Mathematical Modeling Questions for Secondary School Curriculum

November 2020
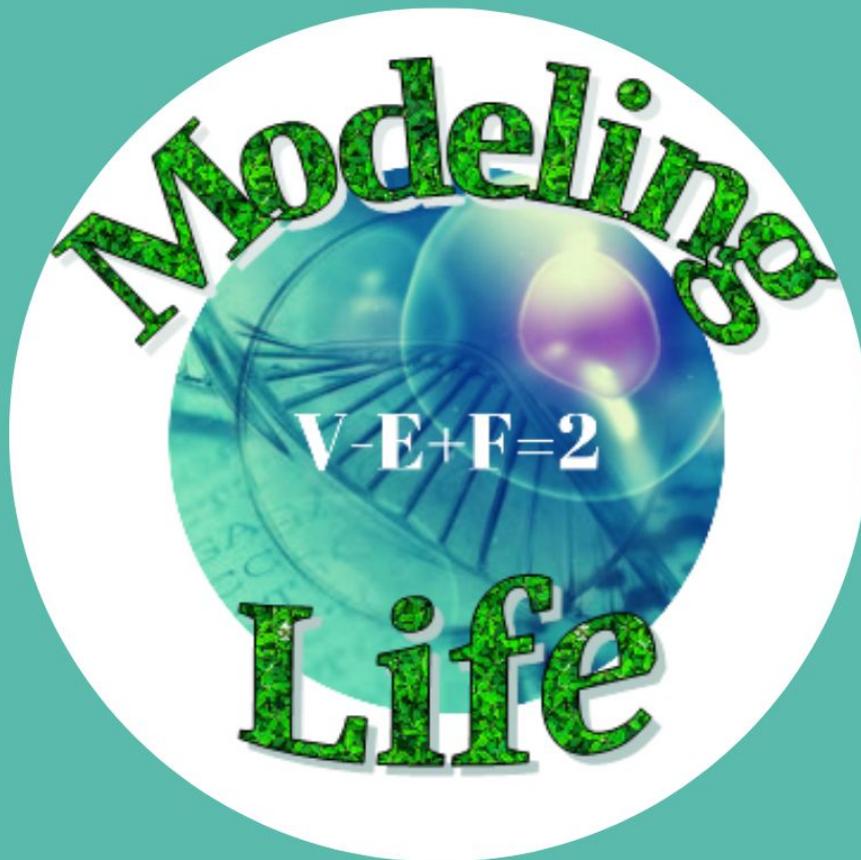
Authors:

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eTwinning

This e-book contains mathematical modeling questions and solutions produced by teachers involved in the project as part of an eTwinning project "Modeling Life".

Project Twinspace:

eTwinning is the community for schools in Europe.

eTwinning offers a platform for staff (teachers, head teachers, librarians, etc.), working in a school in one of the European countries involved, to communicate, collaborate, develop projects, share and, in short, feel and be part of the most exciting learning community in Europe.

About the "Modeling Life"
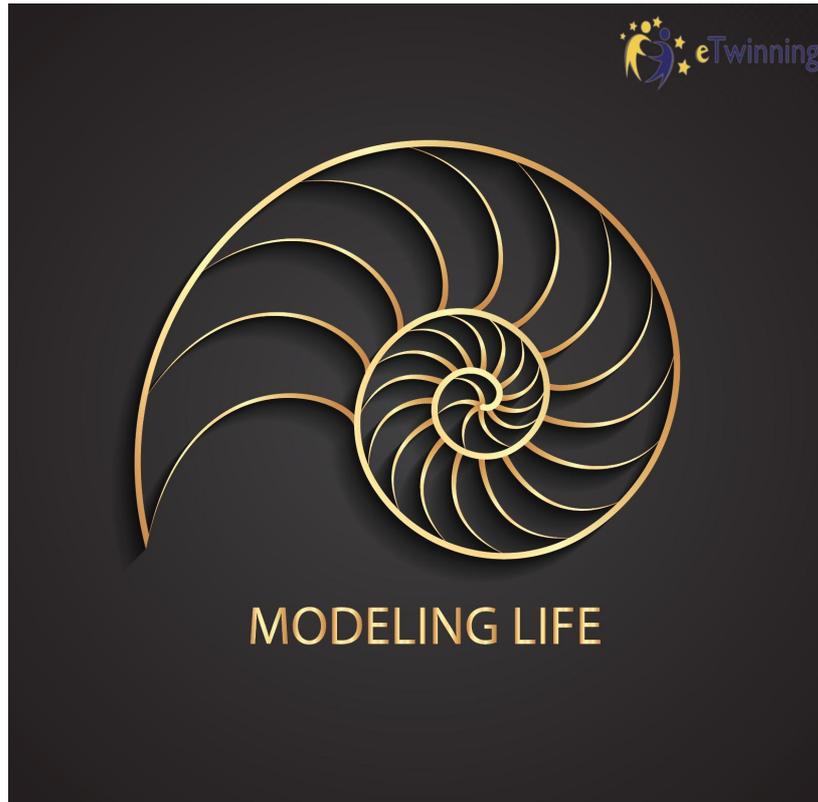
      In this project our students will apply mathematics to solve real-life problems, make a mathematical model that describes, solve and validates their problem, and improve their model. During the project, mathematical modeling questions will be produced by the teachers involved in the project and these questions will be applied in mathematics lessons with teachers and students in different schools. At the end of the project, expected that students write their own mathematical modeling questions and solution reports.

The project language: English

Age range: From 16 to 19

Subject: Mathematics / Geometry, Biology, Physics, Informatics / ICT

The duration of the project: Nine months

The academic year of the project: 2020-2021

Newspaper Sales

Author: *Özlem Kahraman*

Newspapers, one of the most important arguments of our world, which has moved into the age of total information, were also greatly affected by this situation. Electronic media reporting is doubling its power every day. When we look at the habits of young people who will determine the conditions of the coming period, we learn that when they first enter the internet, they click on news sites more. As such, newspapers shrink in terms of circulation and this situation requires new measures to be taken. In the process that starts with a decrease in the number of pages, the owners make a price reduction in order to make the newspaper more attractive.

The circulation of a newspaper with a sales price of 1.6 Euros is 150 000. According to a study conducted among readers, the number of people who buy the newspaper increases by 15000 for every 0.1 euro discount on the sale price. What should the newspaper's new sale price be in order to make the most profit?

Related Topics and Concepts: Quadratic Functions

Real Life Context: Price policy, circulation, economy

The water bill

Author: *Inmaculada Illán*

Invoicing concepts:

CHARGES	FIX	VARIABLE			VAT
SEWER	1.6624€	0.1247 €/m³			10%
WATER TREATMENT	4.0938€	0.1689 €/m³			10%
DISTRIBUTION	2.7143€	**BLOCK**	**LIMIT**	**€/m³**	10%
		1	$x \leq 35$	0.3128	
		2	$35 < x < 50$	0.8462	
		3	$x \geq 50$	1.4462	
EXPLOTATION WATER TREATMENT STATION	7.1479€	0.2184€/m³			10%
INVESTMENT REDEMPTION	0€	0.1117€/m³			10%
GARBAGE COLLECTION	11.86€	0			0%

In Spain, due to the drought, city council fixes different prices depending on your consumption. The table above shows the fees

The blue part means the following: If your consumption is less or equal to 35m³ then you pay 0.3128 €/m³ , but if your consumption is a number between 35 and 50 then you pay 0.3128 €/m³ for the 35 m³ first and 0.8462 €/m³ for the rest. Finally if your consumption is bigger than 50 then then you pay 0.3128 €/m³ for the 35 m³ first and 0.8462 €/m³ for 15 m³ and the rest is paid at 1.4462 €/m³

Calculate:

a) How much do you have to pay if you don't consume water this quarter?

b) How much do you have to pay if your consumption is 23 m³?

c) How much do you have to pay if your consumption is 40 m³.?

d) How much do you have to pay if your consumption is 53 m³.?

e) I paid € 59.94 last quarter, how many m³did I consume?

f) Find out a piecewise function that reflects this situation

Related Topics and Concepts: Piecewise functions, percentages

Real Life Context: Price policy, economy

Highway Speed Corridor Application   *Author: Özlem Kahraman*

In Turkey, to avoid the traffic accidents occurred due to excessive speed, to minimize the damage of exhaust emissions to nature and to prevent increased fuel consumption due to high speed, Highway Speed Corridor Application is in force.

According to the application, the minimum arrival time is determined by measuring the distance between the entrance and exit toll booths on the highways. If the drivers enter the highways from the toll booths and pass the exit before the specified time, it is considered that they have exceeded the maximum speed limit and a traffic ticket is applied. Speed limit set for highways; 120 km / h for cars and 90 km / h for trucks. The speed limit penalties determined in the regulation are 235 TL if the speed limit is exceeded between 10% and 30%, and 488 TL if more than 30% is exceeded.

Accordingly, for the 450 km long Istanbul-Ankara TEM highway,

a) Determine the minimum arrival times for cars and trucks in accordance with the speed limit.

b) Determine whether a truck driver who completes his journey in 4 hours will pay a traffic ticket, and if so, how much he will pay.

c) Determine whether a car driver who completes his journey in 3 hours and 15 minutes will pay a traffic ticket, and if so, how much he will pay.

Related Topics and Concepts: Speed problems, percentages

Real Life Context: Traffic rules, speed limits

Holiday Plan

Author: Özlem Kahraman

Antalya is Turkey's most popular holiday region and welcomes millions of tourists every year. With early booking, tourism companies offer holidaymakers the opportunity to purchase their summer vacation months in advance with attractive payment options and high discount rates. A person planning a 10-day vacation in Antalya in August wants to take advantage of the early booking discount. Can you help him determine the 10-day date range where he can benefit most from the sun by examining the 10-year average temperature data of Antalya?

27 33° 25°	28 33° 27°	29 33° 26°	30 34° 27°	31 33° 27°	1 33° 27°	2 34° 27°
3 33° 26°	4 33° 26°	5 32° 26°	6 32° 26°	7 33° 26°	8 33° 27°	9 33° 27°
10 32° 26°	11 32° 26°	12 32° 26°	13 33° 26°	14 33° 27°	15 34° 27°	16 33° 27°
17 33° 26°	18 33° 26°	19 33° 26°	20 33° 26°	21 33° 26°	22 33° 26°	23 33° 26°
24 33° 26°	25 34° 25°	26 33° 26°	27 33° 26°	28 32° 26°	29 30° 24°	30 31° 24°
31 31° 24°	1 31° 24°	2 31° 24°	3 31° 24°	4 31° 24°	5 31° 24°	6 31° 24°

10-year average temperature data of Antalya for August (highest and lowest average temperature data for the days of the month)

Related Topics and Concepts: Arithmetic mean, maximum value

Real Life Context: Tourism, early booking, meteorology

Area of Cardboards

Author: Tatjana Vinokurova

You will determine the area of any shape of cardboard without using any formula.

© Can Stock Photo - csp6581180

Required tools:

- regular and irregular pieces of cardboard
- scales
- a pocket calculator
- ruler to measure the length of the sides of the squares

Your task:

1. Measure the weight of square sheets of cardboard. Measure the lengths of their sides and calculate the area. Fill in the table.

	Square 1	Square 2	Square 3
Weight (g)			
Area ( )			
Area to weight ratio			

2. Find a function to calculate the area for a given (x) weight of a piece of cardboard. Draw a graph of this function.

Related Topics and Concepts: Functions and Areas of Various Shapes

Real Life Context: Construction, engineering calculations

Defective Bulbs

Author: Özlem Kahraman

The Poisson distribution is used to model the number of events occurring within a given time interval.

The formula for the Poisson probability mass function is

$$p(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x = 0, 1, 2, \dots$$

λ is the shape parameter which indicates the average number of events in the given time interval.

A company estimates that an average of 10 bulbs are defective every day during the final control of the bulbs it produces.

Calculate the probability that 4 bulbs will be defective on a control day using the Poisson distribution.

Related Topics and Concepts: Probability, Poisson distribution

Real Life Context: Industrial engineering

Patient information leaflet

Author:Inmaculada Illán

THE IBUPROFEN.

   Mr. and Ms. Martínez went to the doctor because their son Ramiro was ill. Ramiro had a temperature and the doctor decided to prescribe him an antipyretic (an antipyretic is a medicine that brings the fever down).

   Ramiro's parents bought the syrup and when they read the patient information leaflet, there appeared the information below:

IBUPROFEN 40mg/ml

Contains: 150 ml.

Dosage: The dose depends on the child's weight and age. For children between 3 months and 12 years old, the daily recommended dose is 25 mg/kg a day (every 6 or 8 hours).

<u>*Ramiro is 8 years old and weighs 28 kg*</u>

.

1.-What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?

2.- Calculate how many mg. Ramiro must consume every day.

3.- Calculate how many ml. Ramiro must consume every day.

4.- How many ml. must Ramiro drink each dose?

5.- Complete the table below:

6.- Find out a formula relating weight and dose (mg.).

7.- Find out a formula showing the relationship between weight and dose (ml.).

8.- Apply the previous formulae and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.

9.- Represent each function using a coordinates system axes.

Weight·(kg.)¤	mg/day¤	ml/day¤	ml/dose¤
5¤	¤	¤	¤
10¤	¤	¤	¤
15¤	¤	¤	¤
20¤	¤	¤	¤
25¤	¤	¤	¤
30¤	¤	¤	¤
35¤	¤	¤	¤
40¤	¤	¤	¤

Related Topics and Concepts: Proportionality and units

Real Life Context: Health

The daily walk

Author:Inmaculada Illán

Elderly people in Hellín usually go for a walk in the afternoon. They want to be fit and healthier.

One of the most usual paths appears in black in the map.

You can also see another black line. This is the graphical scale.

1) Measure the length of the line that shows the scale.
2) Measure the length of the line which indicates the walk.
3) Calculate the real length of the walk.
4) Using Google Maps calculate the length of the walk.
5) If one person's velocity is 4 Km/h, How long is the walk?
6) One person spent one hour and a quarter in his/her walk. What was his/her average velocity?
7) The City Council wants to point a milestone every 250 meters. How many milestones do they need?
8) The walk is clockwise. Put a red point in each place in the map where a milestone is placed.
9) The City Council wants to carve the milestones indicating the numbers of meters that you have walked from the starting point, which is placed in front of the railway station - yellow point in the map-, remember that the walk is clockwise.
   The prices are the following:
   One milestone: € 68.
   A carving digit: €2 (except if the number is 0, in this case the price is 1 Euro)
   Raising a milestone: € 20 each.
   Calculate the total amount.
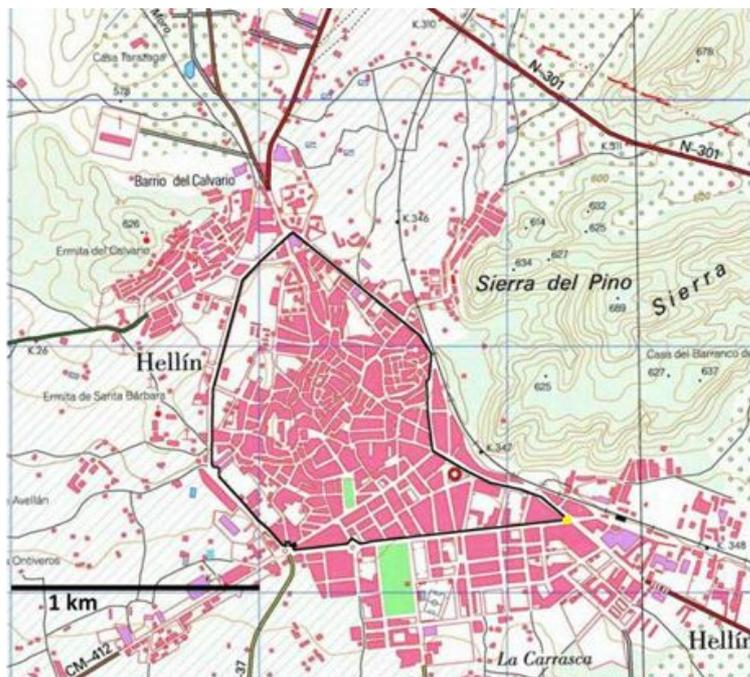
Related Topics and Concepts: Proportionality, measurements,
**Real Life Context:** Health and sport

AQUARIUM DESIGN

Author: *Mihaela Git*
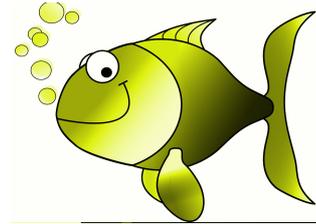
A manufacturer is designing an aquarium whose base is a regular hexagon.

The aquarium should have a volume of 24 cubic feet and use the least amount of material possible. Let l be the length (in feet) of a side of the base, and let h be the height (in feet).

1. Write an equation that gives h in terms of l.
2. Determine the function that expresses the surface of the aquarium depending on the side of the base.
3. Use the GeoGebra program to graphically represent this function.
4. Find the dimensions s and h that minimize the amount of material used.
5. For l and h determined at 4., how many liters of water must be put in the aquarium for the water to wet three quarters of the edge of the aquarium?
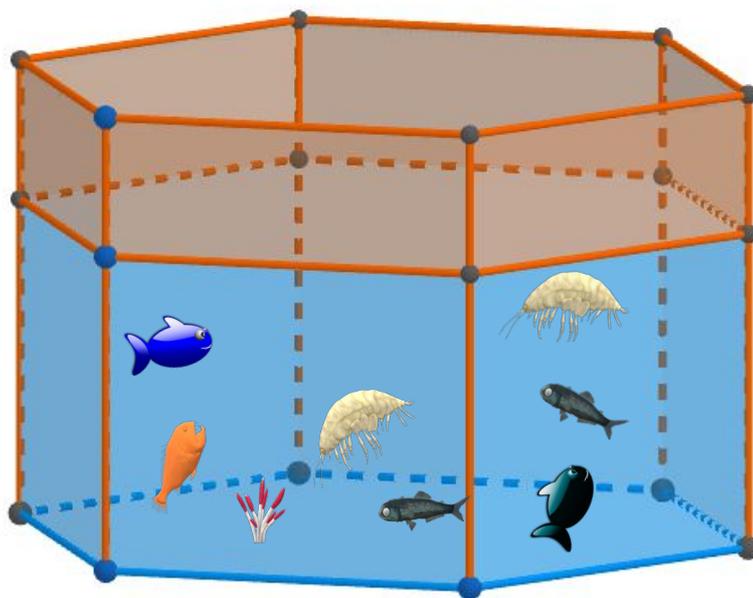
Related Topics and Concepts: Geometry, Functions
Real Life Context: Construction engineering

Taxi Fare Calculation

Author: Özlem Kahraman

Taxis are one of the most widely used means of transportation today. Taxis are frequently preferred by citizens, allowing them to travel more comfortably and in a faster time than public transport. The taximeter, which is used to calculate the fare in taxis, calculated by adding the price / kilometer charge on the opening price.

Citizens living in Istanbul pay 3.1 TL per kilometer in taxis. Taximeter opening fee is determined as 5 TL and the minimum taxi fee is 13 TL.

Accordingly,

a. Write the mathematical relation that gives how much will be paid when traveling x km by taxi in Istanbul, and draw its graph.

b. Calculate how much a person who takes a taxi from Istanbul Airport pays to go to Taksim Square, which is 43 km away.

