranges varies between 0 m and 120 m. The distance 0-120 is called the **range** of the function.



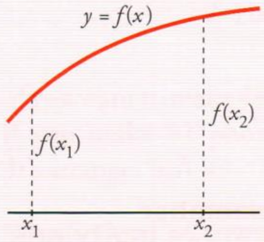
FUNCTION

A **function** is a relationship between two variables, which we generally call x and y.

The function associates each value of x with a **unique** value of y.

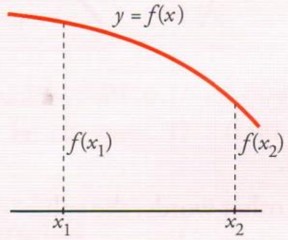
The function f is **increasing** in this interval because:

$$\text{if } x_1 < x_2, \text{ then } f(x_1) < f(x_2)$$



In contrast, the function f is **decreasing** in this interval because:

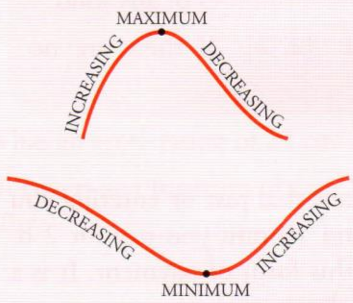
$$\text{if } x_1 < x_2, \text{ then } f(x_1) > f(x_2)$$



A function can be increasing in some intervals and decreasing in others.

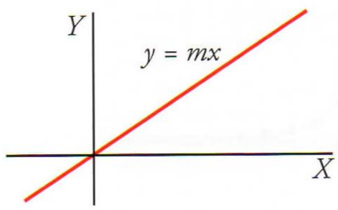
A function has a **relative maximum** at a point when the value of that point is higher than the values of all the points close to it. In such cases, the function is increasing up to the maximum, and decreasing after it.

Conversely, if f has a **relative minimum** at one point, it is decreasing before that point and increasing after it.



Linear functions:

FUNCTIONS OF PROPORTIONALITY: $y = mx$



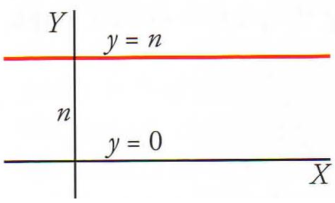
Functions of proportionality are graphed with straight lines that pass through the origin. They describe a ratio between the values of both variables.

The slope of the straight line is the constant of proportionality, m.

For example, the *space covered* at a constant velocity as a function of *time* is:

$s = v \cdot t$, where v is the slope of the straight line that relates s to t.

CONSTANT FUNCTION: $y = n$



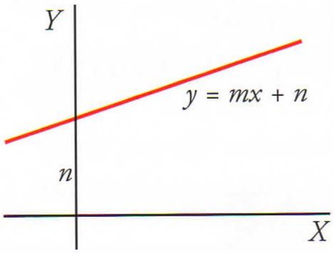
A constant function is represented with a straight line parallel to the X axis.

Its slope, or gradient, is 0.

Line $y = 0$ coincides with the X axis.

■ **GENERAL EXPRESSION: $y = mx + n$**

This is represented on a graph as a straight line with slope m that intersects the Y axis at point $(0, n)$. The number n is called the **ordinate at the origin.**



For example, the straight line $°F = 32 + 1.8\ °C$, represented in the graph in the margin, allows us to change from a temperature in Celsius (centigrade), $°C$, to the corresponding temperature in Fahrenheit, $°F$.

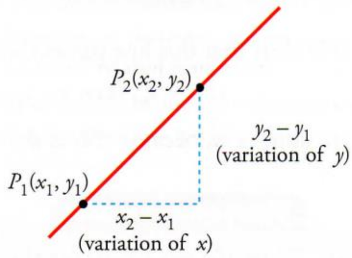
Point: $P(x_0, y_0)$ Slope: m Equation: $y = y_0 + m(x - x_0)$

4.1 Finding the slope using two points

We can use the graph of a straight line that passes through two known points to find its slope. By measuring (or counting squares) we can find the variation of x and y.

However, it is quicker and more efficient to calculate it like this:

$$\left.\begin{array}{l} P_1(x_1, y_1) \\ P_2(x_2, y_2) \end{array}\right\} \rightarrow m = \frac{\text{variation of } y}{\text{variation of } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



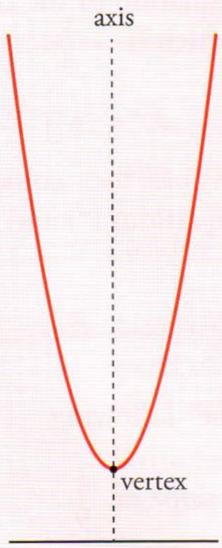
Other functions:

The functions $y = ax^2 + bx + c$, with $a \neq 0$, called **quadratic functions,** are all graphed using **parabolas** and are continuous in all $\mathbb{R}$.

Each of these parabolas has an axis parallel to the Y axis.

Their shape depends on a, the coefficient of x^2, in the following way:

- If two quadratic functions have the same coefficient of x^2, their corresponding parabolas are identical, although they can be located in different positions.
- If $a > 0$, their branches point upwards, and if $a < 0$, they point downwards.
- The larger the magnitude of $|a|$, the narrower the parabola.



Worked example

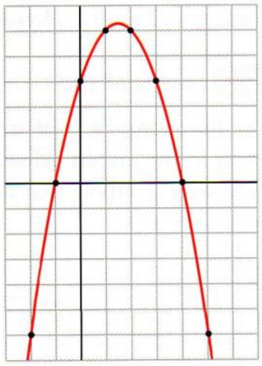
Graph the parabola for the equation $y = -x^2 + 3x + 4$.

First, we find the vertex:

Abscissa: $p = -\frac{3}{-2} = 1.5 \rightarrow$ Ordinate: $f(1.5) = 6.25 \rightarrow$ Vertex: (1.5; 6.25)

Then, we find points close to the vertex:

x	–2	–1	0	1	2	3	4	5
y	–6	0	4	6	6	4	0	–6



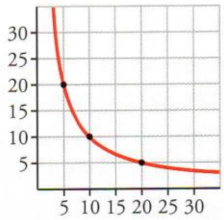
Note how the symmetry of the parabola with regard to its axis means the points at the same distance from the vertex coincide in the value of their ordinate. In other words, as the vertex is at $x = 1.5$, then $f(1) = f(2)$; $f(0) = f(3)$...

We see that $-x^2 + 3x + 4 = 0$ has two solutions, $x = -1$ and $x = 4$; and that $f(0) = 4$. However, these points of intersection with the axes already appear in the table.

3.1 Inversely proportional functions

A rectangle has an area of 100 cm^2, but we do not know the length of its sides. We call them x and y. Obviously, $xy = 100$. We write it in this way:

$y = \frac{100}{x}$ (While the area remains the same, the sides are inversely proportional).

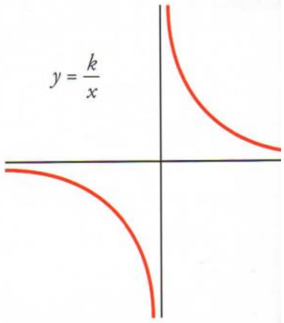


Relationships of inverse proportionality, like the one above, are often found in nature, physics, economics, etc. Let's look at the theory.

The functions $y = \frac{100}{x}$ have the following characteristics:

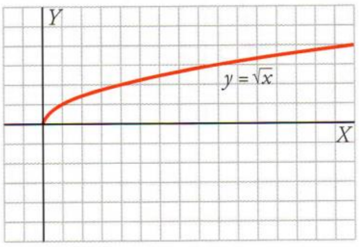
- They are not defined in $x = 0$.
- If x gets close to 0, y takes on ever-larger values. This is why we say that the Y axis is an **asymptote.**
- If x takes ever-increasing values, y gets closer to 0. This is why the Y axis is another asymptote.

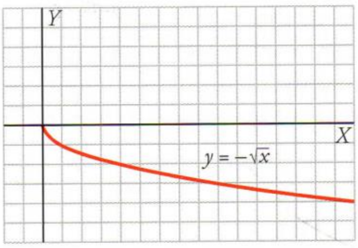
This curve is called a **hyperbola.**

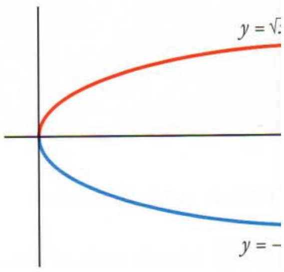


3.2 Radical functions

The functions $y = \sqrt{x}$ and $y = -\sqrt{x}$ can be plotted point by point and result in the graphs below. They are halves of parabolas and together they describe a parabola that is identical to $y = x^2$, but with the X axis as its symmetry axis.







The domain of definition for these functions is $[0, +\infty)$.

4.1 Increasing exponential functions: $y = a^x$, $a > 1$

The graph of the exponential function with base 2: $y = 2^x$ is in the margin.

$x \geq 0$:

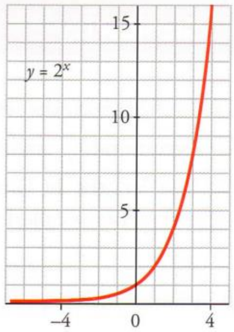
x	0	1	2	3	4	…
2^x	1	2	4	8	16	…

When x takes on increasingly larger values, 2^x tends towards infinity.