Related Topics and Concepts: Piecewise functions

Real Life Context: Price policy, economy

Water Slide problem

Author: Tatjana Vinokurova

The owners of a water park decided to create a new attraction - a spiral slide around a cylindrical aquarium with fish. The height of the cylindrical aquarium is 10 meters, but the diameter can vary from 2.5 meters to 5 meters. According to safety requirements, the angle of inclination of the slide relative to the horizontal level should be 30 degrees. Which cylinder diameter better satisfies the conditions: 1) the maximum possible length of the slides; 2) material consumption of the cylindrical aquarium is minimal?

Water Slide (Source: https://unsplash.com/photos/e2VveZKIZyQ)

Related Topics and Concepts: Geometry, Functions
Real Life Context: Construction engineering

Mobile phone companies

Author: *Mihaela Git*
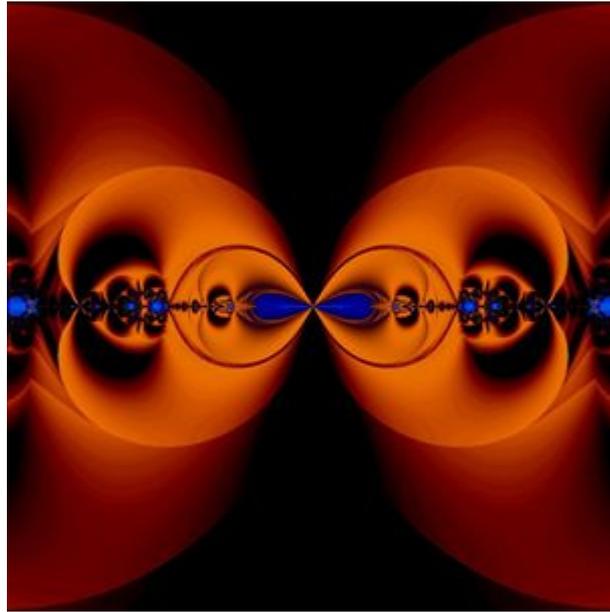
Two different mobile phone companies announced their subscriptions' offers. The first one's is about 9€ per month and it has 900 minutes and every extra minute means 30 cents more to pay. The second one's should be paid 12€ per month for 400 included minutes and calling is billed with 0,20 Euro per minute.

a) Which of these companies have the most efficient offer, for the needs of a person that works in business ?

b) Define the functions that express the cost variation per number of minutes.

c) Represent the plots of the two functions in the same coordinate system.

Related Topics and Concepts: Functions, graphs

Real Life Context: Economy, Price policy

Math and Art

Author: *Mihaela Git*
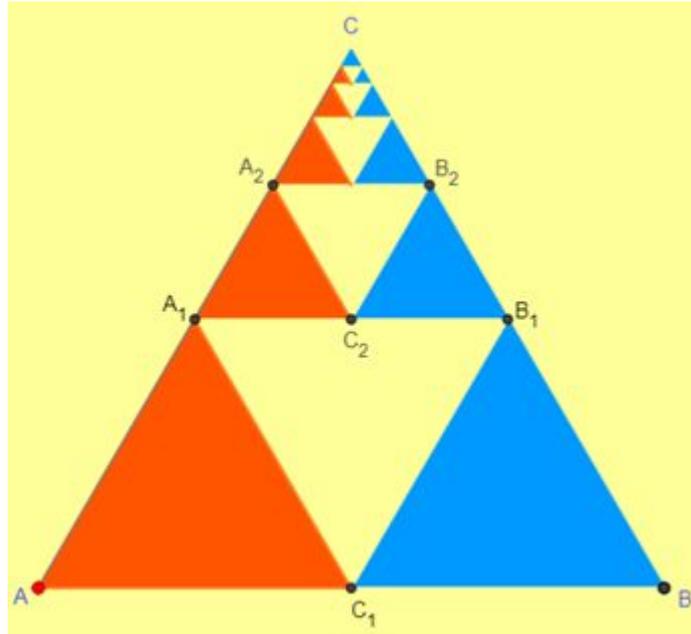
Consider an equilateral triangular surface ABC with unit area. Construct A_1, B_1, C_1 the midpoints of the sides of the triangle ABC, as in the drawing above.

a)   Prove that the 4 triangles formed are congruent.

b)   Repeat the construction for the triangle CA_1B_1, CA_2B_2 and so on.

   Calculate the sum of the areas of the blue triangular surfaces after 5 steps. Calculate the sum of the areas of the blue triangular surfaces after n steps. Approximate this value for a very large number n.

See also the construction in GeoGebra: https://www.geogebra.org/classic/zfpvrd44

The source of inspiration for the problem:

https://geometrydaily.tumblr.com/post/22586826330/128-ad-astra-a-new-minimal-geometric

Related Topics and Concepts: Congruent triangles, Areas, Geometric progression

Real Life Context: Art

Spiral of equilateral triangles

Author: *Mihaela Git*

This is a spiral formed by equilateral triangles.
Each equilateral triangle, starting with the second,
has the side equal to the height of the previous triangle.
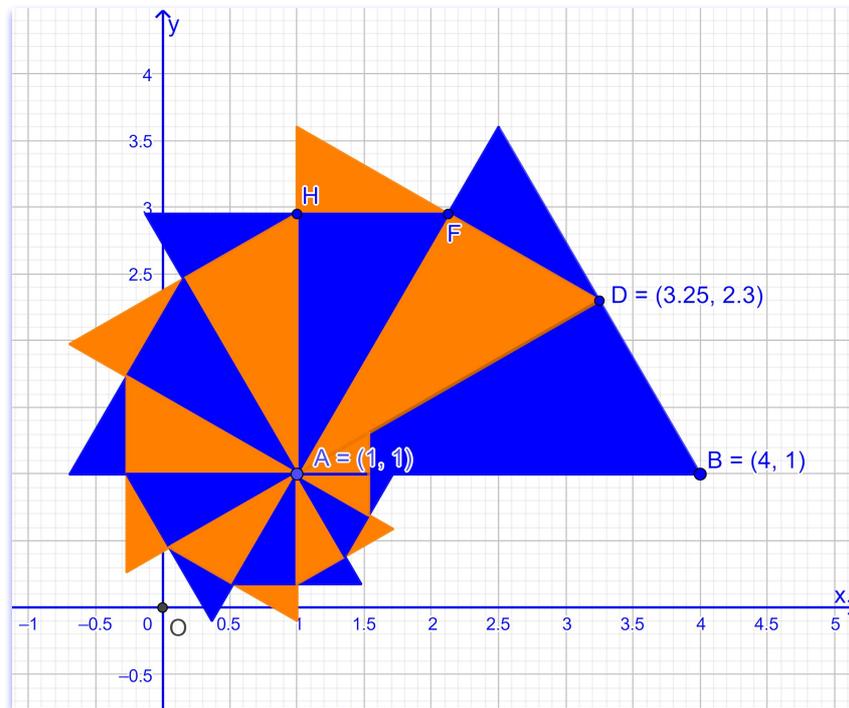
ABC is equilateral triangle and A(1; 1), B(4;1). AD⊥ BC , D∈BC, ADE equilateral triangle, etc.

a) Determine the equation for the line BC.
b) Determine the equation for the line AC.
c) Find the coordinate for the point C.
d) Determine the equilateral triangle height equation that passes through A.
e) Calculates the area of the ABC triangle.
f) Determine the equation for the line AE.
g) What is the position of the line AH and AB?
h) Calculates the area of the ADE triangle.

Related Topics and Concepts: Areas, Analytical geometry: the equations of a line

Real Life Context: Art

The ship & the islands

Author: Mihaela Git

A ship must arrive from one island to another in t = 3 hours.

It travels at speeds of v = 45 km/h and owing to a storm outages d_0 = 6km. Knowing that the movement of the ship represents a function with the correspondence law $f(v) = vt + d_0$.

Calculate:

a) What is the distance between islands?

b) What distance would be between the islands if the boat would have to travel at 48 km/h?

c) What minimum speed could the ship have if the islands were 150km away?

d) Represent the plot of the function in the coordinate system.

Related Topics and Concepts: speed, distance

Real Life Context: economy

Pressure and volume of gases   *Author: Abel Carenas Velamazán*
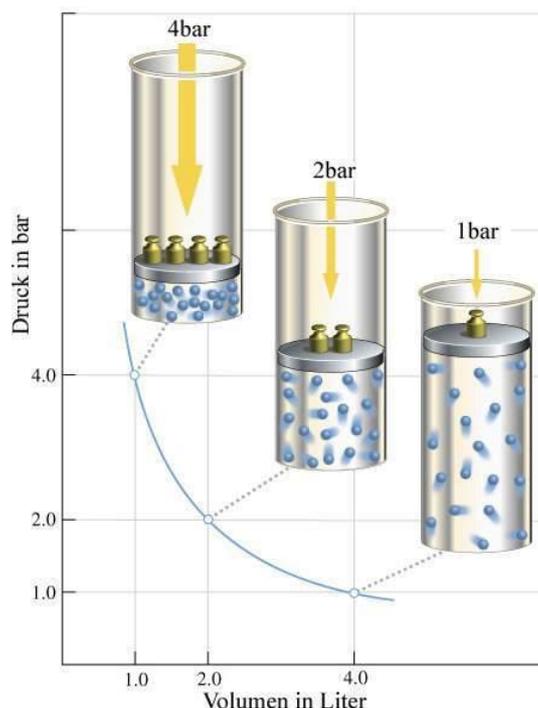
Boyle's law is an experimental gas law that describes how the pressure of a gas tends to increase as the volume of the container decreases. A modern statement of Boyle's law is:

"The absolute pressure exerted by a given mass of an ideal gas is inversely proportional to the volume it occupies if the temperature and the quantity of gas remain unchanged within a closed system"

A)   Explain what happens in the graph

B)   Write the expression that relates between volume and pressure at constant temperature.

C)   A quantity of gas occupies a volume of 70 L at a pressure of 0.8 atm. What volume will it occupy at a pressure of 1.2 atm if the temperature does not change?

D)   A balloon explodes if the volume inside it exceeds 5 L. If for a pressure of 1.25 atm the volume of the balloon is 3 L, at what pressure will the balloon explode?

Related Topics and Concepts: State variables in gases

Real Life Context: Physics and Fluid Technology

Pressure and Temperature   *Author: : Abel Carenas Velamazán*

Joseph Louis Gay-Lussac in the early 1800s, established the relationship between the temperature and the pressure of a gas when the volume is constant. Obtaining the following graph:
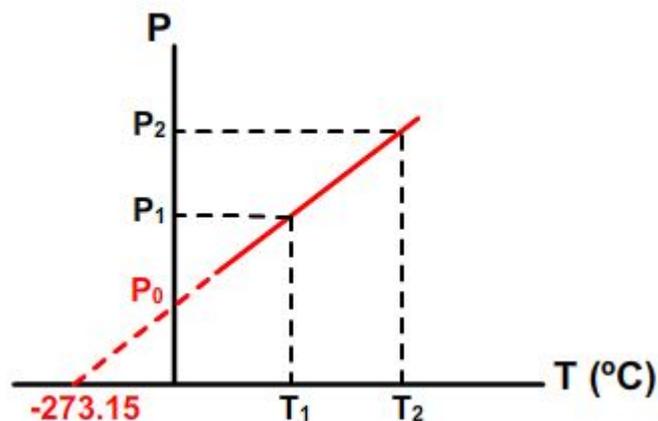
A) What is the relationship between pressure and temperature?

B) Write the expression that relates pressure and temperature

C) A gas is at a pressure of 2 atm and at a temperature of 27°C. Up to what temperature do we have to heat the gas so that the pressure triples? The volume of the gas does not change.

D) The wheel of a car contains air at a pressure of 2.5 atm and the temperature is 20°C. After a long journey the air temperature rises to 55°C. What pressure will the air in the wheel have?

Related Topics and Concepts: State variables in gases

Real Life Context: Physics and Fluid Technology

The echo

Author: *Abel Carenas Velamazán*

Echo is an acoustic phenomenon produced when a wave reflects off a surface and returns to its emitter.

An observer is 510 m. of a wall. Between the observer and the wall, and at an equal distance from both, a shot is fired.After how many seconds will the observer perceive:

a) Direct sound.

b) The echo

Sound speed 340 m / s.

Related Topics and Concepts: Movement, sound

Real Life Context: Physics

Elasticity

Author: Abel Carenas Velamazán

Hooke's law of elasticity or Hooke's law establishes the relationship between elongation or longitudinal stretch and the applied force. Elasticity is the physical property in which objects are capable of changing shape when a deformation force acts on an object and subsequently the object has the ability to return to its original shape when the cause that causes the deformation ceases.

The applied force is proportional to the deformation produced and the constant of proportionality is K, which depends on each elastic material.
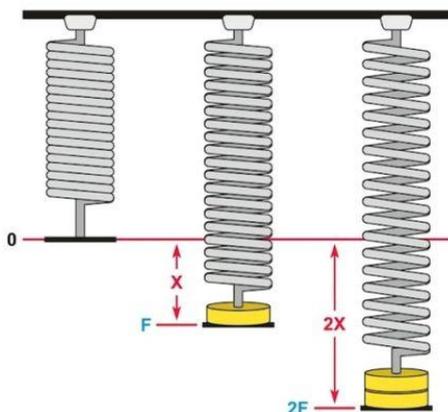
A body is hanging from a spring, so that the length of the same when hanging a body of 6N weight is 5 cm. If it is added 5N more, it becomes 8 cm. What is the spring constant of the spring?

Related Topics and Concepts: Elasticity, proportions

Real Life Context: Physics

Fuel Consumption

Author:Aslıhan Arıcıoğlu

FUEL CONSUMPTION

Diesel fuel is one of the most common fuel used in today's vehicles.As diesel contains more energy than the gasoline, diesel promotes fuel economy much more.Gasoline-powered engines' fuel efficiency rates between 20-25% whereas diesel-powered engines' effiency goes up to 40%.That's why diesel-powered engines can go further endurance wise with the same amount of fuel when compared with gasoline-powered ones.

In addition to this, diesel-powered engines have a more elaborate and detailed structure with their manufacturing technology and extra equipments.

As a result, diesel-powered engines are sold with a higher price.

Fuel Consumption

Author:Aslıhan Arıcıoğlu

Gas Engine   VS   Diesel Engine

**Mr. Adam and Mis. Sarah want to decide on whether to buy gasoline-powered or diesel-powered engines.You can see the data they have collected by talking to the salesperson.

-Numbers for the prices and fuel consumption per 100 km's for them are given below in the chart.

	General	Urban roads	Extra urban roads	Price of the vehicle
Gasoline Vehicle	5,9 lt/100km	6,6 lt/100km	5,5 lt/100km	430500 TL
Diesel Vehicle	4,3 lt/100km	4,7 lt/100km	4,0 lt/100km	461900 TL

-The amount of distance Mr. Adam and Mis. Sarah covered in a year has given below in the chart.

	Urban roads	Extra urban roads
Miss. Sarah	3500 km	2500 km
Mr. Adam	2000 km	20800 km

Fuel Consumption

Author:Aslıhan Arıcıoğlu

The cost per litre of gasoline is 7,07 and diesel is 6,45 TL. So;

· Which vehicle is more advantegous for Mis. Sarah?

· Which vehicle is more advantegous for Mr. Adam?

Prepared by : Aslıhan Arıcıoğlu

Related Topics and Concepts: Profit and loss problems

Real Life Context: Price policy, economy
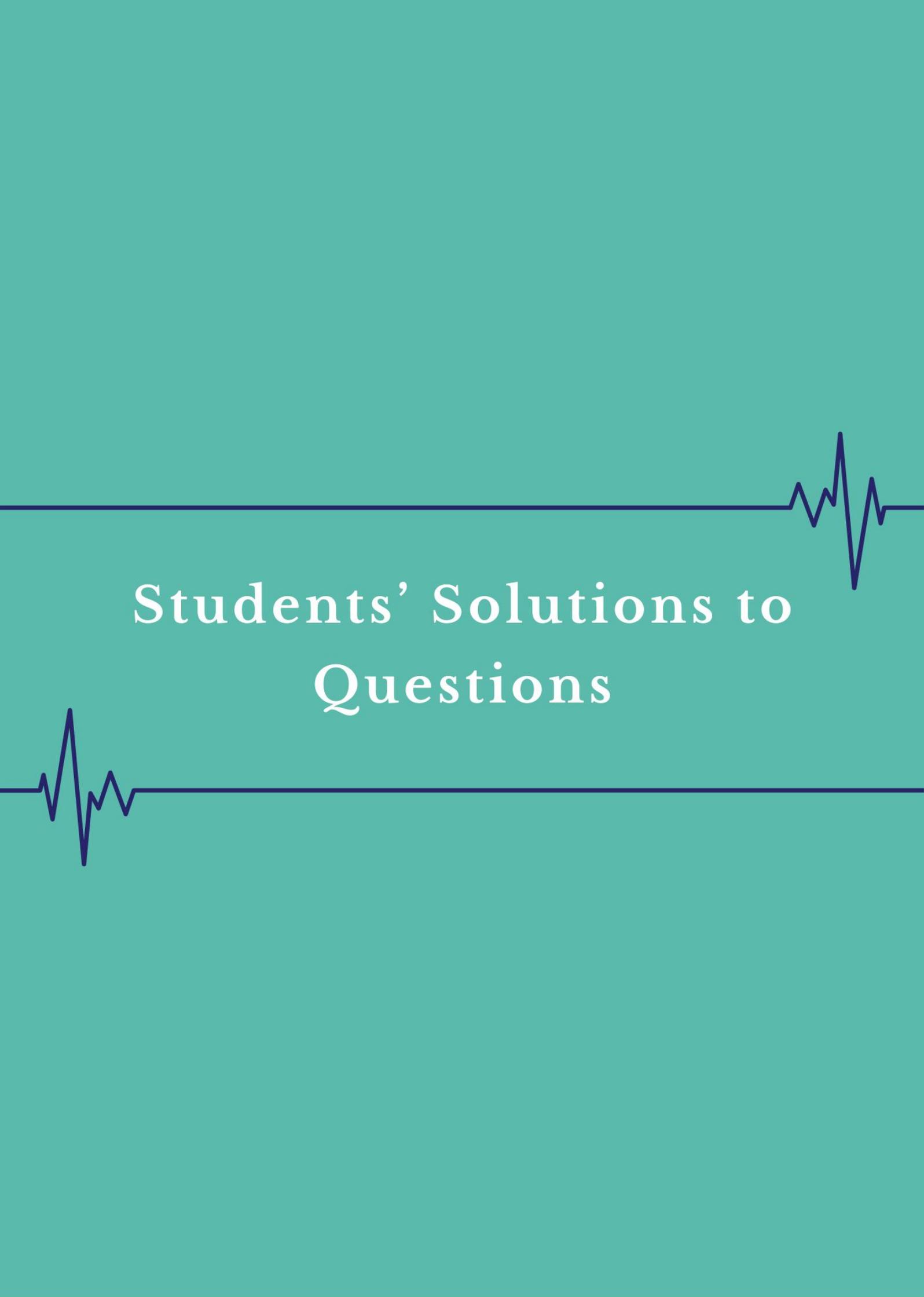
Students' Solutions to Questions

Solution of the problem, Newspaper Sales

The revenue from the newspaper sale is equal to the price of the newspaper multiplied by the number of newspapers sold. Let x be the number of 0,1 Euro discounts on the newspaper's selling price. Let's write the revenue from the newspaper sale as a function that depends on x.

$$f(x)=(1,6-0,1\,x)\cdot(150000+15000\,x)$$

$$f(x)=240000+24000\,x-15000\,x-1500\,x^2$$

$$f(x)=-1500\,x^2+9000\,x+240000$$

This is a quadratic function of x, and the graph opens downward, so the highest point on the graph is the vertex.

x coordinate of the vertex;

$$\frac{-9000}{2\cdot(-1500)}=3$$

Accordingly, if a 0,3 Euro discount is made on the newspaper's selling price, the profit from the sale will be the highest.