$x \leq 0$:

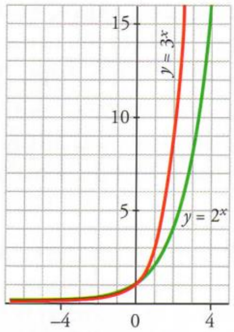
x	-1	-2	-3
2^x	$2^{-1} = \frac{1}{2} = 0.5$	$2^{-2} = \frac{1}{4} = 0.25$	$2^{-3} = \frac{1}{8} = 0.125$

When x takes the values $-4, -5, -6, -10, \ldots$, 2^x becomes very small. In other words, to the left, 2^x tends towards zero.



Exponential functions are those that have the equation $y = a^x$.

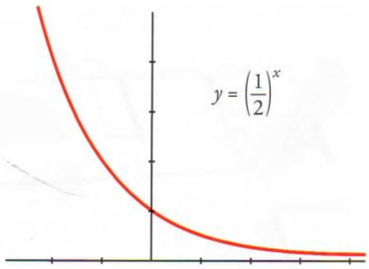
- All of them are continuous, are defined in all $\mathbb{R}$ and pass through points (0, 1) and (1, a).
- If the base is greater than 1 $(a > 1)$, then they are increasing.
- They increase more quickly the greater a is.



$y = 3^x$ increases more quickly than $y = 2^x$.

4.2 Decreasing exponential functions: $(0 < a < 1)$

The function $y = \left(\frac{1}{2}\right)^x$ is also an exponential function. As its base (1/2) is less than 1, the function is decreasing.



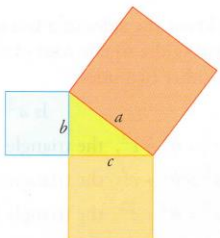
The functions $y = a^x$ with $0 < a < 1$ also pass through (0, 1) and (1, a), are continuous and are defined in all $\mathbb{R}$, but they are decreasing. They decrease more quickly the closer a is to zero.

Geometry:

1 Pythagoras' theorem

The two shortest sides of a right-angled triangle form a right angle. They are called the **legs.** The longest side is called the **hypotenuse.**

In general, we say that a is the hypotenuse and b and c are the legs.



According to **Pythagoras' theorem:** $a^2 = b^2 + c^2$

This means that the area of the square of the hypotenuse is equal to the sum of the areas of the squares of the legs.

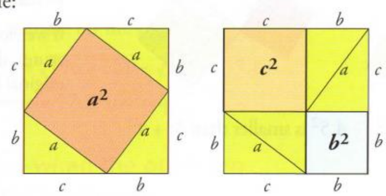
This relationship is only true if the triangle is right-angled.

Amazing fact

This observation was made by the Chinese 400 years before Pythagoras was born.

Look at the following example:

Two identical squares have $b + c$ as sides. If we compare both decompositions, it is clear that $a^2 = b^2 + c^2$.



On the Web

Graphical demonstration of Pythagoras' theorem.

2.1 Thales' theorem

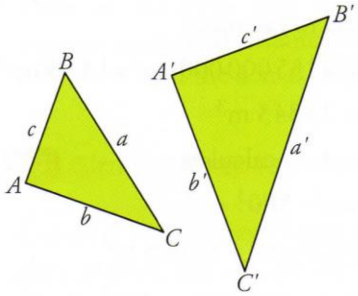
If a, b and c are parallel lines and they intersect with another two straight lines, r and s, then the segments they create are in proportion.

$\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{A'B'}}{\overline{B'C'}}$. As a result, we can verify that: $\frac{\overline{AB}}{\overline{A'B'}} = \frac{\overline{BC}}{\overline{B'C'}} = \frac{\overline{OA}}{\overline{OA'}}$

The opposite also occurs: if the segments $\overline{AB}$ and $\overline{BC}$ are proportional to $\overline{A'B'}$ and $\overline{B'C'}$ and the lines a and b are parallel, then line c is parallel to them.

Thales' theorem can be used to study the similarity of triangles.

2.2 Similar triangles



Two **similar triangles** have:

- Proportional sides:

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = \text{similarity ratio}$$

- Equal angles:

$$\hat{A} = \hat{A}',\ \hat{B} = \hat{B}',\ \hat{C} = \hat{C}'$$

2.3 Thales' theorem

Triangles ABC and $AB'C'$, on the right, have a common angle, $\hat{A}$. In other words, the smaller triangle fits into the larger one.

In addition, the sides opposite $\hat{A}$ are parallel.

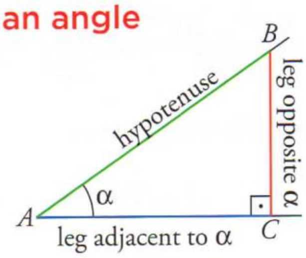
We say that these two triangles are in **Thales' position.**

Two triangles in Thales' position are similar.

1.1 The sine, cosine and tangent of an angle

We draw a right-angled triangle, ABC, on an acute angle, α, like the one on the right.

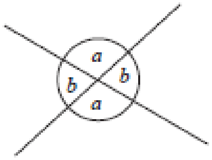
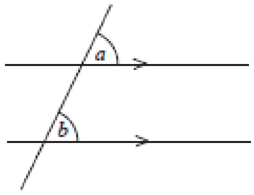
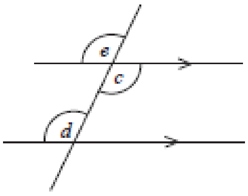
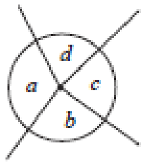
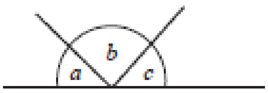
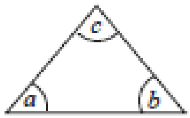
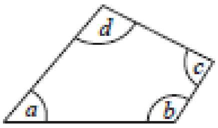
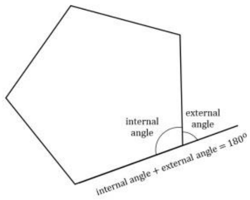
Look carefully at the following relationships called the **trigonometric ratios** of the angle α.

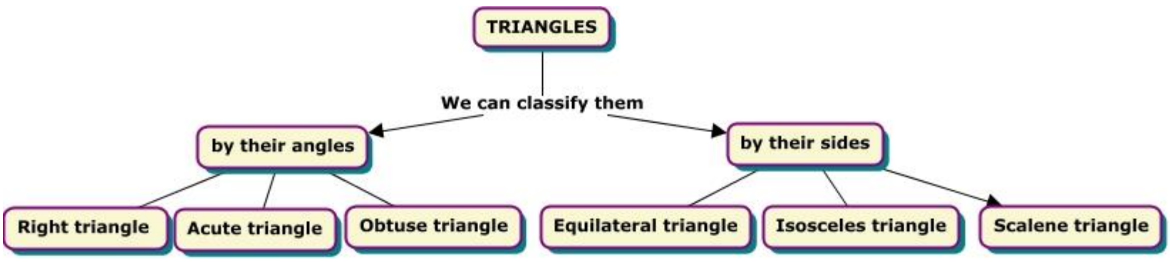


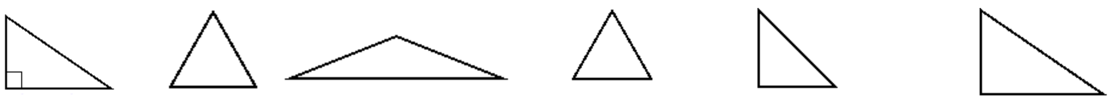
sine of $\alpha = \dfrac{\text{length of leg opposite } \alpha}{\text{length of the hypotenuse}}$ $\qquad sin\ \alpha = \dfrac{\overline{BC}}{\overline{AB}}$

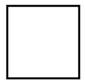
cosine of $\alpha = \dfrac{\text{length of leg adjacent to } \alpha}{\text{length of the hypotenuse}}$ $\qquad cos\ \alpha = \dfrac{\overline{AC}}{\overline{AB}}$

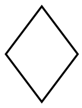
tangent of $\alpha = \dfrac{\text{length of leg opposite } \alpha}{\text{length of leg adjacent to } \alpha}$ $\qquad tan\ \alpha = \dfrac{\overline{BC}}{\overline{AC}}$

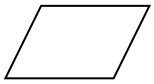
ANGLES	
Opposite angles	***Corresponding angles***
	
Whe any two lines cross, two pairs of equal angles are formed. The two angles marked **a** are a pair of opposite equal angles. The angles marked **b** are also a pair of opposite equal angles.	When a line crosses a pair of parallel lines, **a** = **b**. The angles **a** and **b** are called corresponding angles.
Alternate angles	***Angles at a point***
	 $a+b+c+d=360°$
The angles **c** and **d** are called *alternate angles*. They are equal.	The angles at a point will always add up to 360 º. It does not matter how many angles are formed at the point – their total will always be 360 º.
Angles on a line	***Angles in a triangle***
 $a+b+c=180°$	 $a+b+c=180°$
Any angles that form a straight line add up to 180º.	The angles in any triangle add up to 180º.
Angles in a quadrilateral	***Angles of a regular polygon (n sides)***
 $a+b+c+d=360°$	
The angles in any quadrilateral add up to 360º.	Sum of interior angles = (**n**-2) × 180**°**

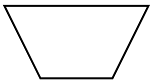




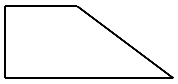

SQUARE

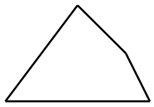

RHOMBUS

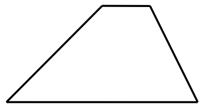

RHOMBOID


ISOSCELES TRAPEZIUM


RECTANGLE


RIGHT TRAPEZIUM


TRAPEZOID


SCALENE TRAPEZIUM

POLYGONS	
3-sided polygon	**4-sided polygon**
triangle	quadrilateral
5-sided polygon	**6-sided polygon**
pentagon	hexagon
7-sided polygon	**8-sided polygon**
heptagon	octagon
9-sided polygon	**10-sided polygon**
nonagon	decagon

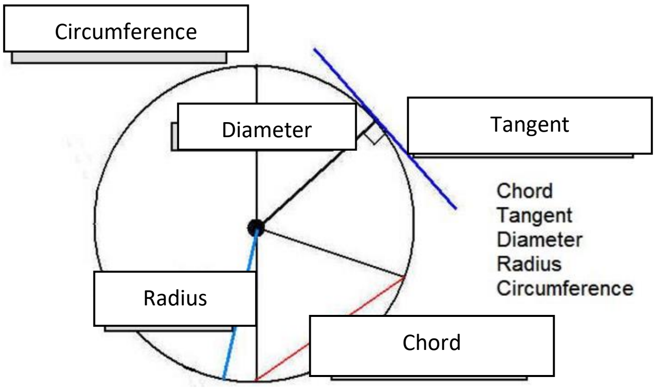
The perimeter of a circle is called the **circumference**. The centre of the circumference is the point from which all the points of the circumference are equidistant.

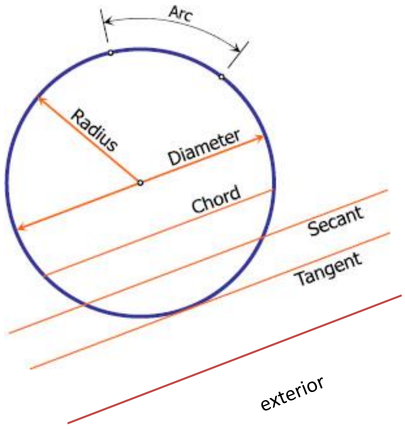
The **radius** of the circumference is the line segment that joins the circumference with the centre.

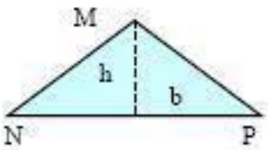
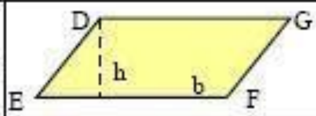
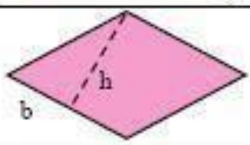
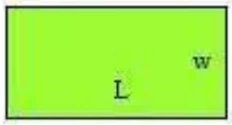
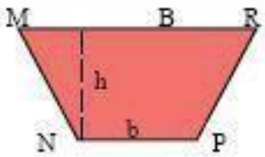
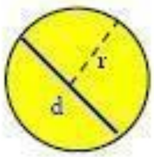
A **chord** is a line segment that joins two points of the circumference.

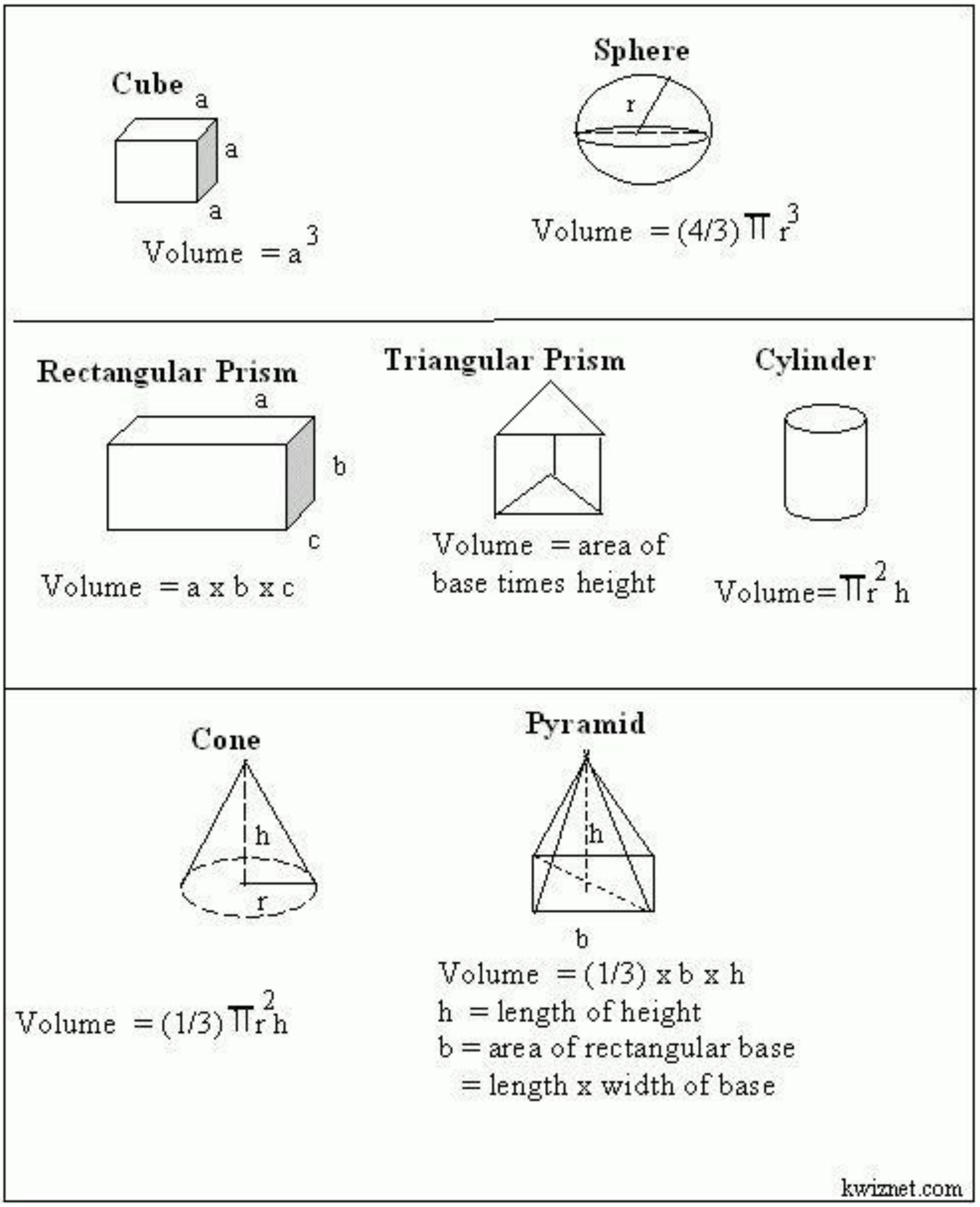
A **diameter** is a cord that passes through the centre of the circumference.

An **arc** is a part of the circumference, the curve line between two points of the circumference.





NAME	FIGURE	AREA	PERIMETER CIRCUMFERENCE
TRIANGLE		$A = \frac{b \times h}{2}$	$P = MN + NP + PM$
PARALLELOGRAM		$A = b \times h$	$P = DE + EF + FG + GD$
RHOMBUS		$A = b \times h$	$P = b + b + b + b$ $P = 4b$
RECTANGLE		$A = L \times w$	$P = L + w + L + w$ $P = 2L + 2w$
SQUARE		$A = l^2$	$P = l + l + l + l$ $P = 4l$
TRAPEZOID		$A = \frac{(B + b) \times h}{2}$	$P = MN + NP + PR + RM$
CIRCLE		$A = \pi r^2$	$C = 2\pi r = \pi d$

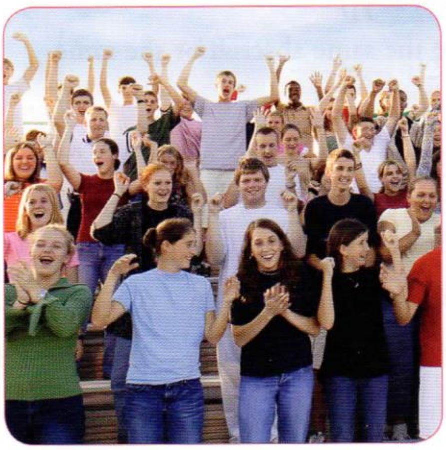

Cube
a
a
a
Volume = a3
Sphere
r
Volume = (4/3) π r3
Rectangular Prism
a
b
c
Volume = a x b x c
Triangular Prism
Volume = area of base times height
Cylinder
Volume=πr2 h
Cone
h
r
Volume = (1/3) πr2h
Pyramid
h
r
b
Volume = (1/3) x b x h
h = length of height
b = area of rectangular base
= length x width of base
kwiznet.com

Statistics:

Statistics aims to develop techniques for numerical understanding of a set of empirical data (collected in experiments or surveys).

Let us go over some of the basics. Some of these concepts you learnt many years ago and are essential to understanding statistics.