Students:

Mehmet Emin C., Kardelen T. and Sarajulhaq Q.
Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

Newspapers are greatly affected by technologies and social media nowadays. Electronic media double their impact every day. When we look at the habits of young people, we learn that on entering the internet, they increasingly choose the news sites. As a result, newspapers shrink in terms of circulation. In the process that starts with a decrease in the number of pages, the owners are forced to lower the price in order to make the newspaper more attractive.

The circulation of a newspaper with a sales price of 1.6 euros is 150 000. According to a study conducted among readers, the number of people who buy newspapers increases by 15 000 for every 0.1 euro discount on the sale price. What should the newspaper's new sale price be in order to make the most profit?

Let's enter the notation: X - this is the number of times for 0.1 euros there should be a discount, P - revenue from the sale of newspapers.

Function:

$$P(x)=(1,6-0.1x)\cdot(150000+15000x)$$

$$P(x)=240000+24000x-15000x-1500x^2=-1500x^2+9000x+240000$$

Since in the function the first coefficient is negative, the function has a maximum

$$x_0=\frac{-9000}{-3000}=3$$

$$0.1x=0.3€$$

Answer: for the maximum profit, the discount on the sale price of the newspaper should be 0.3€

Ņečajevs Mihails 11th grade student

Private secondary school "KLASIKA"

Riga, Latvia

Newspaper sales

MADE BY: SIMONA GEORGIEVA

perfect

$$P(x)=(1,6-0.1x)\cdot(150000+15000x)$$
$$P(x)=240000+24000x-15000x-1500x$$
$$2=-1500x2+9000x+240000$$

X - this is the number of times for 0.1 euros there should be a discount, P - revenue from the sale of newspapers.

Since in the function the first coefficient is negative, the function has a maximum
$$x0=-9000-3000=3 \quad 0.1x=0.3€$$
The discount on the sale price of the newspaper should be 0.3€

THE WATER BILL

Process:

· In subparagraph a) we take the fixed part of the table and add it with the variable part, that part is multiplied by 0 because we don´t consume water and multiply it by VAT which is 10%. We do this same process in all, but the distribution part that according to what we have spent we have to choose between three different blocks and in garbage collection since we
don´t have to increase the VAT.

· In subparagraph b) we take the fixed part of the table and add it with the variable part, that part is multiplied by 23 because the water consumption is 23m³ and multiply it by the VAT which is 10%. We do this same process in all but, the distribution part that as we have spent 23m³ we have to choose block 1 multiplied by the said cubic meters and in the garbage collection since we don´t have to increase the VAT.

· In subparagraph c) we take the fixed part of the table and add it with the variable part, that part is multiplied by 40 because the water consumption is 40m³ and multiply it by the VAT which is 10%. We do this same process in all but, the distribution part that as we have spent 40m³ we have to choose block 1 and 2, in this case we have to start the 40 meters in two blocks, that is, 35 in block 1 and 5 in block 2, add and multiply by VAT and in garbage collection since we don´t have to increase the VAT.

· In subparagraph d) we take the fixed part of the table and add it with the variable part, that part is multiplied by 53 because the water consumption is 53 m³ and multiply it by the VAT which is 10%.
We do this same process in all but, the distribution part that as we have spent 53 m³ we have to choose block 1 and 2, in this case we have to start the 53 meters in three blocks, that is, 35 in block 1, 15 in block 2 and 3 in block 3, add and multiply by VAT and in garbage collection since we don´t have to increase the VAT.

· In subparagraph e) we have to take out which block of the distribution are the €59, 94, we know that it is in block 1. We removed the variable part with VAT (29, 04) which are 30, 9. 30, 9 we divide it by VAT (1, 1) which is
 28, 09 without VAT. We take out what is the variable part to know the result of 1 m³ which is 0, 9365. The VAT-free part (28, 09) is divided by the result of 1 m³ (0, 9365) which result are 30 m³ consumed.

Student: Goizane Jiménez Izquierdo, Spain

Solution of the Problem, Water Bill

We have created a spreadsheet that calculates a water bill based on the amount of water used in Excel. According to this,

a. If we do not consume water during this quarter, we will pay 29.04 Euros for fixed fees and their taxes.

CONSUMPTION: 0

CHARGES	FIX	VARIABLE				VAT	TOTAL
SEWER	1,6624	0				0,16624	1,82864
WATER TREATMENT	4,0938	0				0,40938	4,50318
			BLOCK	CONSUMPTION	PRICE		
DISTRIBUTION	2,7143		1	0	0	0,27143	2,98573
			2	0	0		
			3	0	0		
EXPLOTATION WATER TREATMENT STATION	7,1479	0				0,71479	7,86269
INVESTMENT REDEMPTION	0	0				0	0
GARBAGE COLLECTION	11,86	0				0	11,86

AMOUNT TO BE PAID: 29,04024

b. If our consumption is 23 m³, we have to pay 52.73 Euros.

CONSUMPTION: 23

CHARGES	FIX	VARIABLE				VAT	TOTAL
SEWER	1,6624	2,8681				0,45305	4,98355
WATER TREATMENT	4,0938	3,8847				0,79785	8,77635
			BLOCK	CONSUMPTION	PRICE		
DISTRIBUTION	2,7143		1	23	7,1944	0,99087	10,89957
			2	0	0		
			3	0	0		
EXPLOTATION WATER TREATMENT STATION	7,1479	5,0232				1,21711	13,38821
INVESTMENT REDEMPTION	0	2,5691				0,25691	2,82601
GARBAGE COLLECTION	11,86	0				0	11,86

AMOUNT TO BE PAID: 52,73369

c. If our consumption is 40 m³, we have to pay 73,17 Euros.

CONSUMPTION: 40

CHARGES	FIX	VARIABLE				VAT	TOTAL
SEWER	1,6624	4,988				0,66504	7,31544
WATER TREATMENT	4,0938	6,756				1,08498	11,93478
			BLOCK	CONSUMPTION	PRICE		
DISTRIBUTION	2,7143		1	35	10,948	1,78933	19,68263
			2	5	4,231		
			3	0	0		
EXPLOTATION WATER TREATMENT STATION	7,1479	8,736				1,58839	17,47229
INVESTMENT REDEMPTION	0	4,468				0,4468	4,9148
GARBAGE COLLECTION	11,86	0				0	11,86

AMOUNT TO BE PAID: 73,17994

d. If our consumption is 53 m³, we have to pay 96,17 Euros.

	CONSUMPTION:	53				
CHARGES	FIX	VARIABLE			VAT	TOTAL
SEWER	1,6624	6,6091			0,82715	9,09865
WATER TREATMENT	4,0938	8,9517			1,30455	14,35005
DISTRIBUTION	2,7143	BLOCK	CONSUMPTION	PRICE	3,06939	33,76329
		1	35	10,948		
		2	15	12,693		
		3	3	4,3386		
EXPLOTATION WATER TREATMENT STATION	7,1479	11,5752			1,87231	20,59541
INVESTMENT REDEMPTION	0	5,9201			0,59201	6,51211
GARBAGE COLLECTION	11,86	0			0	11,86

AMOUNT TO BE PAID: 96,17951

Students:

Mehmet Emin C., Kardelen T. Sarajulhaq Q. and Umut S.
Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

Solution of the Problem, Water Bill

Let's write the water bill to be paid as a function dependent on x, where x is the amount of water consumed.

$0 \leq x \leq 35$, f(x) = 29,04024 + 1,1.(0,6237.x + 0,3128.x)

f(x) = 29,04024 + 1,03015.x

35<x<50, f(x) = 29,04024 + 1,1.[0,6237.x + 0,3128.35 + (x-35).0,8462]

f(x) = 29,04024 + 1,1.(0,6237.x + 10,948 + 0,8462.x -29617)

f(x) = 29,04024 + 1,1.(1,4699.x -18,669)

f(x) = 8,50434 + 1,61689.x

$x \geq 50$, f(x) = 29,0424 + 1,1.[0,6237.x + 0,3128.35 + 0,8462.15 + (x-50).1,4462]

f(x) = 29,0424 + 1,1.[0,6237.x + 10,948 + 12,693 + 1,4462.x – 72,31]

f(x) = 29,0424 + 1,1.(2,0699.x – 48,669)

f(x) = -24,49566 + 2,27689.x

If we calculate the desired values by plotting this function with desmos,

a) We will pay 29.04 euros bill if we do not consume any water

b) If we consume 23 cubic meters of water, we will pay 52.73 Euros.

c) If we consume 40 cubic meters of water, we will pay a bill of 73.17 Euros.

d) If we consume 53 cubic meters of water, we will pay a bill of 96.17 Euros.

e) In order to pay 59.94 Euros, we must have consumed 30 cubic meters of water.
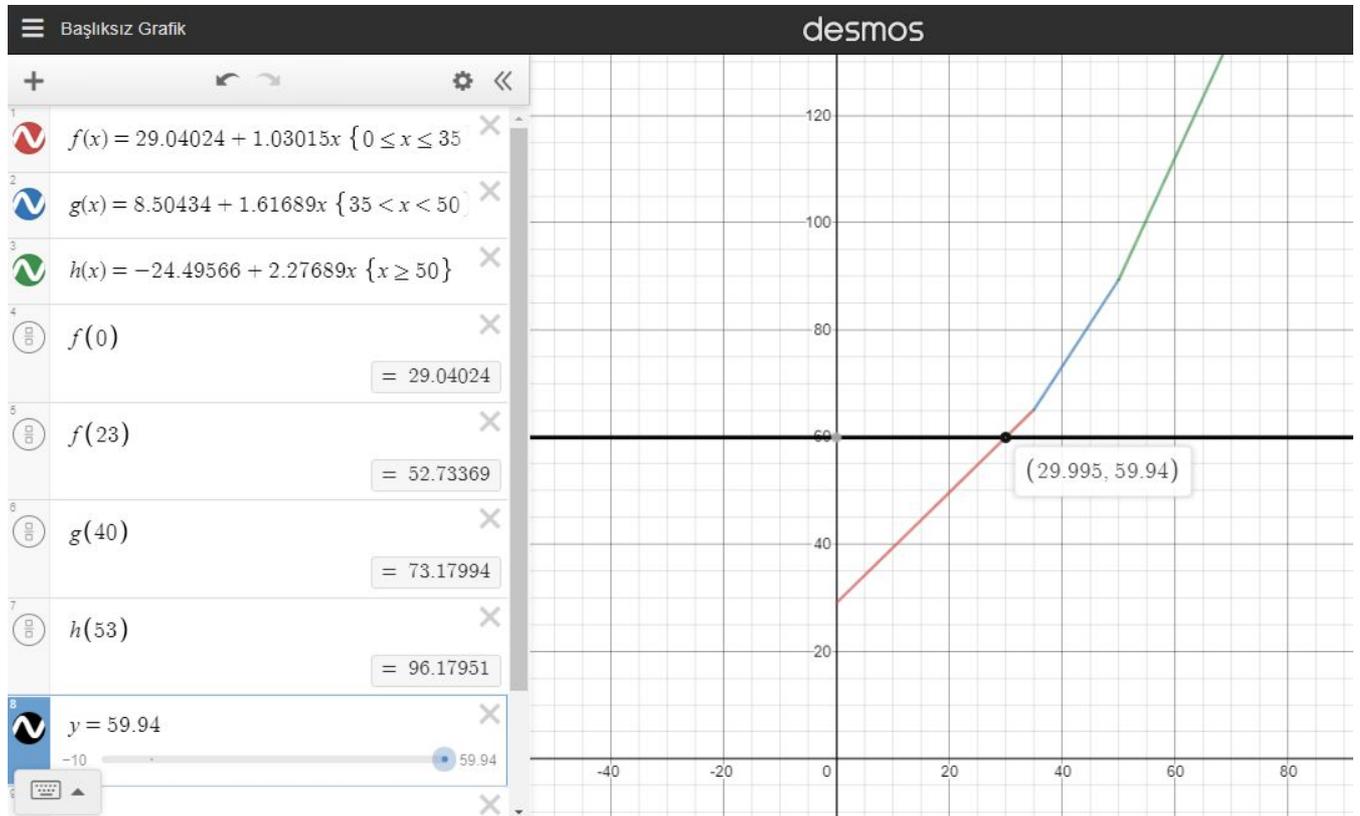
Students:

Sümeyye B., Hilal K. and Medine K.

Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

The water bill solution

By Eugene Gofenshefer from Latvia

Invoicing concepts:

CHARGES	FIX	VARIABLE			VAT
SEWER	1.6624€	0.1247 €/m³			10%
WATER TREATMENT	4.0938€	0.1689 €/m³			10%
DISTRIBUTION	2.7143€	**BLOCK**	**LIMIT**	**€/m³**	10%
		1	$x \leq 35$	0.3128	
		2	$35 < x < 50$	0.8462	
		3	$x \geq 50$	1.4462	
EXPLOTATION WATER TREATMENT STATION	7.1479€	0.2184€/m³			10%
INVESTMENT REDEMPTION	0€	0.1117€/m³			10%
GARBAGE COLLECTION	11.86€	0			0%

a) If I didn't consume a single cubic meter of water I still have to pay 0.3128€, as

$x \in [0; 35]$

b) 23≤35, therefore 23x0.3128=7.1944

c) 35≤40<50, therefore 35x0.3128+5x0.8462=15.17

d) 53≥50, therefore 35x0.3128+15x0.8462+3x1.4462=27.9796.

e) 54.94€>27.98€, therefore x≥50, so 35x0.3128+15x0.8462+1.4462x=54.94€ from which we get that x≈21.64 cubic meters.

f) A piecewise-defined function (y asis – price, x asis – amount of cubic meters)
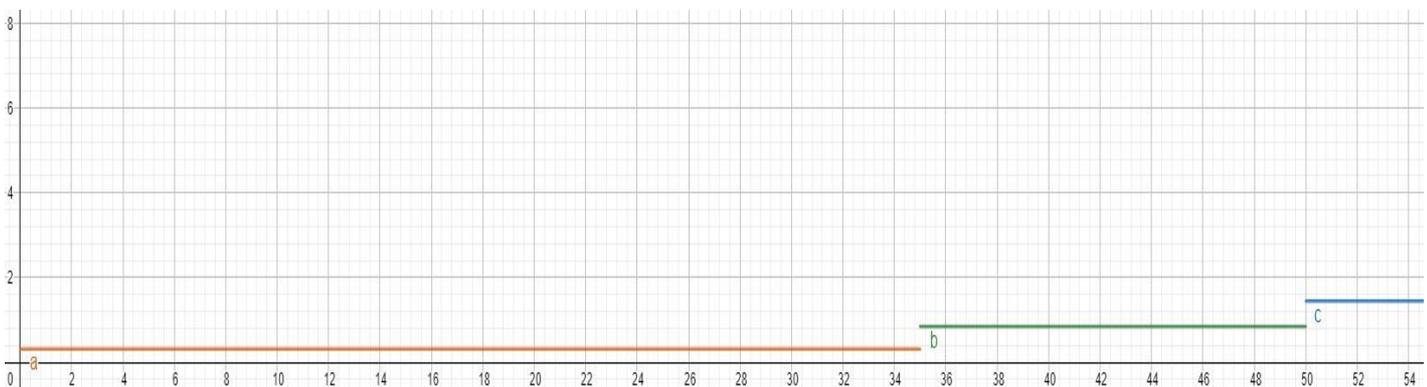
Solution of the problem, Highway Speed Corridor Application

V: Speed

X: Distance $V = \dfrac{X}{t}$

t: Time

a) The minimum arrival times for cars in accordance with the speed limit,

$$120 = \frac{450}{t} \Rightarrow t = \frac{450}{120} \Rightarrow t = 3\,h\,45\,min$$

The minimum arrival times for trucks in accordance with the speed limit,

$$90 = \frac{450}{t} \Rightarrow t = \frac{450}{90} \Rightarrow t = 5\,h$$

b) $\dfrac{450}{4} = 112{,}5\,km/h$, $\dfrac{90 \cdot 110}{100} = 99$ and $\dfrac{90 \cdot 130}{100} = 117$

Accordingly, the truck driver who completes his journey in 4 hours pays a penalty of 235 TL for exceeding the speed limit between 10 % and 30 %.

c) 3 h 15 min = 3,25 h

$\dfrac{450}{3{,}25} \approx 138\,km/h$, $\dfrac{120 \cdot 110}{100} = 132$ and $\dfrac{120 \cdot 130}{100} = 156$

Accordingly, the car driver who completes his journey in 3 hours and 15 minutes pays a penalty of 235 TL for exceeding the speed limit between 10 % and 30 %.

Students:

Mehmet Emin C., Kardelen T., Öner M., Sarajulhaq Q. and Umut S.
Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

Highway Speed Corridor Application Solution

V- Speed

d- Distance $\qquad v=\dfrac{d}{t}$

t-Time

a) $120=\dfrac{450}{t}$

$t=\dfrac{450}{120}$ $\qquad$ } $\qquad$ ⇒The minimum arrival times for cars

$t= 3$ hours & 45 minutes

$90=\dfrac{450}{t}$

$t=\dfrac{450}{90}$ $\qquad$ } $\qquad$ ⇒The minimum arrival times for trucks

$t= 5$ hours

b) $\dfrac{450}{4}= 112,5$ km/hour

$\dfrac{90\cdot110}{100}=99$

$\dfrac{90\cdot130}{100}=117$

The truck driver pays a penalty of 235 TL for exceeding the speed limit between 10% and 30%

c) $\dfrac{450}{3,25}\approx138\dfrac{km}{h}$

$\dfrac{120\cdot110}{100}=132$

$\dfrac{120\cdot130}{100}=156$

The car driver pays a penalty of 235 TL for exceeding the speed limit between 10% and 30%

Students:

Gaciu Isabela (Lisa) & Maria Bedau (Raisa)

"Jean Monnet" highschool; Romania

Speed Problems, Percentages

Daniel Klimkevics Grade 11.b, Private Secondary School , Klasika'' Riga, Latvia

Condition

Speed limit is set on motorways; 120 km / h for cars and 90 km / h for trucks. The speed limit penalties defined in the rules are TL 235 if the speed limit is exceeded between 10% and 30%, and TL 488 if more than 30% is exceeded.

a) Determine the minimum arrival time

truck time = 450km/90km/h
car time = 450km/120km/h

Answer
For truck - 5 hours
For car - 3 hrs 45 min

b) Determine if the truck driver who completes the trip in 4 hours will pay a traffic ticket, and if so, how much he will pay.

450/4= 112,5km/h   90/112,5=100/x
90 km/h - 100%   0,8x=100
112,5 km/h - x%   x=125% - attached speed

Answer
Fine = 235 lir because attached speed is 25%

c) Determine if the driver of the car who will make the trip in 3 hours 15 minutes will pay a traffic ticket

450 : 13/4 = 450 * 4/13 = 1800/13= 138,5
120 km/ h - 100%
138,5 km/h - x%
120/138,5 - 100/x
x=1385/12
x = 115,5

Answer
Fine = 235 lir because speed limit is attached - 15,5% (less than 30%)

Solution of the problem, Holiday Plan

According to the 10-year August temperature data, the average temperatures calculated in excel for 10-day intervals are as follows.

Days	Max. Temp	Min. Temp.	Average Daily Temp.
1	33	27	30
2	34	27	30,5
3	33	26	29,5
4	33	26	29,5
5	32	26	29
6	32	26	29
7	33	26	29,5
8	33	27	30
9	33	27	30
10	32	26	29
11	32	26	29
12	32	26	29
13	33	26	29,5
14	33	27	30
15	34	27	30,5
16	33	27	30
17	33	26	29,5
18	33	26	29,5
19	33	26	29,5
20	33	26	29,5
21	33	26	29,5
22	33	26	29,5
23	33	26	29,5
24	33	26	29,5
25	34	25	29,5
26	33	26	29,5
27	33	26	29,5
28	32	26	29
29	30	24	27
30	31	24	27,5
31	31	24	27,5

10 day Intervals	Average Temp.
1 – 10	29,6
2 – 11	29,5
3 – 12	29,35
4 – 13	29,35
5 – 14	29,4
6 – 15	29,55
7 – 16	29,65
8 – 17	29,65
9 – 18	29,6
10 – 19	29,55
11 – 20	29,6
12- 21	29,65
13 – 22	29,7
14 – 23	29,7
15 – 24	29,65
16 – 25	29,55
17 – 26	29,5
18 – 27	29,5
19 - 28	29,45
20 – 29	29,2
21 – 30	29
22 – 31	28,8

The highest 10-day average temperature of August is between 13-22 August or 14-23 August. This person should have his holiday between 13-22 August or 14-23 August.

Students:

Mehmet Emin C., Kardelen T. and Sarajulhaq Q.

Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

Holiday Plan Solution

					1-33	2-34
3-33	4-33	5-32	6-32	7-33	8-33	9-33
10-32	11-32	12-32	13-33	14-33	15-34	16-33
17-33	18-33	19-33	20-33	21-33	22-33	23-33
24-33	25-34	26-33	27-33	28-32	29-30	30-31
31-31						

The maximum temperature

					1-27	2-27
3-26	4-26	5-26	6-26	7-26	8-27	9-27
10-26	11-26	12-26	13-26	14-27	15-27	16-27
17-26	18-26	19-26	20-26	21-26	22-26	23-26
24-26	25-25	26-26	27-26	28-26	29-24	30-24
31-24						

The minimum temperature

					1-30	2-30,5
3-29,5	4-29,5	5-29	6-29	7-29,5	8-30	9-30
10-29	11-29	12-29	13-29,5	14-30	15-30,5	16-30
17-29,5	18-29,5	19-29,5	20-29,5	21-29,5	22-29,5	23-29,5
24-29,5	25-29,5	26-29,5	27-29,5	28-29	29-27	30-27,5
31-27,5						

the average

10 day time interval	Average temperature
1-10	29,6
2-11	29,5
3-12	29,35
4-13	29,35
5-14	29,4
6-15	29,55
7-16	29,65
8-17	29,65
9-18	29,6
10-19	29,55
11-20	29,6
12-21	29,65
13-22	29,7
14-23	29,7
15-24	29,65
16-25	29,55
17-26	29,5
18-27	29,5
19-28	29,45
20-29	29,2
21-30	29
22-31	28,8

The best period for booking a holiday is between 13-22 August or 14-23 August beacause then the average temperature is the highest.

Students : Gaciu Isabela (Lisa) & Maria Bedau (Raisa)
"Jean Monnet" Hightschool; Romania

The solution to Area of Cardboards

First of all, we selected some random cardboard pieces and we calculated the area of each of them. These are the shapes that we selected for this problem:

The 1'st shape is resembles a hexagon, but it has curved edges. We added that in to show how this would help you in day to day life.

	A	B	C	D	E
1		Shape 1	Shape 2	Shape 3	Shape 4
2	wheight (g)	1	2	3	4
3	Area cm^2	x	64	84	100
4	Weight to Area ratio	$\dfrac{1}{x}$	$\dfrac{2}{64}$	$\dfrac{3}{84}$	$\dfrac{4}{100}$

After that, we plugged our numbers into the table provided and in GeoGebra soon after and these were our results.

Because the points can't form a straight line, we got around that by using a regression line. A regression line is a line that shows the approximation of the sum of all the values of a set amount of data. We use it to get a near perfect prediction of the area the cardboard has for a known weight.

GeoGebra Calculator Suite

- B = (2, 64)
- C = (3, 84)
- D = (4, 100)
- f : YDreaptăRegresie({B, C, D})
  → y = 18x + 28.6666666666667
- A = (1, 47)
- De intrare...
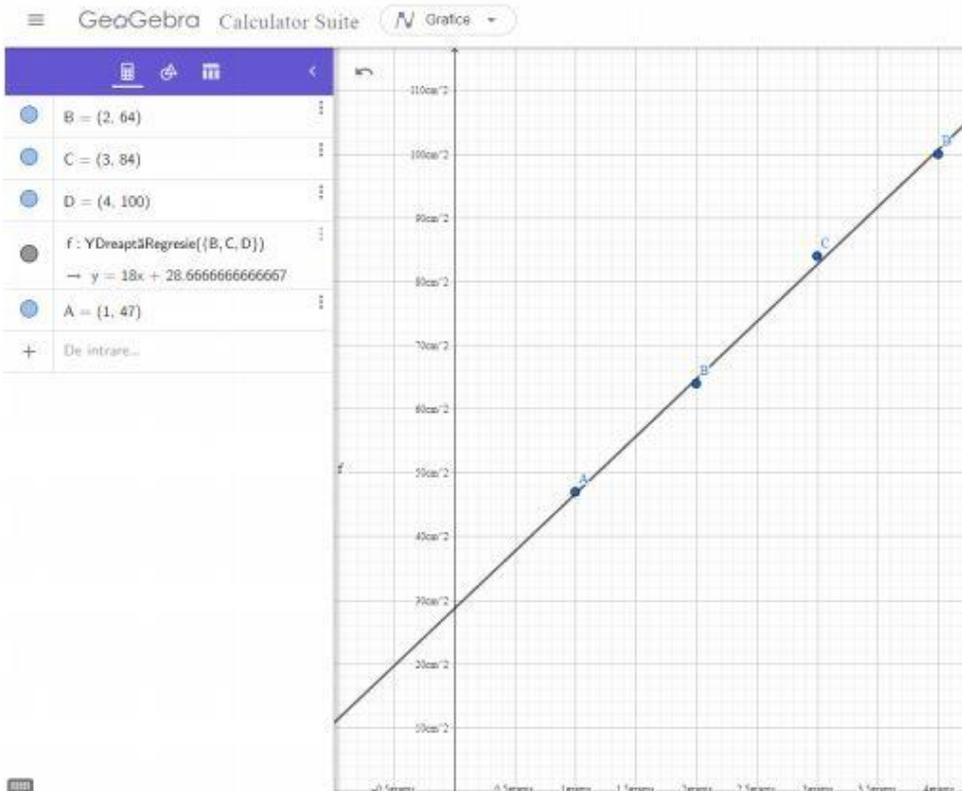
So, the function for the formula for calculating the weight to area of the cardboard is f(x)=18x+80:3. The Ox axis represents the weight of the cardboard shape and the Oy axis represents the area of the cardboard.

Therefore, the approximate area of the first shape is around 47 cm^2.

Students: Niculescu Vlad and Morarescu Matei
from Jean Monnet High School

Solution of the Problem, Defective Bulbs

In Poisson distribution, we should take $\lambda = 10$ and $x = 4$.

$$p(x;\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x!} \qquad p(4;10)=\frac{e^{-10}10^{4}}{4!}\approx 0.019$$

The probability that 4 bulbs are out of order on a control day is 0.019.

The solution of the problem with spreadsheet software is as follows.

İşlev Sihirbazı ✕

Fonksiyonlar Yapı

POİSSON.DAĞ   İşlev sonucu   0,018916637401035

Ara

Poisson dağılımı değerleri.

Kategori

İstatistiksel ⌄

Birikimli (isteğe bağlı)

İşlev

Olasılık yoğunluk işlevi 0 ya da YANLIŞ olarak hesaplanır. Diğer değerler, DOĞRU olarak ya da ihmal edilmişse birikimli dağılım işlevi olarak hesaplanır.

OLASILIK
ORTALAMA
ORTALAMAA
ORTANCA
ORTSAP
PEARSON
PERMÜTASYON   Sayı   *fx*   4
PERMÜTASYONA
PHI   Ortalama *fx*   10
POISSON
POİSSON.DAĞ   Birikimli *fx*   0
RANK
RANK.EŞİT
RANK.ORT   Formül   Sonuç 0,018916637401035
RKARE
SAPKARE   =POİSSON.DAĞ(4;10;0)
STANDARTLAŞTIRMA
STDSAPMA
STDSAPMA.P ⌄

☐ Dizi   Yardım   < Geri   İleri >   Tamam   İptal

Students:

Mehmet Emin C., Sarajulhaq Q. and Umut S.
Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

Defective Bulbs

The Poisson distribution is used to model the number of events occurring within a given time interval.
The formula for the Poisson probability mass function is

$$p(x;\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{for x = 0, 1, 2, ...}$$

λ is the shape parameter which indicates the average number of events in the given time interval.
A company estimates that an average of 10 bulbs are defective every day during the final control of the bulbs it produces.
Calculate the probability that 4 bulbs will be defective on a control day using the Poisson distribution.

$\lambda =$ 10

$x =$ 4   P(X = x) = ∨ 0.01892
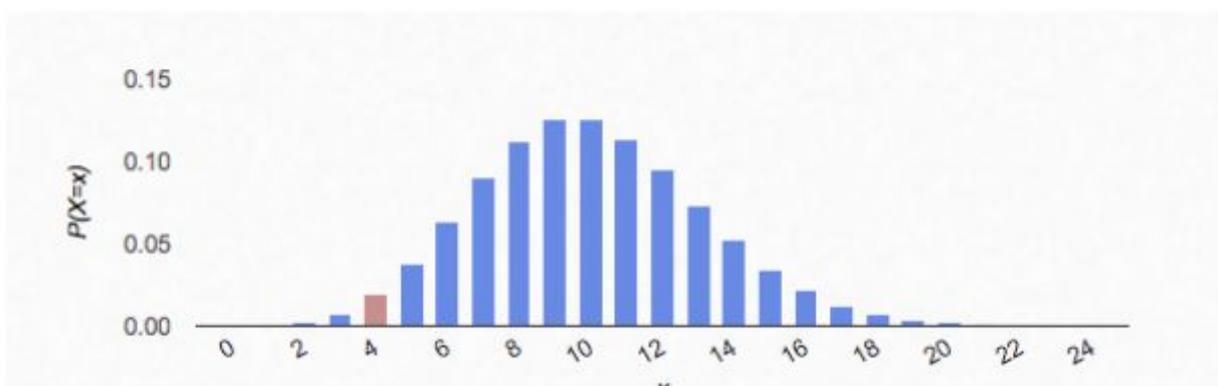
Answer : 0.01892 the probability that 4 bulbs will be defective on a control day which we found using the Poisson distribution.

By Sofija Ter-Oganesova, Latvija

IBUPROFEN 40mg/ml

Contains: 150 ml.

Contains: 40 x150 mg = 6000mg ibuprofen

Dosage: The dose depends on the child's weight and age. For children between 3 months and 12 years old, the daily recommended dose is 25 mg/kg a day (every 6 or 8 hours).

Ramiro is 8 years old and weighs 28 kg

1. What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?

1 ml of syrup contain 40 mg ibuprofen

2. Calculate how many mg. Ramiro must consume every day.

1 kg .... 25 mg

28 kg ..... x = 25 · 28 = 700 mg ibuprofen for one day

3. Calculate how many ml. Ramiro must consume every day.

40mg .... 1 ml

700 mg........ x = $\frac{700}{40}$ = 17.5 *ml* syrup every day

4. How many ml. must Ramiro drink each dose?

17.5 *ml*: 4 = 4.375 *ml*

5. Complete the table below:

Weight(kg)	mg/day	ml/day	ml/dose
5	125	3.125	0.78125
10	250	6.25	1.5625
15	375	9.375	2.34375
20	500	12.5	3.125
25	625	15.625	3.90625
28	700	17.5	4.375
30	750	18.75	4.6875
35	875	21.875	5.46875
40	800	20	6.25

$125 \text{ mg} \dots \frac{125}{40} = 3.125 \text{ ml}$

$250 \text{ mg} \dots \frac{250}{40} = 6.25 \text{ ml}$

$700 \text{ mg} \dots \dots \frac{700}{40} = 17.5 \text{ ml}$

6. Find out a formula relating weight and dose (mg.).
   1kg......25 mg
   X kg.....y $\Rightarrow y = 25x$ *each day.* $\Rightarrow$ *The dose is done through the function:*
   $$f: [0; 40] \to R, f(x) = \frac{25x}{4}$$

7. Find out a formula showing the relationship between weight and dose (ml.).
   $$g(x) = \frac{25x}{4} : 40 = \frac{25x}{4 \cdot 40} = \frac{5x}{32},$$
   $$g: [0; 40] \to R, f(x) = \frac{5x}{32}$$

8. Apply the previous formulae and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.
   $$g(22.5) = \frac{5 \cdot 22.5}{32} = \frac{112.5}{32} \approx 3.5$$

9. Represent each function using a coordinates system axis.
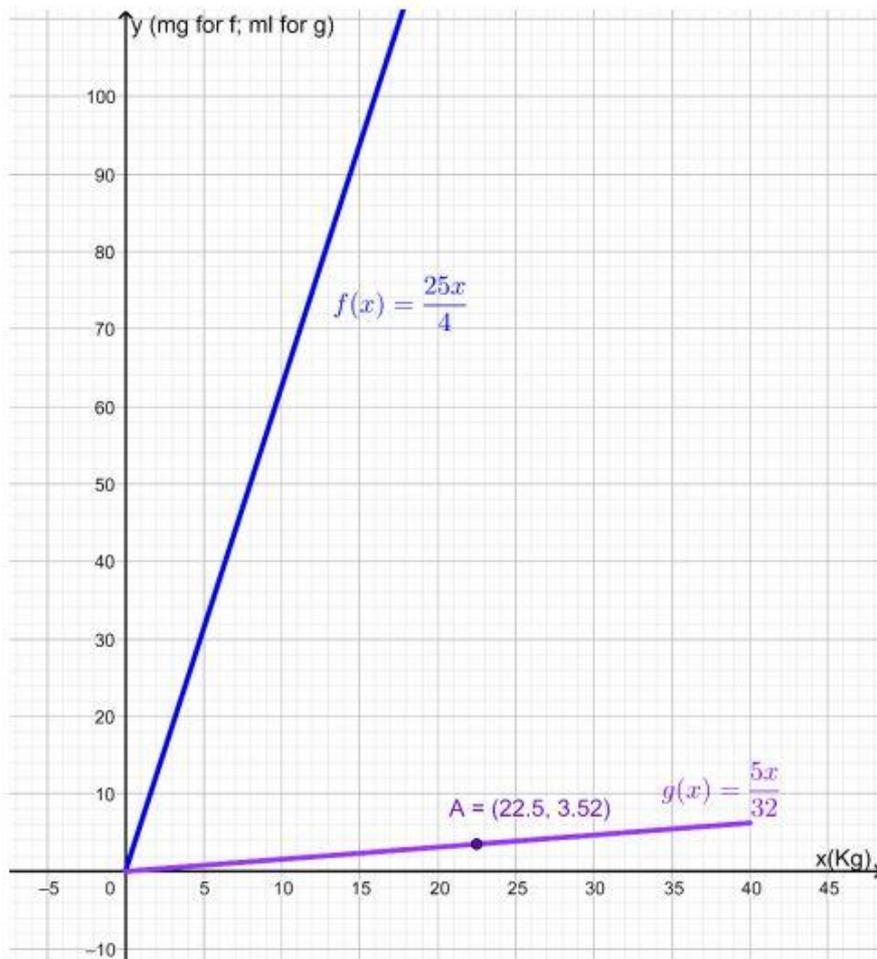
IBUPROFEN 40mg/ml - Solution

Contains: 150 ml.
Contains: 40 x150 mg = 6000mg ibuprofen
Dosage: The dose depends on the child's weight and age. For children between 3 months and 12 years old, the daily recommended dose is 25 mg/kg a day (every 6 or 8 hours).

Weight(kg)	mg/day	ml/day	ml/dose	mg/dose
1	**25**	**0,625**	0,15625	6,25
5	125	3,125	0,78125	31,25
10	250	6,25	1,5625	62,5
15	375	9,375	2,34375	93,75
20	500	12,5	3,125	125
25	625	15,625	3,90625	156,25
28	**700**	**17,5**	**4,375**	**175**
30	750	18,75	4,6875	187,5
35	875	21,875	5,46875	218,75
40	1000	25	6,25	250

The relationship between weight and dose (mg.)

The relationship between weight and dose (ml.)

Alex & Maria, LT Jean Monnet
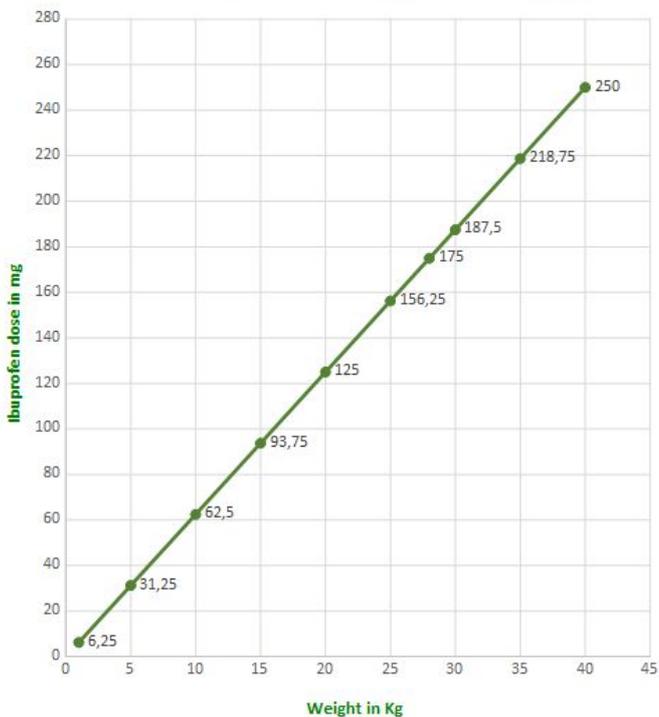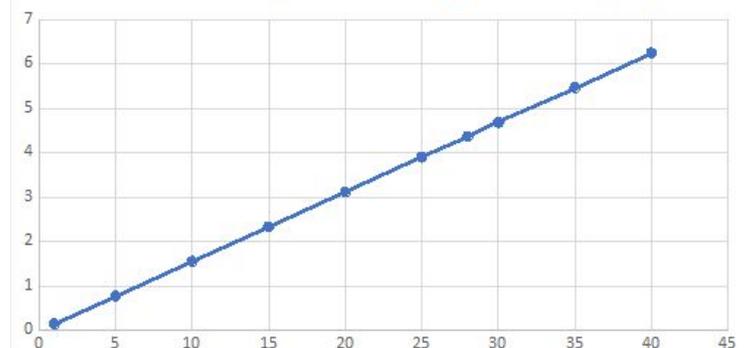
Patient information leaflet-THE IBUPROFEN

Contains: 150 ml.
Dosage: The dose depends on the child's weight and age. For children between 3 months and 12 years old, the daily recommended dose is 25 mg/kg a day (every 6 or 8 hours).