1.1 Basic concepts



- **The population** is the entire set of elements about which we wish to learn and which shall serve as the object of our study.

  For example, a population can be the 1 580 students in a school.

- **An individual** is each of the elements in a population.

  In the case of the school, each student is an individual.

- **A sample** is a subset of the population. Studies of a subset provide information on the characteristics of the population as a whole.

  Continuing with the example above, a sample could be a relevant group of 50 students.

- **Characteristics** are the aspects which we wish to study in the individuals in a population. Each characteristic or attribute can have different values.

  With reference to the students, we could analyse characteristics such as age, height, weight, hair colour, how they get to school (means of transport), the number of people living in their home, their preferred genre of books, etc.

- **A variable** covers all the values of a given characteristic. A variable can be:
  - **Quantitative** if it takes on numerical values.
    - **Discrete:** only takes on isolated values.
    - **Continuous:** it can take on any value within an interval.
  - **Qualitative** (or categorical) if it does not take on numerical values.

  The number of people who live at home takes on discrete quantitative values. Age, height and weight all take on continuous quantitative values. Hair colour and the means of transport used to get to school assume qualitative values.

Descriptive statistics presents and analyses some characteristics of the individuals in a given group (population) without coming to any conclusions with regards to a larger group.

■ **MEAN:** $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$ $\Sigma f_i x_i \rightarrow$ the total sum of all the data; $\Sigma f_i = N \rightarrow$ total no. of individuals

For example, in the distribution in the margin:

$\Sigma f_i = 288$. There are 288 individuals (those who did the test).

$\Sigma f_i x_i = 766$. This is the total sum of the results of all the individuals.

The mean is $\bar{x} = 766/288 = 2.66$.

■ **VARIANCE** $Var = \dfrac{\Sigma f_i (x_i - \bar{x})^2}{N}$ or $Var = \dfrac{\Sigma f_i x_i^2}{N} - \bar{x}^2$

The two expressions are the same (refer to the demonstration on the web).

— In the first case, the meaning of variance is clear: the average of the squared deviations from the mean.

— It is easier to do the calculations in the second.

Let us use the second formula to obtain the variance based on this table:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	12	0	0
1	31	31	31
2	86	172	344
3	92	276	828
4	48	192	768
5	19	95	475
	288	766	2 446

$$Var = \frac{2\,446}{288} - 2.66^2 = 1.42$$

■ **STANDARD DEVIATION:** $\sigma = \sqrt{variance}$

Standard deviation is a more reasonable parameter than variance since it is expressed in the same magnitude as the data and the mean (for example, if the data is in centimetres, the standard deviations will also be in centimetres; however, variance would be expressed in centimetres square).

In the example: $\sigma = \sqrt{1.42} = 1.19$

■ **COEFFICIENT OF VARIATION:** $C.V. = \dfrac{\sigma}{\bar{x}}$

The coefficient of variation is used to compare dispersion in heterogeneous populations. It indicates *relative variability.*

In the example: $C.V. = \dfrac{1.19}{2.66} = 0.447$. Or 44.7 %.

Chance and probability:

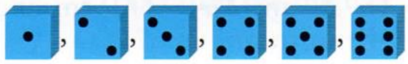
In everyday life, many things happen that cannot be predicted. They depend on chance. When an event depends on chance, we call it a **random event**. For example:

DEPENDS ON CHANCE	DOES NOT DEPEND ON CHANCE
It will rain tomorrow.	The sun will rise tomorrow.
My team will win the league.	We will play football.
When I toss a coin, it will land heads up.	When I toss a coin, it will fall.
I will win the lottery.	Someone will win the lottery.

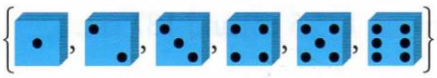
1.1 Random experiences

To study chance and its properties, we can try **random experiences**. In other words, experiments whose results depend on chance. For example, let's study the random experience of rolling a die and observing the result.

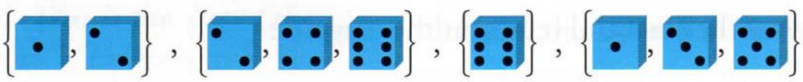
- **Case.** Each of the results that we can get when we complete a random experience is called a **case**.

  When *we roll a die,* the possible cases are: 

- **Sample space.** The group of all possible cases is called the **sample space,** which we represent as E.

  On a die, the sample space is: $E =$ 

- **Events.** Sub-groups of the sample space are called **events.** Many different events are possible as a result of *rolling a die*. Some of them are:



We perform a random experience with a regular instrument.

The sample space has n elements (cases) so, the probability of each case is $1/n$.

S is an event that includes k elements.

So, the probability of S is: $P[S] = \dfrac{k}{n}$

This can be expressed in the following way:

$$P[S] = \frac{\text{number of cases which favour } S}{\text{total number of cases which are possible}}$$ LAPLACE THEORY

If you want to learn more…

Saying numbers:

1 $11 + 3 - 6 = 8$; $7 \times 6 = 42$; $27 \div 3 = 9$

2 0.0305; 0.2222222222 (correct to 10 dp) or $0.\overline{2}$

3 $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{5}{8}$; $\frac{9}{8}$

4 $\frac{1}{7} \approx 0.142857$

5 $\frac{x}{y}$

6 2,718; 1,618

7 $\pi \approx 3.14$; $e \approx 2.718$; $\varphi \approx 1.618$

8 43.5%

9 -35°C; 89°F; 0°C ≡ 32°F

10 $2\pi r$; πr^2

11 x^2; x^3; x^n

12 $\frac{1}{1000} = 0.001 = 10^{-3}$

13 $\sqrt{8} \approx 2.828$

14 $\sqrt[3]{8} = 2$

15 $\sqrt[n]{x}$

16 4:3

17 $x \neq 0$; $\therefore\ x \to \infty$; $\because\ y = 3$

18 $x > y$; $y < z$; $3 < x < 4$

19 $x + y + z \leq 1$; $n \geq 1$

20 $x \propto y$; Intensity $\propto \frac{1}{\text{distance}^2}$

Saying numbers (2) (pages 71-73)

1 eleven plus (add) three minus (take away, subtract, negative) six equals (is equal to, makes, is) eight; seven times (multiplied by) six equals forty two; twenty seven divided by three equals nine.
+, –, x and ÷ are *operators* and are symbols to show which operation needs to be done.

2 nought /nɔ:t/ point oh three oh five; nought point two two two etc. correct to ten decimal places, or nought point 2 repeating/recurring. This is a repeating/recurring/periodic decimal. The bar can be above a single repeating digit or repeating block of digits.

3 a half, a quarter, three quarters, one eighth, five eighths, nine eighths (It is also possible to say one over two, one divided by two, etc.). These are *common fractions*. The line that separates the fraction is called the *fraction bar*. The top number is the *numerator* and the bottom number is the *denominator*. If the numerator is smaller than the denominator it is a *proper fraction*. If the numerator is larger than the denominator, it is an *improper fraction*.

4 one over seven (or the *reciprocal* of seven) is approximately equal to nought point one four two eight five seven.

5 x over y, or x divided by y, or x upon y (less common).

6 two thousand seven hundred and eighteen; one thousand six hundred and eighteen. Note that the word "hundred" is followed by "and". Note also that the comma signifies thousands. However, in the SI (Système International) system of units, the comma denotes the decimal marker.

7 pi /paɪ/ is approximately equal to three point one four; *e* is approximately equal to two point seven one eight (and is known as Euler's number/constant, or Napier's constant, and is the base of natural logarithms); phi /faɪ/ is approximately equal to one point six one eight (and is the *golden ratio/mean/section*). Note that we say a decimal is "correct to three decimal places (3dp)".

8 forty three point five per cent

9 minus thirty five degrees Celsius/centigrade; eighty nine degrees Fahrenheit; zero degrees Celsius/centigrade is identical to thirty two degrees Fahrenheit.

10 two pi /paɪ/ r (the circumference of a circle); pi r squared (the area of a circle)

11 x squared (or x multiplied by itself, or x to the power of two, or x to the second power); x cubed (or x to the power of three, etc.); x (raised) to the power of n, x to the nth power, x to the nth. The word *power* is identical to *exponent*.

12 one thousandth equals nought point oh oh one equals ten to the (power of) minus (negative) three. NB: 1/1000 is a *decimal fraction*, i.e. the denominator is a power of ten.

13 the square root of eight is approximately equal to two point eight two eight

14 the cube root of 8 (the third root of eight) equals two

15 the nth root of x

16 the ratio of four to three

17 x is not equal to (does not equal) zero; therefore x tends to infinity;
because y equals three [Note: $\neq$ $>$ $\geq$ $<$ $\leq$ are called *inequality symbols*]

18 x is greater than y; y is less than z; x is greater than 3 and less than 4, i.e. it is somewhere between 3 and 4.

19 x plus y plus z is less than or equal to one; n is greater than or equal to one

20 x is proportional to y; intensity is inversely proportional to the square of the distance from the source of that physical quantity. This is the inverse square law.

Saying numbers (3)

1 OH, ZERO, NOUGHT

The above are all ways of saying 0 in English.