1-) 1 ml syrup equals is 40mg

2-) The weight of Ramiro is 28 kg and the syrup is used as 25 mg / kg daily. We should multiply 28 by 25.

28.25 = 700 mg/daily

3-) If 1 ml syrup is 40 mg, 700 mg syrup is 700 : 40 = 17,5 ml Ramiro should use 17.5 ml syrup per day.

4-) If the drug is used every 6 hours, it should be used 4 times a day. Accordingly, Ramiro must drink 17,5 : 4 = 4,375 ml syrup for each dose.

5-)

Weight (kg)	mg/day	ml/day	ml/dose
5	125	3,125	0,78125
10	250	6,25	1,5625
15	375	9,375	2,34375
20	500	12,5	3,125
25	625	15,625	3,90625
30	750	18,75	4,6875
35	875	21,875	5,46875
40	1000	25	6,25

6-) The amount of syrup that a child weighing x kg should use for each dose is calculated with the following formula.

$$f(x) = 25x : 4 = 6{,}25x \text{ mg/dose}$$

7-) The amount of syrup a child weighing x kg should use for each dose is as follows in ml, depending on her/his weight.

$$g(x) = (25x : 40) : 4 = 0{,}15625x \text{ ml/dose}$$

8-) The amount of syrup that a child weighing 22.25 kg should use for each dose is calculated as follows.

$$f(22{,}25) = 6{,}25.22{,}5 = 175 \text{ mg/dose}$$
$$g(22{,}25) = 0{,}15625.22{,}25 = 3{,}4765625 \text{ ml/dose}$$

9-) The graphs of these functions drawn with GeoGebra are as follows.
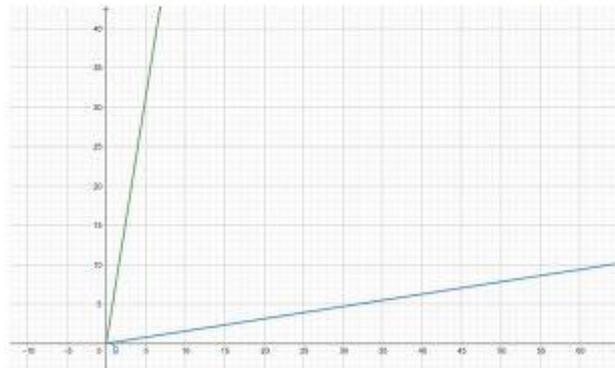
Students:

Mehmet Emin C., Kardelen T., Betül K., Umut S. and Sarajulhaq Q.
Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

PATIENT INFORMATION LEAFLET

THE IBUPROFEN.

   Mr. and Ms. Martínez went to the doctor because their son Ramiro was ill. Ramiro had a temperature and the doctor decided to prescribe him an antipyretic (an antipyretic is a medicine that brings the fever down).
   Ramiro's parents bought the syrup and when they read the patient information leaflet, there appeared the information below:

IBUPROFEN 40mg/ml

Contains: 150 ml.
Dosage: The dose depends on the child's weight and age. For children between 3 months and 12 years old, the daily recommended dose is 25 mg/kg a day (every 6 or 8 hours).

Ramiro is 8 years old and weighs 28 kg

1.-What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?

$40 \, mg$

2.- Calculate how many mg. Ramiro must consume every day.

$25 \times 28 = 700 \, mg$

3.- Calculate how many ml. Ramiro must consume every day.

$700 \div 40 = 17'5 \, ml$

4.- How many ml. must Ramiro drink each dose?

$17'5 \div 3 = 5'83 \, ml$

5.- Complete the table below:

Weight (kg)	mg/day	ml/day	ml/dose
5	125	3'125	1'04
10	250	6'25	2'083
15	375	9'375	3'125
20	500	12'5	4'16
25	625	12'625	5'2083
30	750	18'75	6'25
35	875	21'875	7'2916
40	1000	25	8'3

6.- Find out a formula relating weight and dose (mg.).

$x \times 25 = y$

$25\,x = y$

7.- Find out a formula showing the relationship between weight and dose (ml.).

$25\,x \div 40 = y$

8.- Apply the previous formulae and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.

$25 \times 22.25 = 556\,{'}25\,mg$

$\dfrac{25 \times 22.25}{40} = 13\,{'}90625\,ml$

9.- Represent each function using a coordinates system axes.
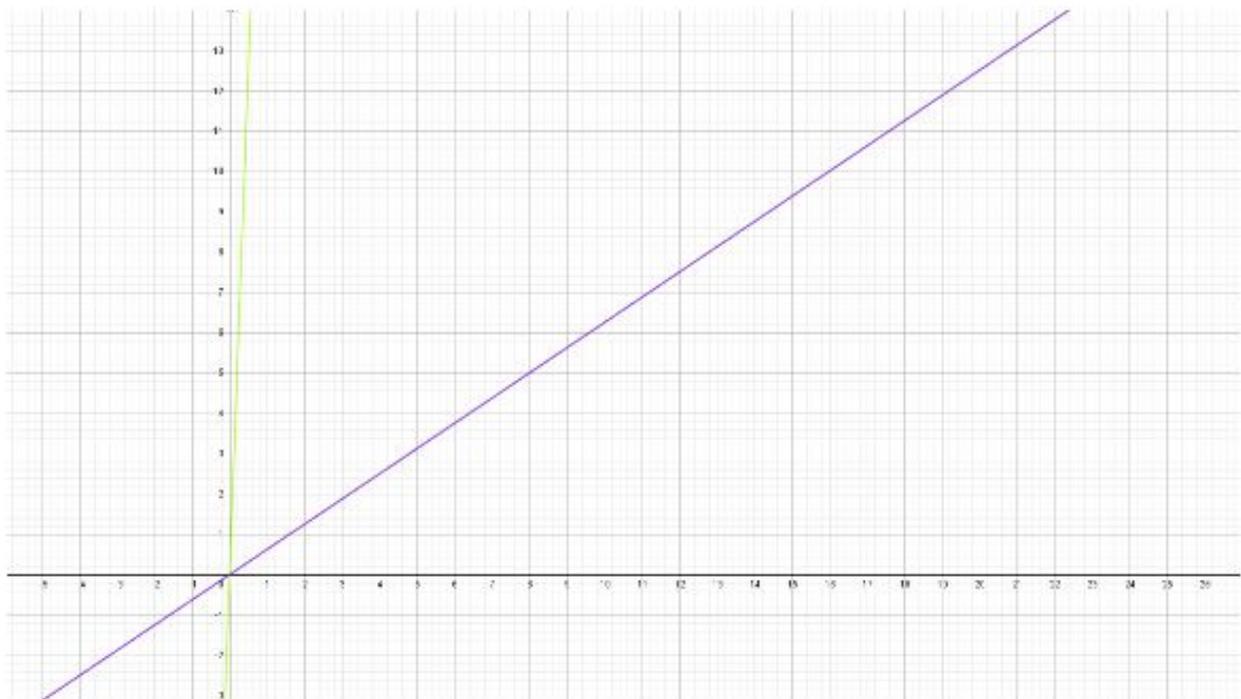
By Goizane Jiménez Izquierdo

SOLUTION OF THE PATIENT INFORMATION LEAFLET

By Juan Alexander

1.-What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?

It means that in each ml of syrup there are 40 mg of ibuprofen.

2.- Calculate how many mg. Ramiro must consume every day.

$28 \cdot 25 = 700$mg.

3.- Calculate how many ml. Ramiro must consume every day.

$700 : 40 = 17'5$ ml.

4.- How many ml. must Ramiro drink each dose?

$17'5 : 3 = 5'83$ ml every day

5.- Complete the table below:

Weight	Mg / day	Ml / day	Ml / dose
5	125	3'125	1'04167
10	250	6'25	2'083
15	375	9'375	3'125
20	500	12'5	4'167
25	625	15'625	5'2083
30	750	18'75	6'25
35	875	21'875	7'29167
40	1000	25	8'3

6.- Find out a formula relating weight and dose (mg.).

$25x = y$

7.- Find out a formula showing the relationship between weight and dose (ml.).

$25x : 40 = y$

8.- Apply the previous formulae and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.

$22'25 \cdot 25 = 556'25$ mg $25 \cdot 22'25 : 40 = 13'90625$ ml

9.- Represent each function using a coordinates system axes.

SOLUTION OF THE PATIENT INFORMATION LEAFLET

By Leticia Jávega

1.-What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?
1 ml contains 40 mg.
2.- Calculate how many mg. Ramiro must consume every day.
He must consume 700mg. (28kg x 25mg/kg= 700 mg)
3.- Calculate how many ml. Ramiro must consume every day.
He must consume 17.5ml. (700/40= 17.5ml)
4.- How many ml. must Ramiro drink each dose?
He must drink 5.83. (17.5/3= 5.83)
5.- Complete the table below:

Weight (Kg)	Mg/day	Ml/day	Ml/dose
5	125	3.1	1.03
10	250	6.25	2.083
15	375	9.375	3.125
20	500	12.5	4.16
25	625	15.625	5.5416
30	750	18.75	6.25
35	875	21.875	7.2916
40	1000	25	8.3

6.- Find out a formula relating weight and dose (mg.).
$Y = 25x$.
7.- Find out a formula showing the relationship between weight and dose (ml.).
$Y = 25x/40$.
8.- Apply the previous formulae and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.
$Y = 22.25kg \times 25 = 556.25mg$ $Y = (22.25kg \times 25)/40 = 13.90625ml$.

9.- Represent each function using a coordinates system axes.
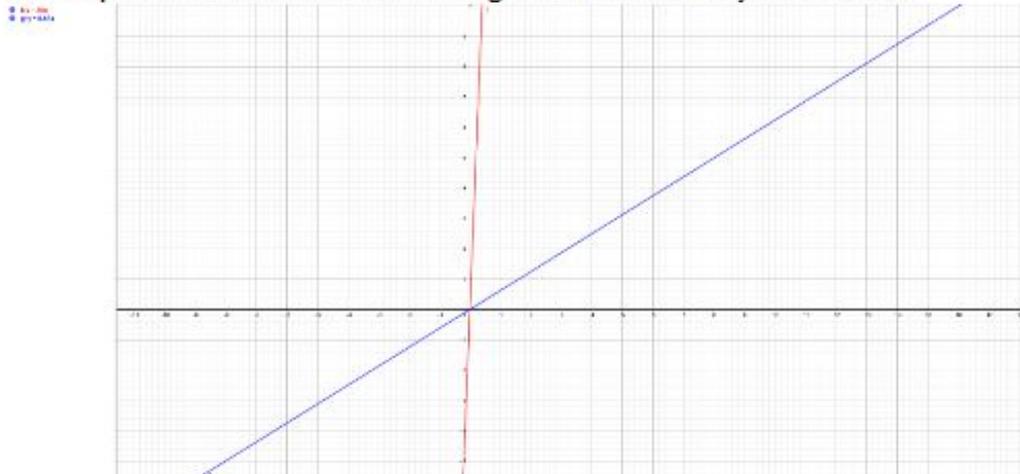
SOLUTION OF THE PATIENT INFORMATION LEAFLET

By Jorge Marín García

1.-What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?

The meaning of the expression 40mg/ml is that in each ml of syrup there are 40 mg of ibuprofen.

2.- Calculate how many mg. Ramiro must consume every day.

28 · 25 = 700mg. Ramiro must consume 700mg of ibuprofen every day

3.- Calculate how many ml. Ramiro must consume every day.

700: 40= 17'5 ml. Ramiro must consume 17'5ml of ibuprofen every day

4.- How many ml. must Ramiro drink each dose?

17'5 : 3 = 5'83 ml every day

5.- Complete the table below:

Weight	Mg / day	Ml / day	Ml / dose
5	125	3'125	1'04167
10	250	6'25	2'083
15	375	9'375	3'125
20	500	12'5	4'167
25	625	15'625	5'2083
30	750	18'75	6'25
35	875	21'875	7'29167
40	1000	25	8'3

6.- Find out a formula relating weight and dose (mg.).

$25x = y$

7.- Find out a formula showing the relationship between weight and dose (ml.).

$25x/40 = y$

8.- Apply the previous formulae and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.

22'25 · 25 = 556'25 mg   25 · 22'25/40 = 13'90625 ml

9.- Represent each function using a coordinates system axes.

SOLUTION OF THE PATIENT INFORMATION LEAFLET

By Oscar Claramonte

1.-What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?
It means that in each ml of syrup there are 40 mg of ibuprofen.
2.- Calculate how many mg. Ramiro must consume every day.
$28 \cdot 25 = 700$mg.
3.- Calculate how many ml. Ramiro must consume every day.
700: 40= 17'5 ml.
4.- How many ml. must Ramiro drink each dose?
17'5 : 3 = 5'83 ml every day
5.- Complete the table below:

Weight	Mg / day	Ml / day	Ml / dose
5	125	3'125	1'04167
10	250	6'25	2'083
15	375	9'375	3'125
20	500	12'5	4'167
25	625	15'625	5'2083
30	750	18'75	6'25
35	875	21'875	7'29167
40	1000	25	8'3

6.- Find out a formula relating weight and dose (mg.).
$25x = y$
7.- Find out a formula showing the relationship between weight and dose (ml.).
$25x : 40 = y$
8.- Apply the previous formulae and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.
$22'25 \cdot 25 = 556'25$ mg $25 \cdot 22'25 : 40 = 13'90625$ ml
9.- Represent each function using a coordinates system axes.

SOLUTION OF THE PATIENT INFORMATION LEAFLET

By Irene Tarancón Martínez

1.-What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?

It means that each ml of syrup, contains 40mg of ibuprofen.

2.- Calculate how many mg. Ramiro must consume every day.

28kg·25mg/kg= 700mg

Ramiro must consume 700 mg of ibuprofen every day.

3.- Calculate how many ml. Ramiro must consume every day.

1ml=40mg

700mg=? ml   700/40=17'5ml

Ramiro must consume 17'5 ml of ibuprofen every day.

4.- How many ml. must Ramiro drink each dose?

17'5 ml/3 times a day= 5'83 ml

Ramiro must drink 5'83 ml in each dose.

5.- Complete the table below:

Weight·(kg)	mg/day	ml/day	ml/dose
5kg	125 mg/day	3'1 ml/day	1'03 ml/dose
10kg	250mg/day	6'3 ml/day	2'1 ml/dose
15kg	375 mg/day	9'4 ml/day	3'1 ml/dose
20kg	500 mg/day	12'5 ml/day	4'2 ml/dose
25kg	625 mg/day	15'6 ml/day	5'2 ml/dose
30kg	750 mg/day	18'8 ml/day	6'3 ml/dose
35kg	875 mg/day	21'9 ml/day	7'3 ml/dose
40kg	1000 mg/day	25 ml/day	8'3 ml/dose

6.- Find out a formula relating weight and dose (mg.).

25x=y

7.- Find out a formula showing the relationship between weight and dose (ml.).

25x/40=y

8.- Apply the previous formula and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.

Weight	Dose in mg	Dose in ml
22'5kg	22'5kg·25mg/kg=562'5mg	562'5mg/40mg/ml= 14'06ml

9.- Represent each function using a coordinates system axes.
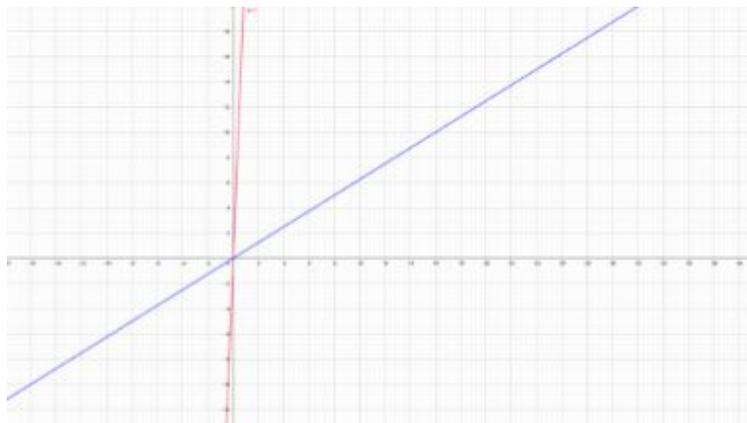
SOLUTION OF THE PATIENT INFORMATION LEAFLET

By Nerea Contreras López

1.-What is the meaning of the expression 40mg/ml? How many mg. of ibuprofen contains each ml of syrup?

40 mg

2.- Calculate how many mg. Ramiro must consume every day.

$25 \cdot 28 = 700$ mg

3.- Calculate how many ml. Ramiro must consume every day.

$700/40 = 17{,}5$ ml

4.- How many ml. must Ramiro drink each dose?

$17{,}5/3 = 5{,}83$ ml

5.- Complete the table below:

kg	mg/day	ml/day	ml/dose
5	125 mg	3,125 ml	1,04 ml
10	250 mg	6,25 ml	2,083 ml
15	375 mg	9,375 ml	3,125 ml
20	500 mg	12,5 ml	4,16 ml
25	625 mg	15,625 ml	5,2083 ml
30	750 mg	18,75 ml	6,25 ml
35	875 mg	21,875 ml	7,29 ml
40	1000 mg	25 ml	8,3 ml

6.- Find out a formula relating weight and dose (mg.).

$x \cdot 25 = y$

7.- Find out a formula showing the relationship between weight and dose (ml.).

$25x/40 = z$

8.- Apply the previous formulae and calculate the dose in mg. and ml. for a child whose weight is 22.25kg.

$25 \cdot 22{,}25 = 556{,}25$ mg

$556{,}25/40 = 13{,}90625$ ml

9.- Represent each function using a coordinates system axes.
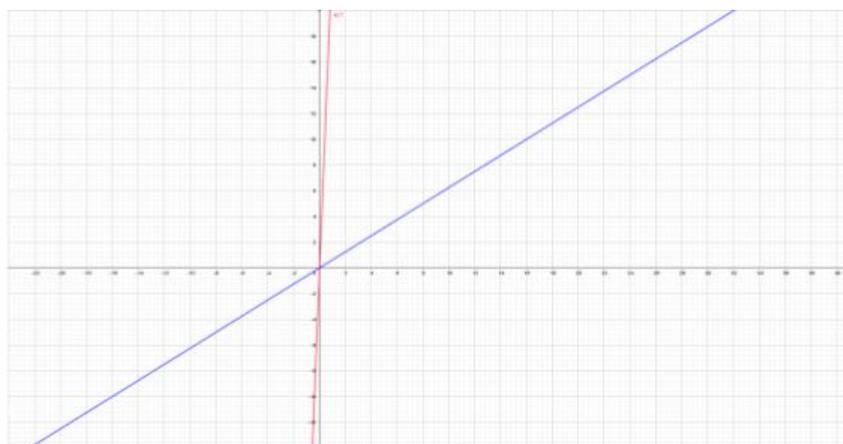
The solution to AQUARIUM DESIGN

A manufacturer is designing an aquarium whose base is a regular hexagon. In geometry, the hexagonal prism is a prism with hexagonal base.

As in all prisms, the volume is found by taking the area of the base, with a side length of l, and multiplying it by the height h, giving the formula:

$$V = A_b \cdot h = \frac{3 \cdot l^2 \sqrt{3}}{2} \cdot h, \text{ where } A_b = \frac{3 \cdot l^2 \sqrt{3}}{2}$$

The aquarium should have:

$V = 24 \text{ ft}^3$

l = lenght of a side of the base

h = height

SOLUTION

1. Write an equation that gives *h* in terms of *l*

A very powerful thing that Algebra can do is to "rearrange" a formula so that another variable is the subject.

$$A_b \cdot h = V \Rightarrow h = \frac{V}{A_b}$$

$$h = \frac{24}{\frac{3 \cdot l^2 \sqrt{3}}{2}} = 24 \cdot \frac{2}{3 \cdot l^2 \sqrt{3}} = \frac{8 \cdot 2}{l^2 \sqrt{3}} \iff h = \frac{16}{l^2 \sqrt{3}}$$

2. Determine the function that expresses the surface of the aquarium

Area of the top of the aquarium: $A = \frac{3 \cdot l^2 \sqrt{3}}{2}$

We do not include the measurement of the top of the aquarium since it is open without glass and does not need to be covered with film.

S of aquarium without the top $= 6l \cdot h + \frac{3 \cdot l^2 \sqrt{3}}{2}$, where $h = \frac{16}{l^2 \sqrt{3}} \Rightarrow$

$$\Rightarrow S = \frac{32\sqrt{3}}{l} + \frac{l^2 \cdot 3\sqrt{3}}{2} = \frac{64\sqrt{3}}{2l} + \frac{l^3 3\sqrt{3}}{2l} \Rightarrow S(l) = \frac{3\sqrt{3}l^3 + 64\sqrt{3}}{2l}$$

3.

$$f(\ell) = \frac{64\sqrt{3} + 3\sqrt{3}\,\ell^3}{2\ell}$$

A = (2.23, 37.77)

4. Find the dimensions l and h that minimize the amount of material used.

So, if l=2.2 ft $\Rightarrow$ h=$\frac{16}{l^2\sqrt{3}}$ $\Rightarrow$ h= $\frac{16}{\sqrt{3}\cdot 2{,}2^2}$

h= **1.9086** ft

5. For l and-h determined at 4, how many liters of water must be put in the aquarium for the water to wet three quarters of the edge of the aquarium?

A cubic measurement is the three-dimensional derivative of a linear measure, so a cubic foot is defined as the volume of a cube with sides 1 ft in length.

In metric terms a cubic foot is a cube with sides 0.3048 meters in length. One cubic foot is the equivalent to approximately 0.02831685 cubic meters, or 28.3169 liters.

$h' = \frac{3}{4} \cdot 1.9086 \approx$ **1.43 ft**

$$1ft^3 \approx 28.31\,l$$

$A_b = 1.43 \cdot \frac{3\cdot 2.2^2\sqrt{3}}{2} \approx$ **12.6 ft²** $\Rightarrow V'=h'\cdot A_b \Rightarrow V' = 12.6\cdot 1.43 \approx 23.94 \;\; (ft^3) \Rightarrow$

$$V' \approx 667.5\,l$$

Student: Florentina Stancu
"Jean Monnet" High School

AQUARIUM DESIGN

A manufacturer is designing an aquarium whose base is a regular hexagon.

The aquarium should have a volume of 24 cubic feet and use the least amount of material possible. Let *l* be the length (in feet) of a side of the base, and let *h* be the height (in feet).

1.Write an equation that gives h in terms of l.

$$S_\Delta(base) = \frac{l^2\sqrt{3}}{4_2}\cdot 6_3 = \frac{3l^2\sqrt{3}}{2}$$

$$V_l(volume) = \frac{3l^2\sqrt{3}}{2}\cdot l$$

$$24 = \frac{3\sqrt{3}}{2}l^2\cdot h$$

$$h = \frac{24\cdot 2}{3l^2\sqrt{3}\sqrt{3}}$$

$$h(l) = \frac{16\sqrt{3}}{3l^2}$$

2.Determine the function that expresses the surface of the aquarium depending on the side of the base.

$$S(lateral) = 6l\cdot h$$

$$S(complete) = S(base) + S(lateral)$$

$$S(complete) = \frac{3l^2\sqrt{3}}{2} + 6lh$$

$$S(complete) = \frac{3\sqrt{3}l^2}{2} + \overset{2}{6l}\cdot \frac{16\sqrt{3}}{3l^2}$$

$$S(complete) = \frac{3\sqrt{3}}{2}l^2 + \frac{32\sqrt{3}}{l}$$

3.Use the GeoGebra program to graphically represent this function.
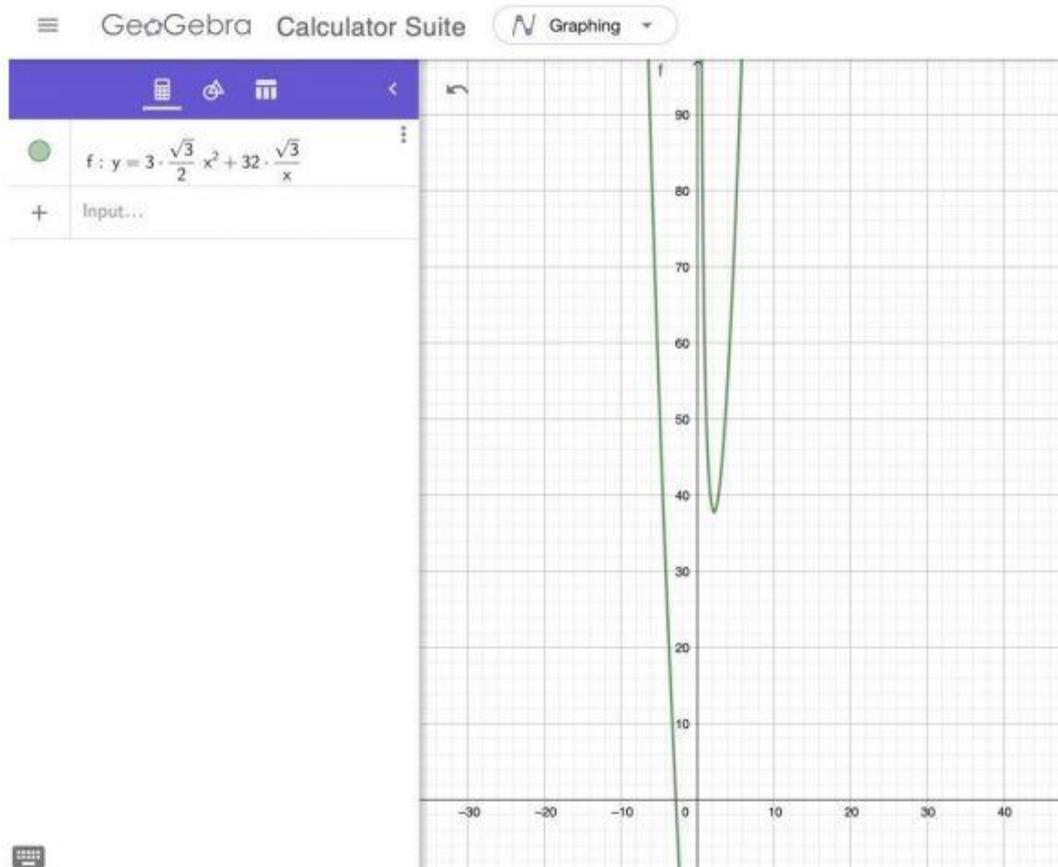
GeoGebra Calculator Suite   Ν Graphing ▾

$f : y = 3 \cdot \dfrac{\sqrt{3}}{2} x^2 + 32 \cdot \dfrac{\sqrt{3}}{x}$

Input...

4. Find the dimensions *s* and *h* that minimize the amount of material used

$S(min) = 38$ (sq. feet)

$l(min) = 2$ (feet)

$h = \dfrac{16\sqrt{3}}{3 l^2} = \dfrac{^4 16\sqrt{3}}{3 \cdot \cancel{4}} = \dfrac{4\sqrt{3}}{3}$ (feet)

$h(min) = \dfrac{4\sqrt{3}}{3}$ (feet)

5. For l and h determined at 4., how many liters of water must be put in the aquarium for the water to wet three quarters of the edge of the aquarium?

$\dfrac{3}{4}$ from $24 = \dfrac{3 \cdot \cancel{24}^{6}}{\cancel{4}} = 18 l$

Answer: 18 litres of water must be put in the aquarium.

Arina Vasiljeva grade 11b, private secondary school Klasika, Riga, Latvia

Solution of the problem, Taxi Fare Calculation

If the taximeter opening fee is 5 TL and the fee to be paid per kilometer is 3,1 TL, the fee to be paid for x km,

$$f(x)=5+3,1.x$$

Since the minimum fee is 13 TL,

$$5+3,1 \cdot x=13 \Rightarrow x=2,58$$

$$f(x) = \begin{cases} 13, & 0 \le x \le 2,58 \\ 5 + 3,1.x, & x > 2,58 \end{cases}$$
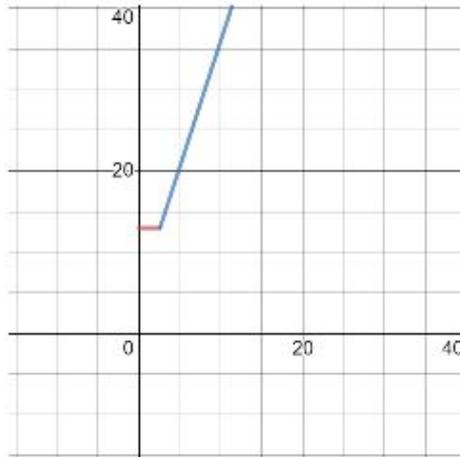
f(43) = 5 + 3,1.43 = 138,3
The taxi fare for a 43 km journey is 138.3 TL.

Students:

Sümeyye B., Hilal K. and Medine K.
Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

Solution for the problem called "Taxi Fare Calculation"

a) Knowing that you pay 3.1 TL/ KM and the taximeter opening fee is 5 TL, results that the mathematical relation is: $f(x) = 3.1x + 5$

Knowing the fact that minimum taxi fee is 13 TL $\Rightarrow 3.1x + 5 = 13$

$$\Rightarrow 3.1x = 8 \Rightarrow x = 2.58$$

The intersection with the AB line, which represents the minimum fee, with the line of the $f(x)$ graph at point C represents the beginning of the line showing the minimum amount accepted by the taxi driver.

$$f: [0; +\infty), f(x) = \begin{cases} 13, & 0 \le x \le 2.58 \\ 3.1x + 5 & > 2.58 \end{cases}$$
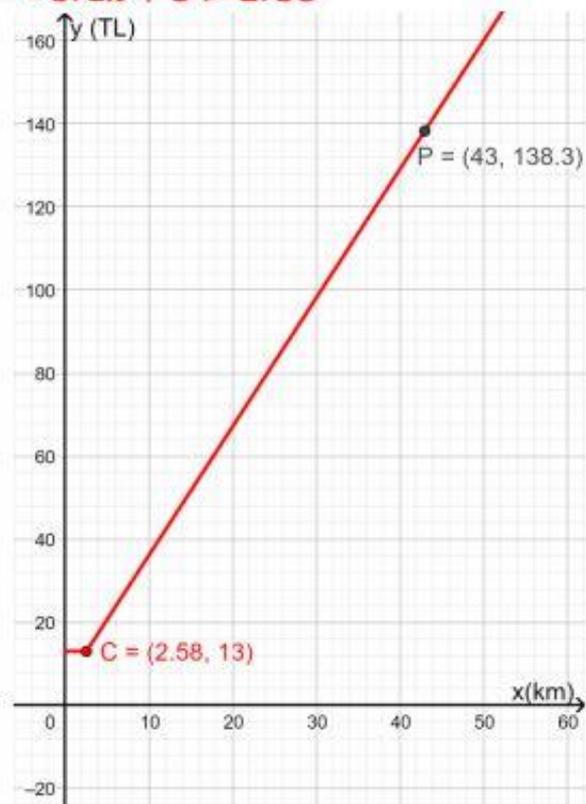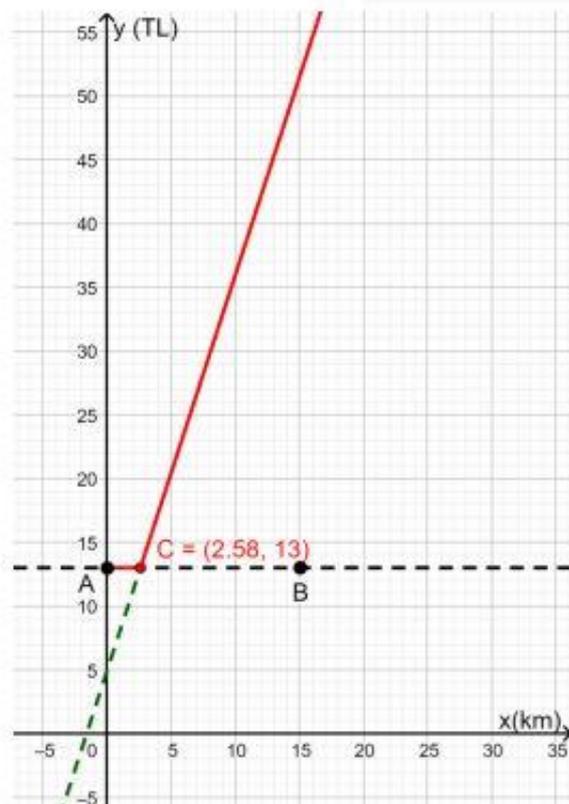
b) At this point we will use the mathematical formula, from above, to find out how much the race for 43 KM cost you.

$$f(x) = 3.1x + 5 \Rightarrow f(43) = 3.1 \cdot 43 + 5 = 138,3 \Rightarrow \text{43 Km cost 138.3 TL}$$

Students: Matei Morarescu and Vlad Niculescu
"Jean Monnet" High School, Bucharest, Romania

"Taxi Fare Calculation" problem

Knowing that you pay 3.1 TL for each km, and the payment for opening a taximeter is 5 TL, we can compose the dependence f(x)

f (x) = 3.1x + 5.

We also know that the minimum trip amount should be 13 TL, from this we can find X.

13 = 3.1x + 5

x = 2.58.

    a) Next, using the formula below, I drew a graph that shows the price change.

$$f(x)= \begin{cases} 13, & \text{if } x \leq 2.58 \\ 3{,}1x+5, & \text{if } x>2{,}58 \end{cases}$$
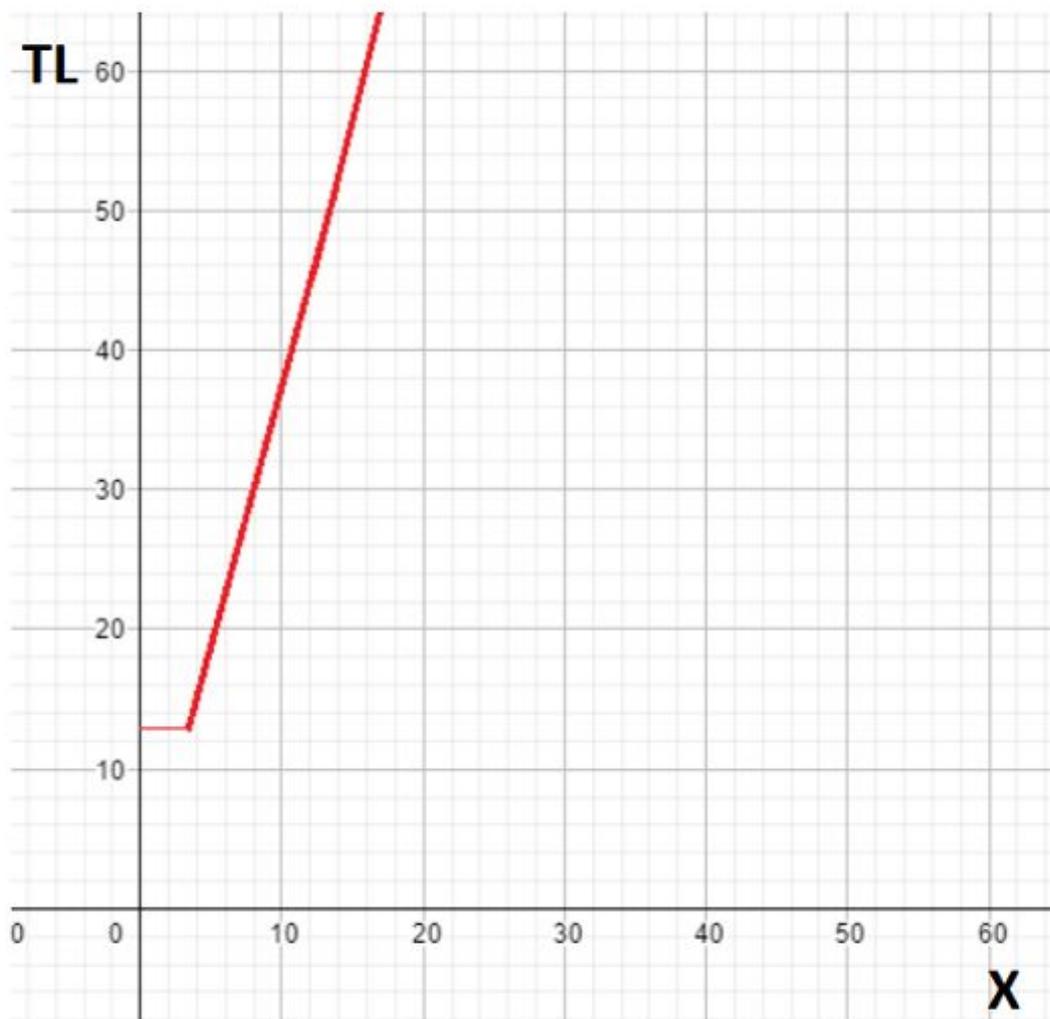
    b) now we will calculate how much a person will pay after driving 43 km by taxi

      f(43) = 3.1*43+5=138,3 TL

      Answer: 138,3 TL person will pay for 43 km

Solution done by **Konstantin Zholudev**
Private secondary school Klasika, Latvia

TAXI FARE CALCULATION

Citizens living in Istanbul pay 3.1 TL per kilometer in taxis.Taximeter fee is determined as 5Tl and the minimal taxi fee is 13TL.

a) Write the mathematical relation that gives how much will be paid when traveling x km by taxi in Istanbul, and draw its graph.

Payment is 3.1TL/Km   =>

Taximeter opening fee is 5TL   => $f(x)=5+3.1x$

The minimum feww is 13TL   =>

$$=> 5+3.1x = 13$$
$$=> x=2.58$$

$f(x)=$ 13, $0 \le x \le 2.58$
         $5+3.1x, x > 2.58$
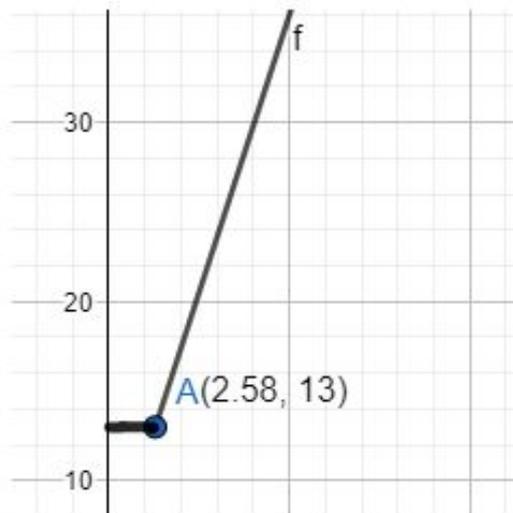
b) Calculate how much a person who takes a taxi from Istanbul Airport pays to go to Taksim Square, which is 43km away.