We say oh	after a decimal point	5.03	five point oh three
We say nought	before the decimal point	0.02	nought point oh two
We say zero	for the number	0	the number zero
	for temperature	-5°C	five degrees below zero

Now say the following:

1 The exact figure is 0.002.

4 Do we have to hold the conference in Iceland? It's 30 degrees below 0!

2 THE DECIMAL POINT

In English, we use a point (.) and not a comma (,) for decimals. We use comas in figures only when writing thousands.

10,001	is ten thousand and one.
10.001	is ten point oh oh one.

In English all the numbers after a decimal point are read separately:

10.66	ten point six six
0.325	nought point three two five
0.001	nought point oh oh one

You will also hear people say:

0.05 zero point oh five OR oh point oh five

But if the number after the decimal point is a unit of money, it is read like a normal number:

£12.50 twelve pounds fifty

Now say the following:

1 It's somewhere between 3.488 and 3.491.

2 Look, it's less than 0.0001 ! It's not worth worrying about.

3 Did you say 0.225 or 0.229?

4 No, I mean 15.005 not 15,005

How to Speak Mathematics

Greek Alphabet

Capital letter	Small letter	Pronunciation	Capital letter	Small letter	Pronunciation	Capital letter	Small letter	Pronunciation	Capital letter	Small letter	Pronunciation
Α	α	/ælfə/	Η	η	/i:tə/	Ν	ν	/nju:/	Τ	τ	/taʊ/
Β	β	/bi:tə/	Θ	θ ϑ	/θi:tə/	Ξ	ξ	/saɪ/	Υ	υ	/ɪpsɪlən /
Γ	γ	/gæmə/	Ι	ι	/aɪəʊtə/	Ο	o	/əʊmaɪkrən/	Φ	φ	/faɪ/
Δ	δ	/deltə/	Κ	κ	/kæpə/	Π	π	/paɪ/	Χ	χ	/kaɪ/
Ε	ε	/epsɪlən/	Λ	λ	/læmdə/	Ρ	ρ	/rəʊ/	Ψ	ψ	/psaɪ/
Ζ	ζ	/zi:tə/	Μ	μ	/mju:/	Σ	σ ς	/sɪgmə/	Ω	ω	/əʊmɪgə/

Substitution tables

Reading mathematics

1, 206, n, π, …	plus		1, 206, n, π, …	equals / is
	minus			
	multiplied	by (with)		
	times			
	divided	by (through)		
	squared / cubed			
	(raised) to	the	power of 0, 1, 206, n, π, … / 4^{th}, …, n^{th} power	
The square root / The cube root / The 4^{th} root / The n^{th} root	of	1, 206, n, π, …		

Giving instructions

Add	1, 206, n, π, …	to	1, 206, n, π, …
Subtract		from	
Multiply		by (with)	
Divide		by (through)	
Square			
Cube			
Raise		to the	power of 0, 5, a, …, z / 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, …, n^{th} power
Take	the square root / the cube root / the 4^{th} root / the n^{th} root	of	1, n, π, …

List of Typical Symbols that May Cause Problems to Read Aloud

Symbol	Meaning / pronunciation	Example
0	nought, zero, /əʊ/	
$=$	equals / is equal to	Five plus three equals eight / Area A is equal to area B.
$\neq$	does not equal / is not equal to	
$\approx$	is about / approximately equal to	One third is about nought (zero) point three
$\cong$	congruent to	
$\triangleq$	corresponds to	
$\sim$, $\propto$	proportional to	
$<$	less than / smaller than	Three is less than five
$>$	greater than	Five is greater than three
$\leq$	less than or equal to / smaller than or equal to	
$\geq$	greater than or equal to	
( )	(round) brackets / parentheses	
[ ]	square brackets	
{ }	curly brackets / wavy brackets	
$\parallel$	parallel to	
$\perp$	perpendicular to	
$A \Rightarrow B$	from A follows B / if A then B	
$\Leftrightarrow$	if and only if / iff / equivalent to	
$\frac{n}{d}$	fraction: n = numerator, d = denominator	$\frac{2}{3}$ = two thirds; n over d
$\sqrt{\ }$	square root of	The square root of nine is three
$\sqrt[3]{\ }$, $\sqrt[4]{\ }$, $\sqrt[n]{\ }$	cube root, fourth root, nth root	The cube root of 8 is 2
$\lvert -5 \rvert$	modulus (absolute value)	The modulus of -5 is 5
$n!$	n factorial	$n! = n\cdot(n-1)\cdot(n-2)\ldots\cdot 3\cdot 2\cdot 1$
$\binom{n}{k}$	n choose k	$\binom{n}{k} = \frac{n!}{(n-k)!\cdot k!}$
π	/paɪ/	$2\pi r$, πr^2
$\sum_{n=1}^{5} n^2$	Sum (Greek sigma)	Sum from n equals 1 to 5 over / of n squared
$f(x)$	/ef əv eks/	
$f'(x)$	f prime of x; f dash	
$f''(x)$	f two prime of x / f double prime of x; f double dash	
$\frac{dy}{dx}$	/di: waɪ əʊvə di: eks/ (Derivative)	
$\int_0^3 f(x)\,dx$	Integral from nought (zero) to three over /ef əv eks di: eks/	

25 Numbers

A Types of numbers

Numbers in a group together may be called a series or **set of numbers**. If the order in which they occur is significant then they may be called a **sequence of numbers**. 1, 4, 9, 16, 25 is a sequence of numbers, for example – it represents the numbers 1 to 5 **squared**.

1, 3, 5, 7 … = **odd numbers**; 2, 4, 6, 8 … = **even numbers**; 2, 3, 5, 7, 11 … = **prime numbers**. The highest number in a group is the **maximum** and the lowest is the **minimum**. *The room holds a **maximum** of 50 and we won't run the class without a **minimum** of 12 students.*

An **approximate** number is one which is roughly correct but is not the precise or exact **number**. *Look at the figures and work out in your head what the **approximate** answer is likely to be. Then use a calculator to find the **exact** number.*

An **aggregate** is a number reached by totalling a set of numbers = the **total**. *The average mark achieved in the exam is calculated by taking the **aggregate** of all the marks and dividing by the number of exam entries.*

A **discrete** number or unit is something which is separate and cannot be divided into smaller numbers or units of the same thing. The opposite of discrete is **continuous**. A bag of apples, for example, could be considered as consisting of discrete items whereas apple sauce could be considered – by mathematicians, at least – as continuous.

A **constant** number or quantity is one that does not change. *In the experiment we varied* [changed] *the amount of water in the beaker but kept the amount of salt added **constant**.* A **random number** is one chosen by chance, i.e. it is not predictable.

B Working with numbers

The word **figure** is often used to refer to the symbol used for a number. *Write the total number in words and **figures**.*
Verbs that are frequently used with the word **number** include **calculate** [work out] a number, **estimate**[1] **a number, round a number up/down**[2], **total** [add up] a set of numbers. Numbers can also **tally**[3]. *My figures don't seem to **tally** with yours.* You can also **deduct** [take away, **subtract**] one number from another number.

[1] make a rough guess at [2] make a **fraction**, e.g. $\frac{1}{8}$ or 0.78 into the nearest **whole number**
[3] match, agree

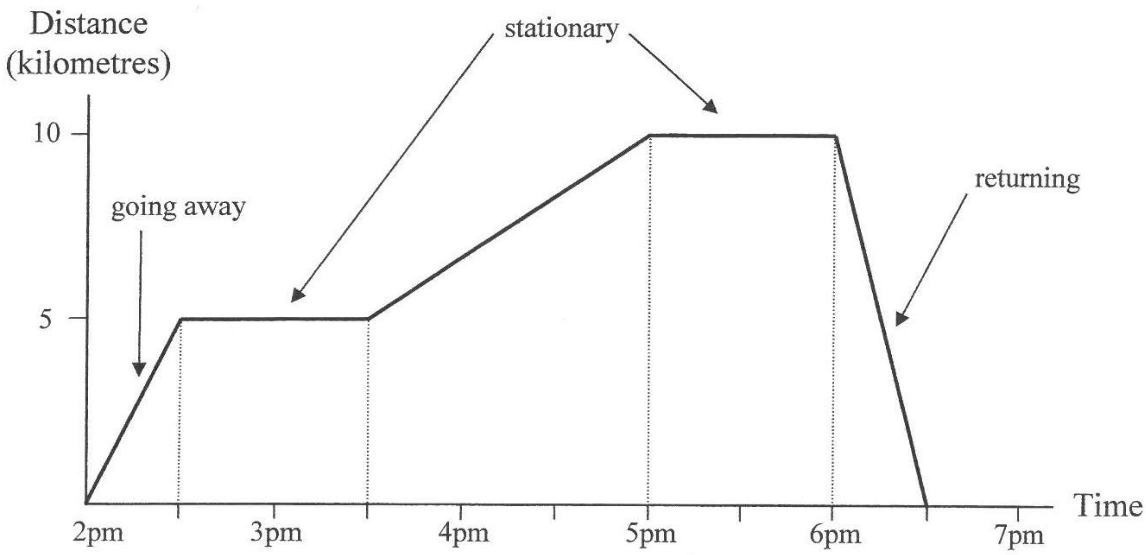
Values and **variables** are also useful terms when working with numbers. **Values** are individual numbers in a set of data. *The graph shows the temperature **values** for different months of the year.* **Variables** are characteristics that can take on different values for different members of a group or set being studied. *In investigating living standards you must take key **variables** such as social provision and cost of living into account.*

The **incidence** of something refers to how frequently it occurs. *The **incidence** of twins in the population is growing.* When talking about numbers, **magnitude** simply refers to the size of something, whereas in other contexts it indicates large size or importance. *Write down the numbers **in order of magnitude**, beginning with the smallest.*

When **making calculations** in, say, an exam, it is often a good idea to make an estimate[4] first of what the answer is likely to be. Then you will see if your final answer is **in the right area**[5] or not. Exam candidates are also often advised to **show their workings**[6] so that the marker can see how they **arrived at their answer** and they may get credit for their method even if the final answer is incorrect.