$f(x)=5+3.1x$
$$=>f(43k)=5+3.1 \cdot 43$$
$$=5+ 133.3$$
$$=138.3 => 43KM \text{ cost } 138.3TL$$

By Ciocoiu Andrei, Romania

WATER SLIDE PROBLEM

By Anastasija Lefkova Municipal high school-„Ljupco Santov`` Kocani, Macedonia

The first patented water slide in the U.S. was the Water-Toboggan Slide, by Herbert Sellner in 1923 in Faribault, MN.

It consisted of a wooden slide which started with a down-ramp and then went out over a lake. People would sit in a wooden sled and slide down the ramp, where they would keep gliding along over the water for up to 100 feet. Before that, however, a very similar contraption was seen in New Zealand during the 1906 International Exhibition, as part of the "Wonderland" attraction.

Water slides have been found in many amusement parks (and probably all water parks) ever since.

Computer models are being used to learn about water flow, friction, and how different masses will behave on a given slide. These models are helping cut down on accidents throughout the industry.

On some rides, like those with steep drops, heavier riders will go faster on average, and that's borne out by computer simulations. The deeply curved sides of some slides let you ride up on the wall as you turn, giving you a thrilling sense that you might fly out but keeping you safe inside.

Innovations in software can now tell us how to space out riders so they never run into each other, or how high to make the wall on a curve to keep everyone in. Water slides are purely recreational, but they occasionally have serious consequences.

The water slide shown in the figure ends at a height of 1.50 m above the pool. If the person starts from rest at point A and lands in the water at point B, which has a horizontal distance L = 2.58 m from the base of the slide, what is the height h of the water slide? (Assume the water slide is frictionless.)

PEi+KEi-W=PEf+KEf

PE=mgh

KE=(1/2)mv2

Vertical displacement after leaving slide: Dy=vot-(1/2)at2

Horizontal displacement after leaving slide Dx=vft

Solution of the Problem, Mobile phone companies

For x minutes of talk in a month, let the tariff charge in the 1st offer be f(x) and the tariff fee in the 2nd offer is g(x).

$$f(x) = \begin{cases} 9, & 0 \leq x \leq 900 \\ 9 + 0{,}3\,(x - 900), & x > 900 \end{cases} \qquad g(x) = \begin{cases} 12, & 0 \leq x \leq 400 \\ 12 + 0{,}2\,(x - 400), & x > 400 \end{cases}$$

Let's draw the graph of both functions with desmos.
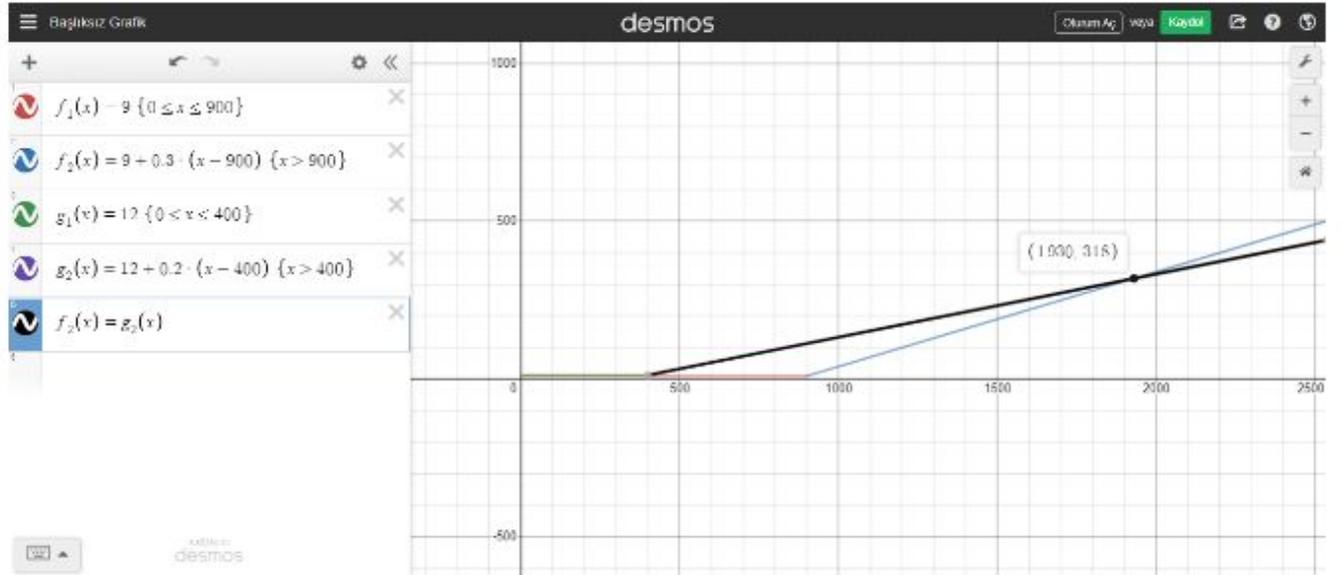
It is paid equally at both offer for 1930 minute call. First tariff for calls less than 1930 minutes, second tariff is more advantageous for calls of more than 1930 minutes.

Students:

Mehmet Emin C., Sarajulhaq Q. and Umut S.
Kızılpınar Belediyesi Çok Programlı Anadolu Lisesi

SOLUTION OF THE MOBILE PHONE COMPANIES

By Mihails Ņečajevs (Latvia)

Functions

X – number of extra minutes

First company

Cost = 9 + 0.30 · x

Second company

Cost = 12 + 0.20 · x

In the graph, we can see that these two companies have the same price with 30 extra minutes.
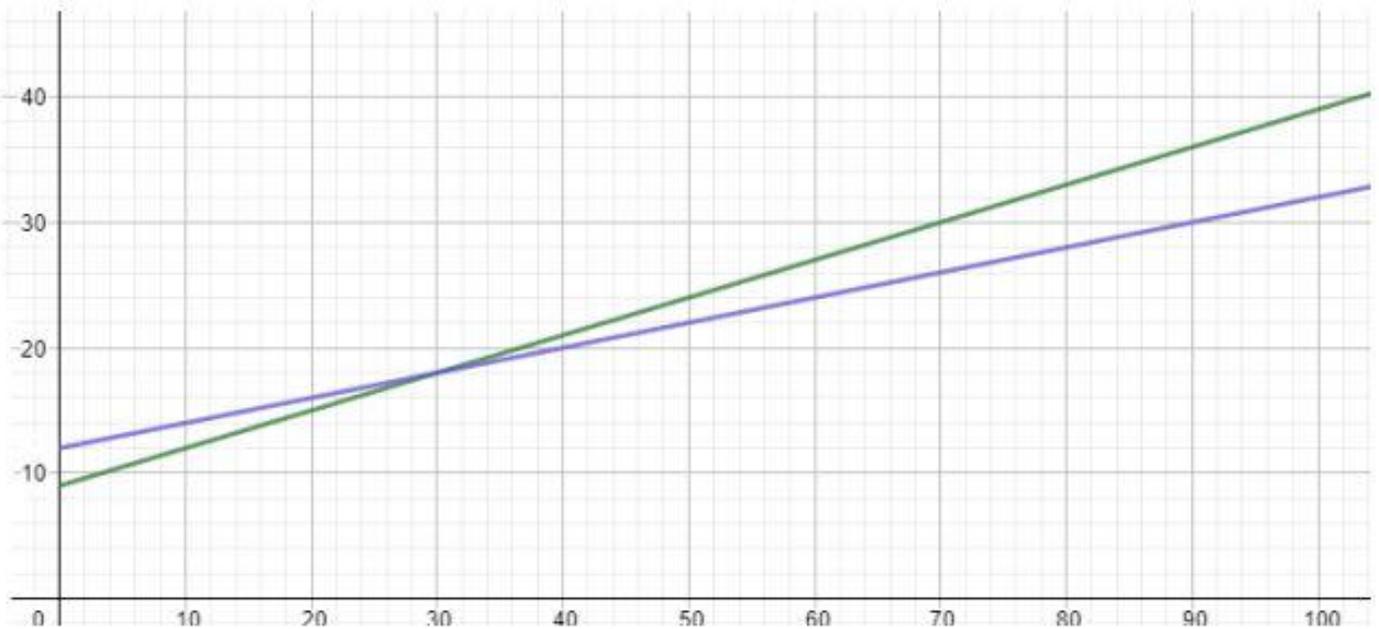
If a company uses the phone for more than 930 minutes (15.5 hours) per month, then the second price will be more profitable for it.

MOBILE PHONE COMPANIES

Two different mobile companies announced their subscriptions offers. The first one is about 9€ per month and it has 900 minutes and every extra minute means 30 cents more to pay. The second one should be paid 12€ per month for 400 included minutes and calling is billed with 0.20 Euro per minute.

The first company:

    9€ per month

    900 minutes   => 100

    minutes = 1€

$$f(x)= \begin{cases} 9, & 0\le x \le 900 \\ 9+0{,}3(x-900), & x>900 \end{cases}$$

The second company:

    12€ per month

    400 minutes   => 33.(3)

$$g(x) = \begin{cases} 12, & 0\le x\le 400 \\ 12+0{,}2(x-400), & x>400 \end{cases}$$
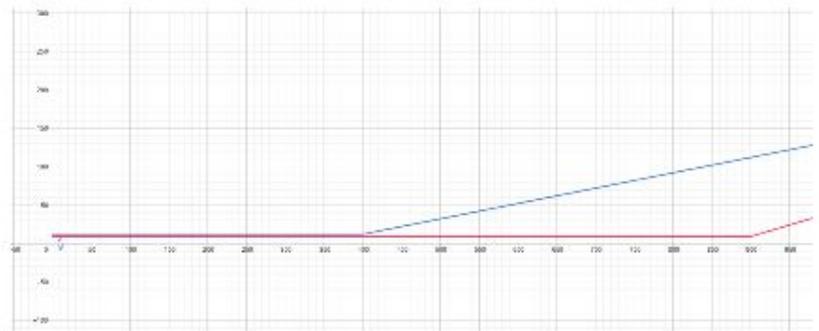
Blue line: G(x), Red line: f(x).
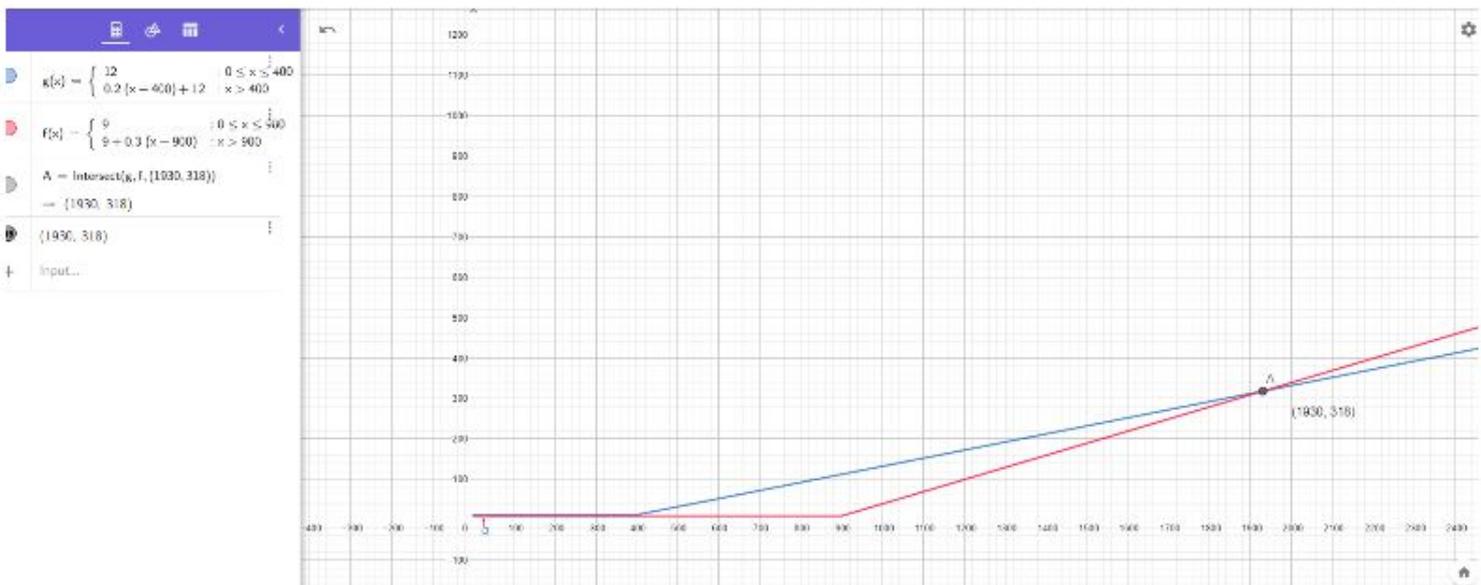
The first company has the most efficient offer **only** if the customer uses less than 1930 minutes. Otherwise, the second company is the most efficient.

By Ciocoiu Andrei, Romania

Solution of the problem Math and Art

a) Prove the 4 triangles are congruent

A_1, B_1,C_1 are midpoints of the sides of the $\triangle ABC$ echilateral =>

A_1B_1 middle line => $A_1B_1 \parallel AB$, $A_1B_1 = \frac{1}{2} \cdot AB$

=> B_1C_1 middle line => $B_1C_1 \parallel AC$, $B_1C_1 = \frac{1}{2} \cdot AC$ => The 4 triangles are congruent (because

A_1C_1 middle line=> $A_1C_1 \parallel BC$, $A_1C_1 = \frac{1}{2} \cdot BC$ $CA_1 = A_1A = AC_1 = C_1B = BB_1 = B_1C$)

b)Calculate the sum of the areas of the blue triangular surface after 5 steps

We observe that the areas are like: The first blue area is $\frac{1}{4} \cdot$ Area $\triangle ABC$, the next one is $\frac{1}{16} \cdot$ Area $\triangle ABC$ and so on. It's creating a geometrical progression.

$Sn = b_1 + b2 + ... + bn = b_1 \cdot \frac{q^n - 1}{q - 1}$

Area $\triangle A_1B_1C_1 = \frac{1}{4} \cdot$ Area $\triangle ABC$ => $Sn = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^n}$

$q = \frac{b_{n+1}}{b_n} = \frac{1}{16} \cdot 4 = \frac{1}{4}$

$Sn = \frac{1}{4} \cdot \frac{\left(\frac{1}{4}\right)^n - 1}{\frac{1}{4} - 1} = \frac{1}{4} \cdot \frac{1 - \frac{1}{4^n}}{\frac{1}{4} - 1} = \frac{1}{4} \cdot \frac{\frac{4^n - 1}{4^n}}{\frac{3}{4}} = \frac{1}{4} \cdot \frac{4^n - 1}{4^n} \cdot \frac{4}{3} = \frac{4^n - 1}{3 \cdot 4^n} = \frac{4^n}{3 \cdot 4^n} - \frac{1}{3 \cdot 4^n} = \frac{1}{3} - \frac{1}{3 \cdot 4^n}$

$S_5 = \frac{1}{3} - \frac{1}{3 \cdot 4^5} = \frac{1}{3} - \frac{1}{3 \cdot 1024} = \frac{1023}{3 \cdot 1024} = \frac{341}{1024}$

Calculate the sum of the areas of the blue triangular surface after n steps

$Sn = \frac{1}{3} - \frac{1}{3 \cdot 4^n}$

Aproximate this value for a very large number n

If n is a very larg number , then the fraction $-\frac{1}{3 \cdot 4^n}$ is going to be close to 0 , which means that

$Sn = \frac{1}{3} - \frac{1}{3 \cdot 4^n}$ is going to be approximately $\frac{1}{3}$.

Feleagă Maria-Alessia
Teodorescu Daria-Alexia
„Jean Monnet" Highschool

SOLUTION OF THE PROBLEM MATH AND ART

By Eugene Angerman (Latvia)

1.Problem
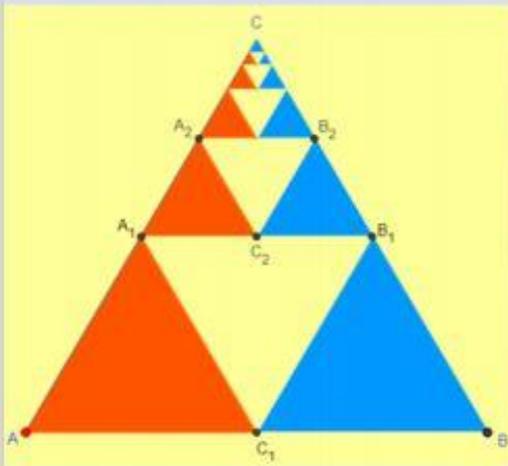
Consider an equilateral triangular surface ABC with unit area. Construct A_1, B_1, C_1 the midpoints of the sides of the triangle ABC, as in the drawing above.

a. Prove that the 4 triangles formed are congruent.

b. Repeat the construction for the triangle CA_1B_1, CA_2B_2 and so on.

Calculate the sum of the areas of the blue triangular surfaces after 5 steps. Calculate the sum of the areas of the blue triangular surfaces after n steps. Approximate this value for a very large number n.

a. Prove that the 4 triangles formed are congruent.
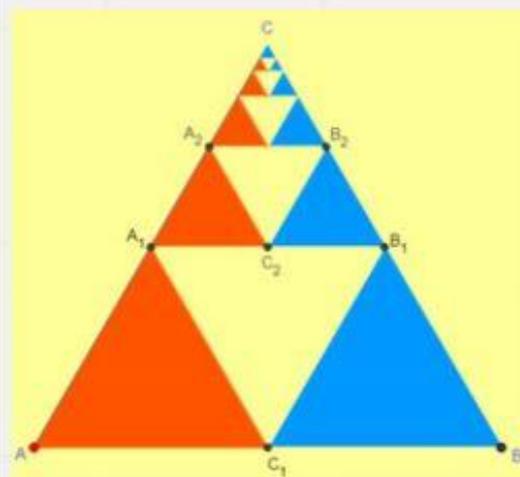
A1 is the midpoint for side AB

Since the AA1C1 triangle is equilateral, all its sides are 2 times smaller than those of ABC

According to the same logic,
A1CB1, C1B1B, A1B1C1 are equilateral.

All equilateral triangles are similar to each other - a sign of similarity about proportional sides.
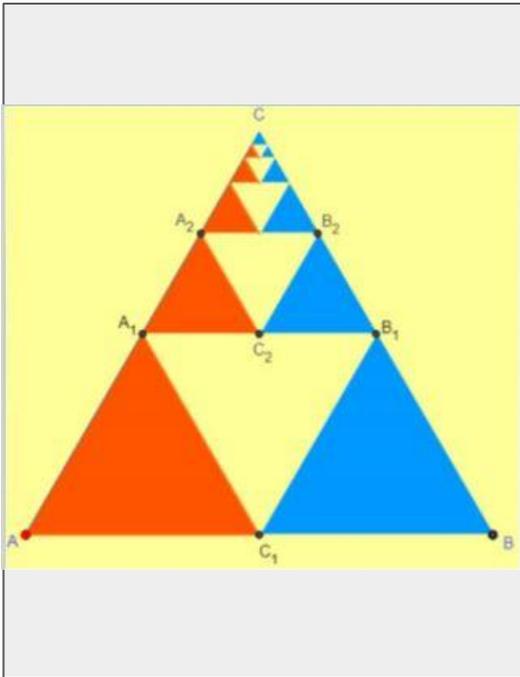
AB / C1B = 2. This means that the coefficient of similarity k between triangles ABC and C1B1B is two.