[4] rough guess [5] approximately the same [6] leave all their calculations on the page

Travel graphs: wall copy



1 A travel graph is always distance against time.

2 The gradient/slope of a line on the graph represents the speed of an object, i.e. how fast it is moving, e.g. 70km/h (kilometres per hour).

3 The speed of an object $= \dfrac{\text{change in the vertical axis (distance)}}{\text{change in the horizontal axis (time)}}$

4 The steeper the graph, the greater the speed of the object.

5 The horizontal lines on the graph are where the object is not moving, i.e. it is stationary.

6 The line representing the return journey slopes downwards.

7 When an object is moving at a constant speed, the line on the graph is straight, but sloped.

NB: The *speed* of an object is how fast it is moving, e.g. 40km/h, and is a scalar quantity, whereas the *velocity* of an object is its speed in a particular direction, and is a vector quantity, e.g. 40km/h east.

Question

Now work with your partner. What is the speed of the return journey?

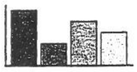
Answer

Speed = gradient = $\dfrac{\qquad\qquad}{\qquad\qquad}$ = _______ metres/minute = ____ km/h.

27 Graphs and diagrams

A Types of diagrams


pie chart

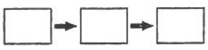

bar chart


histogram

Number	Amount
1	10
2	5
3	20

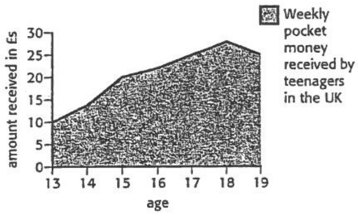
table


cross-section


flowchart

Diagrams are visual ways of **presenting data** concisely. They are often also called **figures**. In an academic article they are usually **labelled** Fig. (Figure) 1, Fig. 2, etc.
A **pie chart** is a circle divided into **segments** from the middle (like slices of a cake) to show how the total is divided up. A **key** or **legend** shows what each segment **represents**.
A **bar chart** is a diagram in which different amounts are represented by thin vertical or horizontal bars which have the same width but **vary** in height or length. A **histogram** is a kind of bar chart but the bar width also varies to indicate different values.
A **table** is a grid with **columns** and **rows** of numbers.
A **cross-section** is something, or a model of something, cut across the middle so that you can see the inside. A cross-section of the earth's crust, for example, shows the different **layers** that make it up. A **label** gives the name of each part of the cross-section. Cross-section can also be used to mean a small group that is representative of all the different types within the total group (e.g. *the survey looked at a **cross-section** of society*).
A **flowchart** is a diagram which indicates the **stages** of a process.

B A graph



The **graph presents** data relating to teenagers and pocket money. A **random sample** of 1,000 teenagers were **surveyed** and the average pocket money received at each age has been **plotted** on the graph. The **x axis** or **horizontal axis indicates** age and the **y axis** or **vertical axis shows** the amount of money received per week. The **graph shows** that 15-year-olds receive twice as much pocket money as 13-year-olds. **From the** graph **we can see** that the amount received **reaches a peak** at the age of 18 and then starts to **decline**. This **decline** can perhaps **be explained by the fact that** many teenagers start earning and stop receiving pocket money at the age of 18.

Graphs are drawn by **plotting** points on them and then drawing a line to join **adjacent** points. If there are two lines on a graph – separate lines, for example, to indicate boys' and girls' pocket money – then the lines would probably **cross** or **intersect** at various points. Lines that **run parallel** to one another never intersect.

Graphs show how numbers **increase** or **decrease**. The nouns **increase** and **decrease** have the stress on the first syllable, but the verbs have the stress on the second syllable. Numbers can also be said to **rise** or **grow** and **fall**, **drop** or **decline**. The nouns **rise**, **growth**, **fall**, **drop** and **decline**, like **increase** and **decrease** are followed by **in** (to explain what is rising) or **of** (to explain the size of the change), e.g. *a **rise** of 10% in the number of cars*. Other verbs used about growth include **double**[1], **soar**[2], **multiply**[3], **appreciate**[4] and **exceed**[5].

[1] grow to twice the size; opposite = **halve** [2] (dramatic word) rapid movement upwards; opposite = **plummet** [3] grow rapidly to a very large number [4] used about the value of something, e.g. a painting or car; opposite = **depreciate** [5] go over, expresses a number in relation to another number; opposite = **fall below**

Note that **graph** is a noun and **graphic** [relating to drawing: vivid, especially when describing something unpleasant] is usually an adjective. *The economics textbook contains a lot of fascinating **graphs**. My nephew studied **graphic** design. The book contains some very **graphic** descriptions of the massacre.* **Graphics** can be used as a plural noun to refer to pictorial material, e.g. *The **graphics** in that computer game are brilliant.*

Language for talking about visuals

Practise saying the phrases below with the correct stress and intonation.

What this visual (chart, graph, etc.) shows is .....

This bar chart shows the distribution of .....

This line stands for (represents) .....

The characteristic features of .... are .....

As you can see, ...

Let's have a look at .....

Let's take a closer look at .....

I'd like you to focus on this point here.

Let's look at this in more detail.

I'd like to draw your attention to .....

Here we can see .....

What are the implications of this?

What is the significance of this?

What conclusions can we draw from looking at this graph ?

Describing change

to fall, to decrease, to drop, to decline, to go down
to show a gradual decrease in, to show a gradual decline in

Adverbs: gradually / slowly / slightly / steadily

to fall, to decrease, to drop, to decline, to go down,
to show a rapid decrease in, to show a sudden decline in

Adverbs: sharply / steeply / rapidly / suddenly / dramatically

to rise, to increase, to go up, to climb, to show a gradual increase in

Adverbs: gradually / slowly / slightly / steadily

to rise, to increase, to go up, to shoot up, to show a rapid increase in

Adverbs: sharply / steeply / rapidly / suddenly / dramatically / exponentially

to peak, to reach a peak / a maximum / a high point

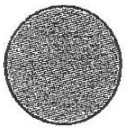
to reach a minimum / a low point

to dip, to be a dip in something

to remain stable, to remain constant, to stay at the same level, to stay the same

Maths shapes

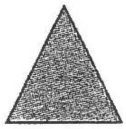
Two-dimensional shapes



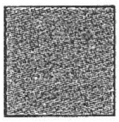
circle
/ˈsɜːkl/

semicircle
/ˈsemi sɜːkl/

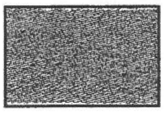
ellipse/oval
/ɪˈlɪps/ /ˈəʊvl/



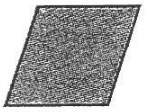
triangle
/ˈtraɪæŋgl/



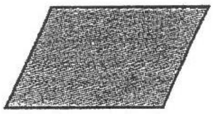
square
/skweə(r)/



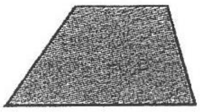
oblong/rectangle
/ˈɒblɒŋ/ /ˈrektæŋgl/



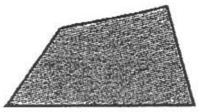
rhombus
/ˈrɒmbəs/



rhomboid
/ˈrɒmbɔɪd/



trapezium
/trəˈpiːziəm/

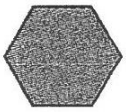


trapezoid
/ˈtræpəzɔɪd/

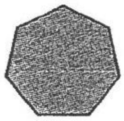
cross
/krɒs/

star
/stɑː(r)/

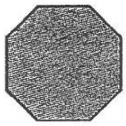
pentagon
/ˈpentəgən/



hexagon
/ˈheksəgən/

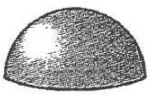


heptagon
/ˈheptəgən/

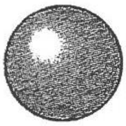


octagon
/ˈɒktəgən/

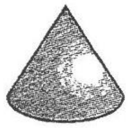
Three-dimensional shapes



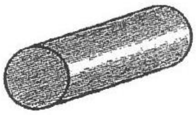
hemisphere
/ˈhemɪsfɪə(r)/



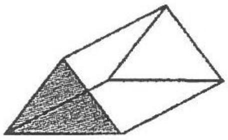
sphere
/ˈsfɪə(r)/



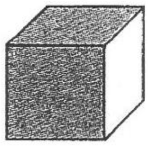
cone
/kəʊn/



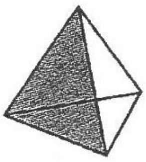
cylinder
/ˈsɪlɪndə(r)/



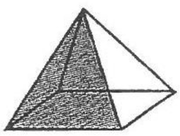
prism
/ˈprɪzəm/



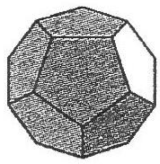
cube
/kjuːb/



tetrahedron
/tetrəˈhiːdrən/



pyramid
/ˈpɪrəmɪd/



dodecahedron
/dəʊdekəˈhiːdrən/

Classroom language for learners

Asking questions

Can I ask a question, please?