If the coefficient of similarity between triangles ABC and C1B1B is two, then the area of triangle ABC is four times the area of triangle C1B1B, since CABC / S C1B1B = k ^ 2 = 2 ^ 2 = 4.

S= C1B1B = 1/4.   S= C1B1B / S C2B2B1 = 4

S= C2B2B1 / S C3B3B2 = 4   and so on

The areas of blue triangles are equal to 1/4, 1/16, 1/64 ... This sequence of numbers is a geometric progression, where b1 = 1/4, q = 1/4.
The sum of the areas of n blue triangles can be found as the sum of the n terms of the specified geometric progression.

Calculating the sum of the progression for n = 5.

$$S_5 = \frac{1}{4} \cdot \frac{1-\left(\frac{1}{4}\right)^5}{1-\frac{1}{4}} = \frac{1-\frac{1}{1024}}{4-1} = \frac{1-\frac{1}{1024}}{3} \cdot \frac{1024}{1024} =$$

$$= \frac{1024-1}{3\cdot 1024} = \frac{1023}{3\cdot 1024} = \frac{341}{1024} = 0,333007812$$

Calculating the sum of the progression for n = ∞.

$$b_n = b_1 \cdot q^{n-1} = \frac{1}{4} \cdot \left(\frac{1}{4}\right)^4 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$$

$$S_n = \frac{b_1}{1-q}$$

$$S_n = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \approx 0,3$$

Spiral of equilateral triangles

Solution:

a) $B(4;1)$; $AB = 3 \Rightarrow h_C = \dfrac{l\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{2} \Rightarrow$

$$C\left(2.5; 1 + \frac{3\sqrt{3}}{2}\right)$$

$B(4;1) \in G_f \Rightarrow f(x) = ax + b \Rightarrow y = ax + b$

$$4a + b = 1$$

$$C\left(2.5; 1 + \frac{3\sqrt{3}}{2}\right) \in G_f \Rightarrow f(x) = ax + b$$

$$\Rightarrow 2.5a + b = \frac{2 + 3\sqrt{3}}{2} \mid \cdot 2$$

$$\Rightarrow$$

$$5a + 2b = 2 + 3\sqrt{3}$$

$$4a + b = 1 \mid \cdot(-2)$$

$$5a + 2b = 2 + 3\sqrt{3}$$

$$\Rightarrow -8a - 2b = -2$$

$$5a + 2b = 2 + 3\sqrt{3}$$

$$-3a = 3\sqrt{3} \Rightarrow a = -\sqrt{3}$$

$$\Rightarrow 4a + b = 1 \Rightarrow -4\sqrt{3} + b = 1 \Rightarrow b = 1 + 4\sqrt{3}$$

$$\Rightarrow f(x) = ax + b \Rightarrow f(x) = -\sqrt{3}x + 1 + 4\sqrt{3} \Rightarrow y = -\sqrt{3}x + 1 + 4\sqrt{3} \ (BC)$$

b) $A(1;1)$; $C\left(2.5; 1 + \frac{3\sqrt{3}}{2}\right)$

$$A(1;1) \in G_g \Rightarrow g(x) = ax + b \Rightarrow a + b = 1$$

$$C\left(2.5; 1 + \frac{3\sqrt{3}}{2}\right) \in G_g \Rightarrow g(x) = ax + b \Rightarrow \frac{5}{2}a + b = \frac{2+3\sqrt{3}}{2} \mid \cdot 2$$

$$a + b = 1 \mid \cdot(-2)$$

$$5a + 2b = 2 + 3\sqrt{3}$$

$$\Rightarrow -2a - 2b = -2$$

$$3a + 2b = 3 + 3\sqrt{3}$$

$$\overline{\qquad\qquad\qquad\qquad}$$

$$a = 3\sqrt{3}$$

$$\Rightarrow 3\sqrt{3} + b = 1 \Rightarrow b = 1 - 3\sqrt{3}$$

$$\Rightarrow g(x) = ax + b \Rightarrow g(x) = 3\sqrt{3}x + 1 - 3\sqrt{3} \Rightarrow y = 3\sqrt{3}x + 1 - 3\sqrt{3} \ (AC)$$

c) $C(x;y)$: $x_C = \dfrac{x_A + x_B}{2} = \dfrac{4+1}{2} = \dfrac{5}{2} = 2.5$

$$y_C = y_A + \frac{3\sqrt{3}}{2} = 1 + \frac{3\sqrt{3}}{2} = \frac{2 + 3\sqrt{3}}{2} \Rightarrow C\left(2.5; \frac{2 + 3\sqrt{3}}{2}\right)$$

d) $A(1;1)$; $D(3.25; 2.3)$

$$A(1;1) \in G_h \Rightarrow h(x) = ax + b \Rightarrow a + b = 1$$

$$D(3.25; 2.3) \in G_h \Rightarrow h(x) = ax + b \Rightarrow \frac{325}{100}a + b = \frac{23}{10} | \cdot 20 \Rightarrow 65a + 20b = 46$$

$$a + b = 1 | \cdot (-20)$$
$$65a + 20b = 46$$
$$\Rightarrow -20a - 20b = -20$$
$$65a + 20b = 46$$

$$\overline{}$$

$$45a = 26 \Rightarrow a = \frac{26}{45}$$

$$\Rightarrow a + b = 1 \Rightarrow b = 1 - \frac{26}{45} = \frac{19}{45}$$

$$\Rightarrow h(x) = ax + b \Rightarrow h(x) = \frac{26}{45}x + \frac{19}{45} \Rightarrow \mathbf{y = \frac{26}{45}x + \frac{19}{45}} \quad (\mathbf{AD})$$

$$\Delta ABC - equilateral, \ AB = 3u \Rightarrow AD \perp BC \Rightarrow h = AD = \frac{l\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}u$$

e) $\Delta ABC - equilateral$
$$AB = AC = BC = 3u$$
$$\Rightarrow A_\Delta = \frac{l^2\sqrt{3}}{4} \Rightarrow \mathbf{A_\Delta = \frac{9\sqrt{3}}{4}u^2}$$

f) $AH \perp AB \,;\, AH \parallel OY$
$$\Rightarrow x = 1$$

g) $\Delta ADE - equilateral \Rightarrow AF \perp DE$
$$\Rightarrow \Delta AFG - equilateral \Rightarrow AH \perp FG$$
$$\Rightarrow HAF = 30°$$
$$\Rightarrow BAH = BAD + DAF + FAH = 30° + 30° + 30° = 90°$$
$$\Rightarrow AH \perp AB$$
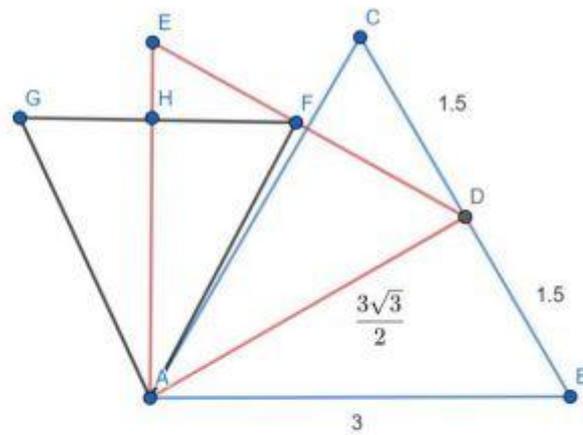
h) $\Delta ADE - equilateral$

$$l = AD = \frac{3\sqrt{3}}{2}u$$

$$\Rightarrow A_\Delta = \frac{l^2\sqrt{3}}{4} = \frac{\left(\frac{3\sqrt{3}}{2}\right)^2\sqrt{3}}{4} = \frac{\frac{27}{4}\sqrt{3}}{4} \Rightarrow \mathbf{A_{\Delta ADE} = \frac{27\sqrt{3}}{16}}$$

Alessia Beja, Diana Ilie, Andrei Ciocoiu
Jean Monnet High School

Spiral of equilateral triangles

In ΔABC, right triangles ABD and ACE
erected outwardly satisfy ∠ABD=∠ACE=30∘
and ∠BAD=∠CAE=90∘; isosceles BCF
(erected inwardly) has base angles of 30∘
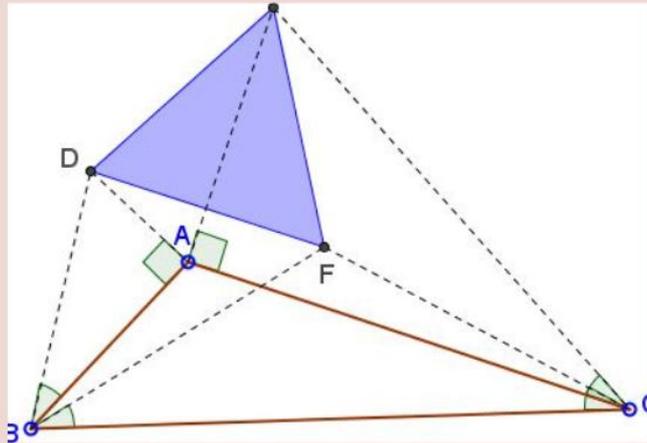
Prove that ΔDEF is equilateral.
Note that an equal triangles is obtained when the words
"inwardly" and "outwardly" interchanged:
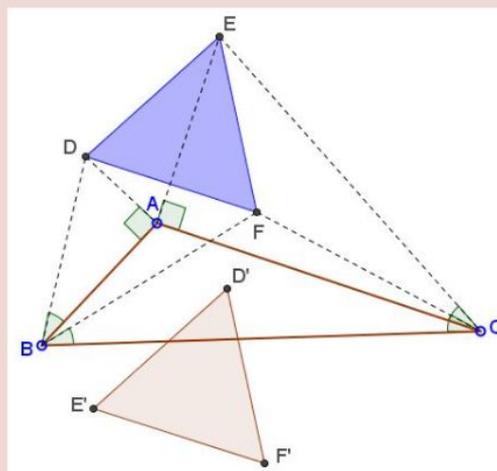
Solution 1.

Complex numbers afford a straightforward solution. Let's place the origin at B and assume that A and C correspond to complex numbers α and 2β. D is obtained from A by a spiral similarity $\gamma=1+i3-\sqrt{3}$ around B, and in the same way F is obtained from α. D is the image of A under the (conjugate) spiral similarity $\overline{\gamma}=1-i3-\sqrt{3}$ around C: $D=\gamma\alpha$, $F=\gamma\beta$, $E=2\beta+\overline{\gamma}(\alpha-2\beta)$. A triangle with vertices D, E, F is equilateral if either $E+jD+j2F=0$ or $E+j2D+jF=0$, where j is a rotation through 120○: $j=-12+i3-\sqrt{2}$, $\overline{j}=j2=-12-i3-\sqrt{2}$. Easy calculations show that $\gamma\cdot j=(1+i3-\sqrt{3})(-12+i3-\sqrt{2})=-1+i3-\sqrt{3}$ and $\gamma\cdot\overline{j}=(1+i3-\sqrt{3})(-12-i3-\sqrt{2})=-i23-\sqrt{3}$. From here, $E+jF+j2D=[2\beta+\overline{\gamma}(\alpha-2\beta)]+\gamma\alpha j+\gamma\beta\overline{j}=2\beta+(1-i3-\sqrt{3})(\alpha-2\beta)+(-1+i3-\sqrt{3})\alpha+(-i23-\sqrt{3})\beta=\alpha(1-i3-\sqrt{3}-1+i3-\sqrt{3})+\beta(2-2+i23-\sqrt{3}-i23-\sqrt{3})=0$.

solution 2.

Let M be the midpoint of BC. We have ∠ABD=∠FBM=30○ and ∠BAD=∠BMF=90○. Also, ΔBAD~ΔBMF, so BDBF=BABM. And, since ∠ABD=∠FBM, ∠FBD=∠MBA. From the proportion and angle equality we get ΔBDF~ΔBAM, so DF=BF·AMBM. Similarly, EF=CF·AMCM. But BF=CF and BM=CM so it follows that DF=EF.

By Jana Ampova

Solution of the problem, Pressure and Volume of Gases

"PRESSURE AND VOLUME OF GASES"

A) The graph tells us that pressure and volume are inversely proportional quantities.

Increasing the pressure decreases the volume.

B) Pressure (P_i)

Volume (V_i)

$$\boxed{P_1 V_1 = P_2 V_2}$$

c) $P_1 = 0,8\ atm$ $P_2 = 1,2\ atm$

$V_1 = 70L$ $V_2 = x$

$$P_1 \cdot V_1 = P_2 V_2 \rightarrow V_2 = \frac{P_1 V_1}{P_2} = \frac{0,8\ atm \cdot 70L}{1,2\ atm} = \boxed{46,7L}$$

d) $V_1 = 5L$ $V_2 = 3L$

$P_1 = x$ $P_2 = 1,25\ atm$

$$P_1 \cdot V_1 = P_2 V_2 \rightarrow P_1 = \frac{P_2 V_2}{V_1} = \frac{1,25\ atm \cdot 3L}{5L} = \boxed{0,75\ atm}$$

Solution of the problem, Pressure and Temperature

	Comunidad de Madrid	Apellidos	
	Consejería de Educación	Nombre	
I.E.S. PALOMERAS - VALLECAS		DNI	
		Asignatura	.

Fecha	Curso	Grupo	Calificación

"PRESSURE AND TEMPERATURE"

A) The grahs tells us that pressure and temperature are directy proportional quantities.
As the temperature increases, the pressure increases.

B) Pressure (P_i)
    Temperature (T_i)

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

C) $P_1 = 2\,atm$ $P_2 = 6\,atm$
    $T_1 = 27 + 273 = 303\,k$ $T_2 = X$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$ $$T_2 = \frac{P_2 T_1}{P_1} = \frac{6\,atm \cdot 303\,K}{2\,atm} = 909\,K$$

D) $P_1 = 2,5\,atm$ $P_2 = X$
    $T_1 = 20°C + 273 = 293\,k$ $T_2 = 55°C + 273 = 328\,K$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$ $$P_2 = \frac{P_1 \cdot T_2}{T_1} = \frac{2,5\,atm \cdot 328\,K}{293\,K} = 2,80\,atm$$

Solution of the problem, Echo

	Comunidad de Madrid	Apellidos	
★★★★ ★★★	Consejería de Educación	Nombre	
I.E.S. PALOMERAS - VALLECAS		DNI	
		Asignatura	

Fecha	Curso	Grupo	Calificación

THE ECHO

The echo

BOOM

255m   255m
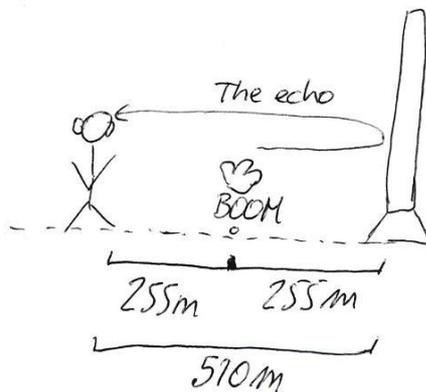
510m

A) DIRECT SOUND

$X = 255m$

$U = 340 \frac{m}{s}$

$U = \frac{X}{t} \rightarrow t = \frac{X}{U} = \frac{255m}{340\frac{m}{s}} = \boxed{0.75s}$

B) THE ECHO

$X' = 255 + 510m = 765m$

$U' = 340\frac{m}{s}$

$U' = \frac{X'}{t'} \rightarrow t' = \frac{X'}{U'} = \frac{765m}{340\frac{m}{s}} = \boxed{2.25s}$

Solution of the problem, Elasticity

	Comunidad de Madrid	Apellidos	
★★★★ ★★★	Consejería de Educación	Nombre	
I.E.S. PALOMERAS - VALLECAS		DNI	
		Asignatura	.

Fecha	Curso	Grupo	Calificación

"ELASTICITY"

Hooke's laws.

$$F = K(\ell_F - \ell_0) \longrightarrow K = \frac{F}{\ell_F - \ell_0}$$

Date: 6N $\qquad \ell_{F_1} = 5 cm \qquad F_2 = 11N \qquad \ell_{F_2} = 8 cm.$

$$\left. \begin{array}{l} F_1 = K(\ell_{F_1} - \ell_0) \\ F_2 = K(\ell_{F_2} - \ell_0) \end{array} \right\} \quad K = \frac{F_1}{\ell_{F_1} - \ell_0} \qquad K = \frac{F_2}{\ell_{F_2} - \ell_0}$$

$$\boxed{\frac{F_1}{\ell_{F_1} - \ell_0} = \frac{F_2}{\ell_{F_2} - \ell_0}}$$

$$\frac{6}{5 - \ell_0} = \frac{11}{8 - \ell_0} \longrightarrow 6 \cdot (8 - \ell_0) = 11(5 - \ell_0)$$

$$48 - 6\ell_0 = 55 - 11\ell_0$$

$$11\ell_0 - 6\ell_0 = 7$$

$$5\ell_0 = 7$$

$$\ell_0 = \frac{7}{5} = 1,4 \ cm.$$

$$K = \frac{F_1}{\ell_{F_1} - \ell_0} = \frac{6}{5 - 1,4} = \frac{6}{3,6} = 1,67 \frac{N}{cm} \left.\right\}$$

$$K = \frac{F_2}{\ell_{F_2} - \ell_0} = \frac{11}{8 - 1,4} = \frac{11}{6,6} = 1,67 \frac{N}{cm} \left.\right)$$

$$\boxed{K = 1,67 \frac{N}{cm}}$$

Solution of the problem, Fuel Consumption

FOR MISS. SARAH

3500 * (6.6/100) = 231

Gasoline per lt.   137,5+231=368.5 lt in a year

25** * (5.5/100)=137,5

3685 * 7,07 = 2605 TL

3500*(4.7/100)=164.5

Diesel per lt.   164,5+100=264.5lt in a year

2500*(4.0/100)= 100

264.5 * 6,45= 1061 TL

461900-430500=**31400** → (Cost of difference between a diesel powered vehicle and a gasoline powered one.

2695- 1061= 1544 TL (Cost of difference between diesel fuel and gasoline fuel in a year.

31400/ 1544 = **20 years**

This means Miss Sarah needs 20 years to recover from the expenses of a diesel powered vehicle. We recommend Miss. Sarah to buy a gasoline powered vehicle or a bicycle as she doesn't travel a lot during a year.

-Umut S.

-Ranya N.

-Saraj Q

- Sude B.

FOR MR. ADAM

2000 * (6.6/100) = 132

Gasoline per lt.   1144+132=1276 lt in a year

20800 * (5.5/100)=1144   1276 * 7,07 = 9021 TL

2000*(4.7/100)=94

Diesel per lt.   832+94=926 lt in a year

2800*(4.0/100)= 832   926 * 6,45=5972 TL

9021- 5972= 3049 TL (Cost of difference between diesel fuel and gasoline fuel in a year.

31400 → (Cost of difference between a diesel powered vehicle and a gasoline powered one.

31400/3049 = 10 years

This means Mr. Adam needs 10 years to recover from the expenses of a diesel powered vehicle.We recommend Mr. Adam to buy a diesel powered vehicle as he travels a lot during a year.

-Umut S.

-Ranya N.

-Saraj Q

- Sude B.