What's the word for ...... in English?

How do I say ..... in English?

How do I write/spell/pronounce this word?

What does .... mean?

What do you call … ?

What's the difference between …… and ……?

Is it correct to say ..... ?

Is there a better way to say this?

Asking for repetition and further explanation

I'm sorry, I haven't understood. Can you explain it again, please?

I'm sorry, I missed what you said. What do you want us to do?

I'm afraid I don't follow. Do you mean … ?

I'm sorry, could you say that again, please?

I'm sorry, I didn't understand the instructions. Can you repeat them, please?

Working in pairs and comparing answers

What was your answer for number 6?

What did you put for question 7?

What did you get? I've got 6F.

Which answer did you choose?

Do you have the same answer as me?

What do you think the answer is?

What did you make it? I made it 452.

Why do you think the answer is that?

Why did you choose that answer?

Why do you think that?

How did you arrive at that answer?

Yes, I agree with you.

Yes, I think you're right.

Yes, I've got the same answer as you.

I'm not sure I've got the right answer.

I've got a different answer from you.

I'm not sure that's correct.

I don't think you're right.

Scaffolding language: expressing functions

Scaffolding language is the language learners need to do a particular task. After selecting the **content language** (vocabulary items, expressions, and collocations related to the topic), you will need to choose the scaffolding language, and this may include functional language. You can select from the boxes below the language that will be most suitable for them.

Sequencing words
First(ly), …
Second(ly), …
Third(ly), …
Then, …
Next, …
After that, …
Finally, …

Adding information
In addition, …
Moreover, …
Another thing to keep in mind is …
Furthermore, …

Comparing and contrasting
but
However, …
Although …, …
On the one hand …, but on the other hand ...
While …,
…, whereas …
… is (not) as …. as …
… is (not) the same as …
… is much (far) greater/smaller than …
… is by far the largest/smallest …
… is different from …
… in comparison with …
… compared to … is more/less …

Giving advantages and disadvantages
This has the (dis)advantage of … (+-*ing*)
One of the advantages of this is … However, …
One argument in favour of/against this is …
One of the drawbacks is …
We need to weigh up the pros and cons of … (+-*ing*)

Describing processes and how things work
This causes the … to …
This has the effect of …
In order to do this, we need to …
If we …, then it will …
By adjusting this, it will …

PTO

Sequencing past events
During this time/period …
It was not long after this that …
Following this, …
This was followed by …

Describing cause and effect
… is/are (was/were) caused by …(+ *-ing*) …
… is/are (was/were) probably caused by …
… is/are (was/were) due to …
… could be (could have been) due to the fact that …
… is/are (was/were) a direct result of …
… and as a result …
…, and consequently …
There is a strong/direct link between … and …
This is (was) connected to …
This led to … (e.g. riot*ing*/the imposition of martial law)
This had the effect of … (e.g. mak*ing* imports more expensive)
This resulted in … (e.g. the fall of the government)
This in turn caused … (e.g. widespread famine)
This culminated in … (e.g. a declaration of war/the greatest works of art)

Hypothesising and expressing different degrees of certainty
There can be no doubt that …
It is almost certain that …
It is highly likely that
It would seem that …
It appears that …
This suggests that …
It is probably true to say that …
It is improbable that …
It is possible to conclude from this that …
It is not certain exactly who/when/where …
It is difficult to say who/where/ when/what …
It is uncertain whether or not ...

Making inferences (examples using the language above)
This would suggest that ... (e.g. our ancestors lived in groups)
It would seem that ... (e.g. food production was a problem)
It is almost certain that … (e.g. food was a constant worry)
This might be the result of ... (e.g. scarce resources)
They probably … (e.g. had poor health as a result)
From this we can conclude that ... (e.g. they lived on a poor diet)

Concluding
Overall, …
To summarise, …
In summary, …
In conclusion, …

Language for giving a presentation

Stating aims
The purpose of this presentation is to show you …
The aim of the next 20 minutes is to give you an overview of …
… so by the end of this presentation you should have a better idea of …

If you have any questions, please feel free to interrupt.
Feel free to ask any questions as we go along.
Please could you hold any questions until the end of the presentation.
I will be happy to answer any questions at the end of my presentation.

Overview
The presentation is divided into four main parts …
First, I'll start by (+ *-ing*) …
Then I'll go on to … look at/show/illustrate/explain/demonstrate …
Next, …
After that, …
Finally, I will speak about …

Signposting
I'd like to begin by looking at …
Now I'd like to move on to …
Now I want to turn to …
This brings me to my next point, which is …
Lastly, I want to look at …

Using visuals
Here you can see …
As you can see from this picture/graph, this shows/illustrates/explains …
I'd like to draw your attention to …
Let's look at this in more detail and focus on …

Inviting questions
Any questions so far?
Are there any questions on that?
Do you have any questions you would like to ask?
If you have any questions, I'll be happy to answer them.

Concluding
So, the main points were …
So, to summarise what I have said, …
Now I would like to summarise briefly what I have said.
In conclusion, …

Closing comments
So, I hope I have demonstrated/shown that …
That covers everything I wanted to say about …
That brings me to the end of my presentation.
Thank you for your attention.

Scaffolding language: solving linear equations

Example 1

$3(4x + 6) = -9 - (9x - 6)$

What do we do first?

We **remove / get rid of / multiply out / expand the brackets.**

This gives us: $12x + 18 = -9 - 9x + 6$

Why does the "–" sign change to "+"? What is the rule? Two minuses make a plus.

What's the next **step**? *OR:* What do we do **in the next step**?

We **isolate** x **by putting / grouping** all the xs **on one side of the equation.**

OR: **We make** x **the subject of the equation.** *OR:* **Collect like terms.**

$12x + 9x = -9 + 6 - 18$

Now we **simplify** and we **get**:

$21x = -21$

Now we can **work out the value of** x. *OR:* We can **arrive at the answer.**

Divide **both sides** by 21.

So:

$x = -1$

<u>NOTE</u>: you can check the answer is correct by **substituting the value of** x **into the equation.**

Example 2 An age problem

Jack's father is 3 times as old as Jack. 8 years ago, he was 11 times as old as Jack. How old are Jack's father and Jack today?

How would you **go about** solv***ing*** this problem?

First, **create a table** and put into it what we know.

Then call what we do not know x.

x is what is **unknown**. In this case, Jack's age now. **So** …

Let Jack's age now **be** x. *OR:* **Let** x **be** Jack's age now.

	NOW	8 YEARS AGO
Jack	x	$x - 8$
Jack's father	$3x$	$11(x - 8)$

How can we **turn this into** a linear equation?

We know that 8 years ago Jack's father was $11(x - 8)$. We also know that if Jack's father is 3x now, then 8 years ago he was $3x - 8$. **So** ... *OR* **Therefore it follows that** ...

$3x - 8 = 11(x - 8)$

Now explain to your partner how to solve this linear equation, using the language above.

Example 3 Another age problem

Ann and Ben are together 65 years old. When Ann is 70 years old, she will be twice as old as Ben. In how many years will Ann be 70?

Task 3 Practise using the scaffolding language to solve this problem.

symbols

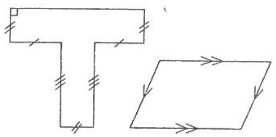
A symbol is a letter or sign used to represent instructions, or a number, in a more concise form. Sometimes a symbol replaces a group of words and can be read directly as it occurs, in these cases the words are shown below in "quotation marks."

$+$	"add" or "positive"	
$-$	"minus" or "subtract" or "negative"	
$\times$	"times" or "multiplied by"	
$*$	"times" or "multiplied by"	*Usually used on a computer*
$\div$	"divided by" or "shared by"	*Example: $6 \div 3$ or 6/3 both mean "6 divided by 3"*
$/$	"divided by" or "shared by"	*or "6 shared by 3" or "How many 3's in 6?"*
$\pm$	"add or subtract" "plus or minus" "positive or negative"	*Example: When $x = 3$ then $x \pm 2$ gives the two answers 5 and 1* *± 6 means the two numbers +6 and ⁻6*
$=$	"equals" or "is equal to"	*Example: $x + 3 = 7$*
$\neq$	"does not equal" or "is not equal to"	
$\approx$	"is approximately equal to"	*Example: $\pi \approx 3.14$*
$\equiv$	"is equivalent to" or "has the same value as"	*Example: \$5.00 ≡ 500¢*
$\equiv$	"is identically equal to"	*Example: $(x + y)^2 \equiv x^2 + 2xy + y^2$*
$<$	"is less than"	*Example: $x < 5$ means x can take any value which is less than 5 but cannot equal 5*
$\leq$	"is less than or equal to"	*Example: $x \leq 7$ means x can take any value which is less than 7 or may equal 7*
$>$	"is greater than"	*Example: $x > 3$ means x can take any value which is greater than 3 but cannot equal 3*
$\geq$	"is greater than or equal to"	*Example: $x \geq 6$ means x can take any value which is greater than 6 or may equal 6*
$\propto$	"varies as" or "is proportional to"	*Example: $y \propto x$ means that y changes in some regular way as x changes.*
.	decimal point	*It is placed on the line and used to separate the whole number part from the fractional part.*
,	decimal marker	*Example: 3,48 is equivalent to 3.48* *The comma is standard in the SI or metric system.*
%	"percent" or "out of a hundred"	
‰	"per mil" or "out of a thousand"	
$[x]$	the largest whole number which is not greater than x	*Example: [3.5] is 3, but [−4.2] is ⁻5*
$\lvert x \rvert$	absolute value; the value of x with no sign attached	*Example: $\lvert ^-8.7 \rvert$ is 8.7*
x^2	"x squared" or "x multiplied by itself"	*Example: $4^2 = 4 \times 4 = 16$*
x^3	"x cubed"	*Example: $2.5^3 = 2.5 \times 2.5 \times 2.5 = 15.625$*

100

symbols

$\sqrt{x}$	"the positive square root of x"	*Example: $\sqrt{9} = 3$, since $3 \times 3 = 9$*
$\sqrt[3]{x}$	"the cube root of x"	*Example: $\sqrt[3]{8} = 2$, since $2 \times 2 \times 2 = 8$*
$\angle$	"angle"	*Example: $\angle ABC$ or $\angle B$*
$\parallel$	"is parallel to"	*Example: $AB \parallel CD$*
$\perp$	"is perpendicular to"	*Example: $AB \perp CD$*
∟	these lines are at right angles to each other	*Examples:*
	these lines are parallel to each other	
	these lines are equal to each other in length	
$\circ$	"degree"	*of angle or temperature*
$'$	"minute"	*one-sixtieth of a degree or an hour*
$''$	"second"	*one-sixtieth of a minute (of angle or time)*
$n!$	"factorial n"	*Multiply together all the integers from 1 to n*
$\{\}$	used to enclose a listed set	*Example: {A, B, C, D, E, F}*
$\in$	"is a member of"	*Example: $D \in \{A, B, C, D, E, F\}$*
$\notin$	"is not a member of"	*Example: $X \notin \{A, B, C, D, E, F\}$*
$\subset$	"is a subset of"	*Example: $\{A, D, E\} \subset \{A, B, C, D, E, F\}$*
$\supset$	"includes"	*Example: $\{A, B, C, D, E, F\} \supset \{A, D, E\}$*
$\cup$	union of two sets	*Example: $\{A, D\} \cup \{B, C, D, E\} \equiv \{A, B, C, D, E\}$*
$\cap$	intersection of two sets	*Example: $\{A, C, D\} \cap \{B, C, E\} \equiv \{C\}$*
$\varnothing$	null or empty set	*Example: $\{A, D\} \cap \{B, C, E\} \equiv \{\} \equiv \varnothing$*
σ	standard deviation	
$\Rightarrow$	"implies"	
$\Leftarrow$	"is implied by"	
$\Leftrightarrow$	"implies and is implied by"	*also written as iff*
$\therefore$	"therefore"	*Example: $AB \perp CD \therefore \angle ABC = 90°$*
∞	"infinity"	
e	≈ 2.71828	
$f(x)$	"a function of x"	*Example: $f(x)$ is $x^2 + 3x - 4$*
π	pi ≈ 3.14159	
( )	parentheses	
[ ]	square brackets	
{ }	curly brackets	



Parentheses or **brackets** are used (in pairs) to enclose an expression that is to be treated as a complete quantity and evaluated before the rest of the expression. Brackets can be nested within each other, and the use of different types allows matching pairs to be seen more readily.
Example: $4\{5x[2x(x+5) - 7]+8\} - 17$ is $40x^3+200x^2 - 40x - 13$

101

abbreviations and mnemonics

abbreviation An abbreviation is a shortened form of a word or phrase, often made by using the initial letter (or letters) of the word (or words). *Some of the more common abbreviations used in mathematics are given below.*

AP arithmetic progression
APR annual percentage rate
cm centimeter(s)
cu cubic (referring to units)
dp or **d.p.** decimal place(s)
GP geometric progression
g gram(s)
gcf or **g.c.f.** greatest common factor
kg kilogram(s)
km kilometer(s)
L or **l** liter(s)
lcd or **l.c.d.** lowest (or least) common denominator
lcm or **l.c.m.** lowest (or least) common multiple
m meter(s)
mm millimeter(s)
mod modulus
QED (quod erat demonstrandum) which was to be proved
sf or **s.f.** or **sig. fig.** significant figures
SI Système International d'Unités (international system of units)
sq square (referring to units)

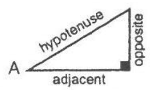
acronym An acronym is an **abbreviation** that is pronounceable and is usually said as a word. *An acronym is often written with capital letters, indicating that it is not a "real" word.*

PEMDAS is an **acronym** that serves as a reminder of the order in which certain operations have to be carried out when working with equations and formulas.

Parentheses Exponents Multiplication Division Addition Subtraction

SOHCAHTOA is an **acronym** that serves as a reminder of how the trigonometric ratios for a right-angled triangle are formed. The meaning of the letters is:

Sine A = Opposite ÷ Hypotenuse
Cosine A = Adjacent ÷ Hypotenuse
Tangent A = Opposite ÷ Adjacent



mnemonic A mnemonic is a device which is intended to help a person's memory. *Some mnemonics are given in the remainder of these two pages.*

order of operations An aid to remembering PEMDAS is:

Please Excuse My Dear Aunt Sally.

abbreviations and mnemonics

Euler's formula A **mnemonic** to help in remembering Euler's formula for graphs giving the relationship between faces, vertices, and edges is:

say	Fred	And	Vera	Took	Eric		Too
to *remember*	Faces	Add	Vertices	Take	Edges	=	Two

SI prefixes A **mnemonic** to help remember the order of some SI prefixes is:

say	to	give	me	kicks	my	musicians	now	play	for	ages
to remember	tera	giga	mega	kilo	milli	micro	nano	pico	femto	atto

colors of the spectrum The order of the seven colors of the spectrum (as seen in a rainbow) is given by one of the most often quoted **mnemonics**:

say ROY G. BIV
to remember red orange yellow green blue indigo violet

trigonometric values The numerical values of the trigonometric ratios of certain angles are given by this easily remembered table:

$\theta =$	0°	30°	45°	60°	90°
$\sin\theta =$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

To get the cosine values, write the second line of values in reverse order.

π The first 8 digits of π are given by the number of letters in each of the words in this question:

may	I	have	a	large	container	of	coffee ?
3.	1	4	1	5	9	2	6

$\frac{1}{\pi}$ The reciprocal of π to 6 decimal places is given by the number of letters in each of the words in this question:

can	I	remember	the	reciprocal ?
.3	1	8	3	10

e The first 8 digits of e are given by the number of letters in each of the words in this sentence:

to	express	e	remember	to	memorize	a	sentence
2.	7	1	8	2	8	1	8

$\sqrt{2}$ The first 4 digits of $\sqrt{2}$ are given by the number of letters in each of the words in the answer to the question:

the root of two?	I	wish	I	knew
	1.	4	1	4

division of fractions An aid to remembering the process for dividing one fraction by another is the rhyme:

The number you are dividing by
Turn upside down and multiply

Example: $\frac{3}{4} \div \frac{9}{16} = \frac{3}{4} \times \frac{16}{9} = \frac{4}{3} = 1\frac{1}{3}$

coordinate pairs (x,y) A phrase to help remember the order for plotting the ordered pair (x,y) is "along the passage and up the stairs," meaning "go along to x and then up to y."

Maths Collocation Quiz 1
(Opposites)

A

1	positive and …	negative
2	odd and …	even numbers
3	maximum and …	minimum
4	add and ...	subtract / take away
5	increase and …	decrease
6	straight and …	curved
7	major and …	minor (axis / arc / sector / segment)
8	approximately and …	exactly
9	dependent and …	independent (variables)
10	terminating and ...	repeating / recurring (decimals)

FOLD --

Maths Collocation Quiz 1
(Opposites)

B

11	plus and …	minus
12	numerator and …	denominator
13	finite and ...	infinite
14	multiply and …	divide
15	rise and …	fall
16	exterior and …	interior (angles)
17	long and …	short (division)
18	definite and …	indefinite (integral)
19	implicit and ...	explicit (functions)
20	domain and ...	codomain (of a function) / range

Maths Collocation Quiz 2
(Verbs)

A

1	to eliminate / get rid of …	the brackets
2	to find the …	answer / solution / value of x
3	to expand the …	expression / brackets
4	to plot a ...	graph
5	to multiply out the …	brackets
6	to calculate the surface …	area
7	to approach …	infinity
8	to bisect a / an …	line / triangle / circle / angle
9	to satisfy an …	equation
10	to cancel out the …	common factors
11	to rationalise a …	denominator / numerator
12	to transpose the …	terms of an equation

FOLD --

Maths Collocation Quiz 2
(Verbs)

B

13	to work out the …	answer / average / area / value of x
14	to solve a / an …	problem / equation
15	to raise something to the …	power of 2, z, … / 2^{nd} , 4^{th}, n^{th} power
16	to simplify a / an ...	equation / expression / fraction
17	to arrive at an ...	answer
18	to sketch a ...	graph
19	to rearrange the …	equation
20	to collect like …	terms
21	to round up/down to the …	nearest … / … d.p./ … s.f./
22	to intersect the …	x-axis / y-axis
23	to find a range of …	values
24	to factorise a / an …	quadratic equation / expression